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Dynamic Direct Long-Short Investing

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AQR

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KEY FINDINGS

- Without the help of additional capital contributions or gifting of appreciated stocks, direct indexing strategies typically realize only a limited amount of net capital losses. In our historical simulations, they reach a maximum average cumulative level of about 30% of the initially invested capital. This maximum level is reached after approximately eight years since inception.
- Tax-aware long-short factor strategies can significantly outperform direct indexing strategies from both a pre-tax and tax perspective. We find that, if implemented with a sufficiently high level of leverage and tracking error, these strategies can realize a cumulative net capital loss of 100% of the invested capital within a few years and, at the same time, substantially outperform the benchmark index before tax, net of implementation costs.
- Leverage and tracking error of tax-aware long-short strategies can be modulated over time without realizing net capital gains. This gives rise to an approach that we call *dynamic direct long-short investing*. Under this approach, an investor's exposure to the strategy's alpha model can be scaled up or down by varying the levels of leverage and tracking error.

ABSTRACT

On average, net losses realized by direct indexing loss-harvesting strategies taper off within the first few years after their inception. In our historical simulations, they reach a maximum average cumulative level of about 30% of the initially invested capital. In addition, direct indexing strategies exhibit a high dispersion of net loss outcomes. Long-short strategies motivated by factor investing can significantly outperform direct indexing strategies from both a pre-tax and tax perspective. We model two types of long-short factor-based strategies: relaxed-constraint and composite long-short. Both types of strategies, if implemented with a sufficiently high level of leverage and tracking error, can realize a cumulative net capital loss of 100% of the invested capital within a few years and, at the same time, substantially outperform the benchmark index before tax, net of implementation costs. We further show that leverage and tracking error of long-short strategies can be managed dynamically in a highly tax-efficient manner. For example, an investor who becomes less optimistic about the prospects of factor investing can reduce the leverage and tracking error substantially, albeit not all the way to zero, without recognizing net capital gains. We find that a full liquidation of the long and short extensions results in realization of most of the previously deferred gains.

Direct indexing provides investors with a highly customizable equity market exposure. In addition, it can be a valuable tax management tool, especially beneficial for those investors whose other investments tend to realize short-term capital gains.[1] However, without the help of additional capital contributions or gifting of appreciated stocks, direct indexing strategies typically realize only a limited amount of net capital losses. For example, Israelov and Lu (2022) show that realized net losses of a long-only tax-loss-harvesting strategy decline rather quickly over time. According to their estimates, the realized net loss is expected to be about 13% in the first year since inception, then decline to high single digits in the second year, and after that gradually decay to mid and low single digits in years three and beyond.[2]

We begin our analysis by confirming the Israelov and Lu result that, on average, net losses offered by direct indexing taper off after the first few years since inception. In addition, we show that direct indexing strategies have a very substantial dispersion of net loss outcomes, including in year one. Sosner et al. (2022), among others, attribute this dispersion in tax outcomes to their sensitivity to market environment variables—market return and stock-level volatility.

The low average level and high uncertainty of the net losses realized by direct indexing present a challenge for investors looking to utilize the strategy's net losses in their tax planning, for example, in dealing with low-basis concentrated positions.[3] In this article, we show that tax-aware long-short factor strategies are much better suited for such situations.

Importantly, Sialm and Sosner (2018) show that long-short factor strategies are significantly better than long-only factor strategies at balancing tax benefits with pre-tax alpha. As a result, in contrast to a more traditional long-only investing, long-short tax-aware factor strategies allow investors to enjoy not only substantial tax benefits but also a significant and highly diversifying pre-tax alpha derived from factor investing.

Prior literature has demonstrated that adding long and short extensions[4] to an existing portfolio may help modify the economic exposures of a low-basis long-only portfolio (Sosner and Krasner 2021) or reduce the tracking error of a concentrated low-basis portfolio (Goldberg et al. 2022b) tax efficiently. Here we look at tax-aware long-short strategies from a different angle: We explore their propensity for realizing *cumulative net capital loss* (hereafter, CNCL) and compare it with that of traditional direct indexing. We also show to what extent the leverage and tracking error of tax-aware long-short strategies can be reduced without realizing net capital gains, thus preserving previously realized CNCL. The ability to modulate leverage and tracking error tax efficiently based on the investor's perceived attractiveness of the strategy's alpha creates opportunities for *dynamic direct long-short investing*.

[1] See, for example, Chaudhuri et al. (2020), Khang et al. (2021), Anderson and Kourtidis (2022), Shalett et al. (2022), and Sosner et al. (2022).

[2] Note that strategic long-term gain realization described in Stein et al. (2008) and Goldberg et al. (2022a) can increase realized short-term capital losses at the expense of realizing larger long-term capital gains. However, these techniques only increase a character benefit, which is derived from realizing long-term gains and short-term losses in matching amounts (see Sosner et al. 2019), not a net loss realization.

[3] Miller (2002), Boyle et al. (2004), and Sosner (2022) discuss the risks associated with holding concentrated stock. Quisenberry and Welch (2005) show how a loss-harvesting strategy can be combined with a variable prepaid forward to increase the tax efficiency of diversification of low-basis concentrated stock. A loss-harvesting strategy can help offset a significant capital gain recognized upon maturity of the variable prepaid forward.

[4] By "extensions" we mean entering into short positions and contemporaneously purchasing additional long positions on margin.

We begin by modeling three types of tax-aware strategies. The first one is a direct indexing strategy. The other two represent alternative approaches to long-short factor investing that differ in the way they implement index beta. Strategies of the first type express index beta using individual stocks. For example, a 150/50 strategy would hold 150% of its net asset value (NAV) in long stocks and 50% of its NAV in short stocks. Following Sialm and Sosner (2018), we refer to these strategies as "relaxed-constraint," abbreviated as RC. In contrast, strategies of the second type implement index beta by holding an index fund, such as a passive mutual fund or an ETF, and only use individual stocks to put on long and short extensions. For example, a 150/50 strategy would hold 100% of its NAV in an index fund, 50% of its NAV in long stocks, and 50% of its NAV in short stocks. Following Liberman et al. (2020) we call these strategies "composite long-short," CLS for brevity.

We first show the results for a direct indexing strategy. We then introduce tax-aware RC and CLS strategies with different levels of leverage and tracking error. In our historical simulations, both types of long-short strategies significantly outperform the direct indexing strategy from both a pre-tax and tax perspective. Directionally, this result is not surprising in light of prior studies on the tax benefits of long-short investing (e.g., Sosner et al. 2019). The new result in this article is quantifying the differences in CNCL between direct indexing and long-short beta-one strategies. We find that the CNCL realization advantage offered by the long-short approach is quite significant. It is particularly remarkable given that the strategy is geared toward delivering pre-tax factor alpha and is simply rebalanced with an eye toward tax-efficiency.

Furthermore, to our knowledge, our side-by-side comparison of the alternative long-short strategy implementations, RC and CLS, is novel and has not been previously explored in the literature. We find that both types of strategies deliver similarly substantial levels of pre-tax alpha. From a CNCL perspective, the average level of CNCL under the two implementations is similar, although the dispersion of outcomes is somewhat larger under CLS than under RC. As a result, we conclude that RC and CLS are comparable approaches to long-short investing, and that, depending on investor's legacy positions and portfolio composition preferences, either of these two approaches can be deployed with similar effectiveness.

Finally, long-short strategies, by construction, expose investors to active risk of the alpha model (measured by tracking error), costs of financing the long and short extensions, and increased t-costs associated with the active management. Investors might want to modulate these risks and costs in accordance with their expectations about the strategy's pre-tax alpha. For example, an investor's confidence in the specific alpha signals used by the strategy may increase or decrease over time. Alternatively, an investor might find the alpha signals of the strategy to be either more or less diversifying with respect to other exposures in the overall investment portfolio going forward. These considerations yield support to dynamic management of tracking error and the associated leverage of tax-aware long-short strategies. Since for taxable investors the pre-tax benefits of adjusting portfolio exposures can be more than offset by the tax costs of rebalancing the portfolio, the key question is: Can dynamic long-short factor investing be implemented tax efficiently?

Sosner and Krasner (2021) show that leverage and tracking error of a low-basis direct indexing portfolio can be increased instantly without incurring a tax burden. We replicated these findings for increasing the leverage and tracking error of low-basis RC and CLS portfolios. For the sake of brevity, we do not report these results in this article because they hold trivially: Leverage and tracking error generally can be increased in just one portfolio rebalance without any adverse tax consequences. Pre-tax alpha of the levered-up strategy increases going forward because of its higher exposure to the alpha model. Tax benefits also increase because new positions with fresh cost basis are created on both long and short sides of the portfolio thereby increasing future opportunities for managing the realizations of capital gains and losses.

Going in the opposite direction, that is, tax efficiently reducing leverage and tracking error of a low-basis long-short portfolio is harder. Portfolio positions must be liquidated, and, given that many of the positions carry built-in gains, such liquidation might lead to substantial gain recognition. Nonetheless, Goldberg et al. (2022b) show that tax-efficient reduction in leverage and tracking error for a low-basis long-short portfolio can still be achieved. However, in contrast to levering up the portfolio, the process of levering down without creating an unnecessary tax burden can take multiple years. For our tax-aware RC and CLS strategies, we find that the de-risking process might indeed take several years but the strategy's portfolio cannot be de-risked all the way to a long-only passive portfolio without recognizing substantial capital gains. However, consistent with Goldberg et al. (2022b), our experiments achieve a meaningful reduction in leverage and tracking error without triggering net capital gains. After de-risking, the level of pre-tax alpha and tax benefits is reduced, which is expected given a lower tracking error and leverage and a large number of appreciated positions remaining in the portfolio. Most importantly, our results indicate that investors in tax-aware long-short factor strategies can meaningfully modulate, both up and down, active risk and expected return of their portfolios while preserving their tax efficiency.

In sum, our analysis shows that tax-aware long-short factor strategies, using either RC or CLS implementation, can significantly enhance investment returns through generating pre-tax factor alpha, are substantially more effective than direct indexing at realizing CNCL, and allow investors to manage leverage and tracking error dynamically in accordance with their time-varying preferences toward alpha-model exposure.

SIMULATION METHODOLOGY

Tax-Aware Strategy Description

This section outlines the conceptual objectives of our strategies. Appendix A provides further technical details of the strategy simulations. All our strategies are constructed over the Russell 1000 index universe and track the Russell 1000 index performance. All the strategies are rebalanced at a monthly frequency. Our direct indexing portfolio construction methodology follows Sosner et al. (2022). In each rebalance, the strategy seeks to defer tax gains and realize tax losses while maintaining a tracking error (TE) of 1% with respect to the benchmark index.

Our RC portfolio construction methodology follows Sosner et al. (2019). In each monthly rebalance, RC strategies maximize expected active pre-tax returns, defer gains, and realize losses subject to TE and leverage constraints. We derive expected active pre-tax returns from an alpha model based on value, momentum, and quality investment themes, or factors, with each factor receiving an equal risk weight.[5] We model three levels of leverage and TE—150/50 at 2% TE, 200/100 at 4% TE, and 250/150 at 6% TE. In contrast to direct indexing, where TE, in part, reflects random and time-varying exposures to industry and style factors, the TE of our long-short portfolios, both RC and CLS, arises from an intended bet on the value-momentum-quality alpha model. The values 150/50, 200/100, and 250/150 stand for the size of long and short positions as a percent of the NAV. All the positions in the RC strategies are held in individual stocks.

[5] Asness et al. (2015) discuss value, momentum, and defensive investment styles and emphasize the importance of combining them together in one portfolio: "While each of the styles employed is strong by itself, they also naturally diversify each other […] to provide even stronger performance."

Finally, our CLS portfolio construction methodology follows Liberman et al. (2020). The CLS strategies consist of a passive index fund, in our case, a hypothetical Russell 1000 fund, and beta-zero long-short portfolios of individual stocks (also known as market-neutral portfolios). In each monthly rebalance, the weights of the index fund and the long-short portfolio are adjusted to maintain a beta-one index exposure as well as a target leverage and TE of the overall CLS strategy. Maximization of expected active pre-tax returns and deferral of tax gains and realization of tax losses is performed only within the long-short component of the strategy.[6] Expected active pre-tax returns come from the same value-momentum-quality alpha model as the one used in the RC strategies. We model the same three levels of leverage and TE as in RC—150/50 at 2% TE, 200/100 at 4% TE, and 250/150 at 6% TE. Similar to RC, 150/50, 200/100, and 250/150 stand for the size of long and short positions as a percent of the NAV. However, in CLS, a 100% long position is always represented by an index fund, and only the extensions—50/50, 100/100, and 150/150, respectively—are implemented with individual stocks.

It is worth clarifying that our choices of leverage-TE combinations are not random. We chose these specific combinations based primarily on pre-tax and partially on tax considerations. From a pre-tax perspective, it has been long understood that over- or under-levering a portfolio relative to its target TE leads to an inferior implementation efficiency of alpha forecasts (e.g., Johnson et al. 2007). From a tax perspective, the ability to realize net tax losses per dollar of the NAV generally increases with leverage simply because position sizes, and thereby potential economic losses, become larger relative to the NAV. We also found that for the leverage-TE combinations modeled in this article, reduction in TE reduces CNCLs and tax benefits as a tighter TE constraint begins to limit the opportunities for deferring the realization of gains. The opposite is also true: by taking a greater economic risk, and thereby a greater exposure to the alpha model, the investor can also increase the tax benefits derived from the strategy. For the sake of brevity, we leave a detailed analysis of the appropriate leverage-TE correspondence outside of the scope of this article.

For each strategy, we simulate 27 ten-year-long histories. The simulations start in January of each year from 1986 to 2012, with the last ten-year simulation beginning in January of 2012 and ending in December of 2021. After a strategy is seeded on the first day of the simulation, there are no contributions or redemptions of capital during the simulation period. For each strategy, we report the mean, and the tenth and ninetieth percentiles of the pre-tax return, tax benefit, and CNCL outcomes across the 27 simulated histories.

Calculation Methodology

CNCL is computed as a percent of the initially contributed capital. Suppose, for example, that \$100 is invested in a strategy that in year 1 realizes a net long-term capital gain of \$0 (since no positions are long-term yet) and a net short-term capital loss of \$20, resulting in a net capital loss of \$20, and in year 2 realizes a net long-term capital gain of \$5 and a net short-term capital loss of \$15, resulting in a net capital loss of \$10 (a \$15 short-term capital loss reduced by a \$5 long-term capital gain). The CNCL for year 1 will be reported as 20%: a \$20 net capital loss in year 1 divided

[6] Whereas there could be opportunities for realizing losses on the index fund component of the CLS strategy, we leave this topic for future research. Realizing a loss on an index fund would require replacing it with another economically similar fund, and whether one fund is "substantially identical" to another for the purposes of wash sale rule is a complex legal issue (see, e.g., Matthews 2016). For example, Aked et al. (2019) mention wash sales as a potential problem but abstract away from it in their analysis of loss harvesting with ETFs.

by the initial investment of \$100. The CNCL for year 2 will be reported as 30%: a \$20 net capital loss in year 1 plus a \$10 net capital loss in year 2, divided by the initial investment of \$100.

In this article, we use a 100% CNCL as a yardstick of tax-loss realization potential of a strategy. While this level of CNCL might look arbitrary, it provides an investor with a useful intuition about how long it would take to realize \$1 of net capital loss for every \$1 invested in the strategy. Moreover, if the strategy is managed by a partnership, losses in excess of the basis of the invested capital (adjusted by the basis of contributions and redemptions) are suspended and cannot be utilized by the investor until the investor increases their basis in the partnership via additional capital contributions or via recognition of net gains and income within the partnership (see Sosner et al. 2018).

For computing tax benefits, we use the top federal tax bracket rates in the year 2022, inclusive of 3.8% net investment income tax: 40.8% for short-term capital gains and ordinary income and 23.8% for long-term capital gains and qualified dividend income. Tax benefits are computed monthly, as a percent of the month's beginning NAV, and then annualized. Following the discussion in Sosner et al. (2022) we assume that capital gains are taxed according to their character (short-term at 40.8% rate and long-term at 23.8% rate), all capital losses, whether long-term or short-term, are used to offset only *long-term* capital gains, and apply a 10% effective tax rate to incremental unrealized gains.[7]

Suppose, for example, that in a given period, as a percent of the period's beginning NAV, a strategy realizes a 12% pre-tax return, 10% long-term capital gain, a 5% qualified dividend income, a 20% short-term capital loss, and a 3% investment expense. The taxable income for the period, taking into account all the realized gains, losses, income, and expenses, is –8% (= 10% + 5% – 20% – 3%). The incremental unrealized gain for the period is pre-tax return of 12% minus taxable income of –8%, or 20%. The tax benefit for the period is computed as –((10% + 5%) × 23.8% – 20% × 23.8% – 3% × 40.8% + 20% × 10%) = 0.4%. Tax benefits of the strategies are shown in excess of the Russell 1000 benchmark tax.[8]

DIRECT INDEXING STRATEGY

We begin by analyzing the loss-realization potential and performance of the direct indexing strategy. Exhibit 1, shows the CNCL of the strategy at annual intervals. On average, the CNCL amounts to 13% in year 1, crosses a 20% mark in year 3, and asymptotes to just under 30% in the last 5 years of our ten-year simulations. These average levels of CNCL are consistent with the level of net capital losses shown in Israelov and Lu (2022), even though these authors used a completely different methodology for realizing net losses and Monte-Carlo simulated data. Notably, we find that there is a great variability in CNCL outcomes across vintages. After half a decade (see, e.g., years 6 to 10), a CNCL realized by the strategy can be as high as approximately 50% (or more) with 10% probability, or as low as approximately 5% (or less) with 10% probability.[9] We show further evidence of dependence of CNCL on market index return in Appendix B. There we divide our 27 ten-year simulation

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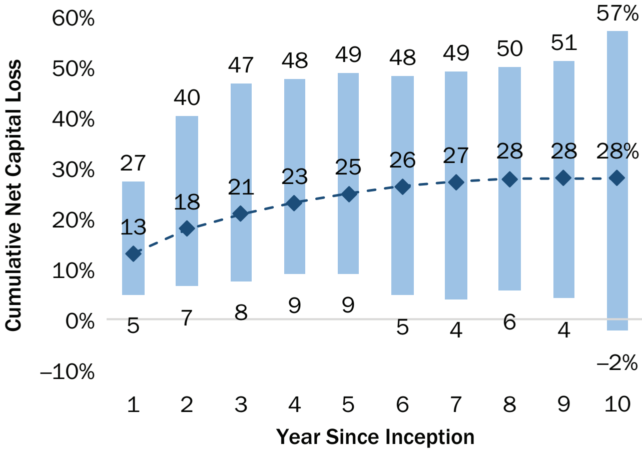
[7] The effective tax rate on current-period unrealized gains reflects the discounted expected tax cost of realizing these gains in the future.

[8] Note that if the short-term capital result were a gain and not a loss, it would have been multiplied by a 40.8% tax rate rather than by 23.8%.

[9] Note that, as shown in Sosner et al. (2022), effectiveness of a direct indexing strategy can be increased by systematic capital contributions or by combining the strategy with a charitable giving plan.

EXHIBIT 1

Direct Indexing Strategy: Cumulative Net Capital Loss Realization



NOTE: The chart shows the mean and the 10th and 90th percentile.

histories into three equally sized groups based on average Russell 1000 index return over the history. The bottom index return tercile has an average annual index return of 5% and achieves an average CNCL of approximately 50% after ten years. The middle tercile has an average annual index return of 11% and reaches an average CNCL of approximately 30%. The top tercile has an average annual index return of 16% and ends the ten-year period with an average CNCL of about 5%.[10]

Exhibit 2 shows the strategy performance statistics. All the results are shown in excess of the benchmark index performance. On average, during the entire ten-year simulation period, the gross-of-costs pre-tax alpha of the strategy is approximately zero, that is, the strategy successfully tracks the index. In years 1 to 5, the average annual turnover of the strategy is 155% of the NAV, which leads to a small negative net-of-costs pre-tax alpha of –12 bps. However, this pre-tax underperformance relative to the benchmark is more than compensated by a tax benefit which we estimate to be 69 bps on average, with the 10th and 90th percentiles at 40 bps and 118 bps, respectively.

As the portfolio matures, the tax benefits are reduced. The tax benefit in years 6 to 10 is on average 9 bps, with the 10th and 90th percentiles at –2 bps and 27 bps, respectively. The average annual turnover of the strategy, which is mostly induced by loss-realization trades, declines accordingly to 55% of the NAV. This is consistent with Exhibit 1, where the CNCL is approximately flat in years 6 to 10, meaning that the strategy realizes minimal net capital losses.

TAX-AWARE LONG-SHORT ALPHA STRATEGIES

Base-Case Results

As a quick reminder, we have selected three long-short strategies for our analysis: 150/50, 200/100, and 250/150 at the TE of 2%, 4%, and 6%, respectively. As opposed to direct indexing, the TE of these strategies is informed by a factor-based alpha model. These strategies are implemented in two ways: RC, which only invests in individual stocks, both long and short, and CLS, which achieves its index exposure by investing in a passive index fund and only uses individual stocks to represent the long-short extensions, in our case 50/50, 100/100, and 150/150, alternatively.

Exhibit 3 shows the CNCLs of long-short strategies. RC strategies are shown in Panel A. RC 150/50, on average, realizes a 100% CNCL in about seven years. After nine years, fewer than 10% of our simulated vintages realize a CNCL less than 100%. The higher leverage strategies achieve a 100% CNCL faster. For RC 200/100, it takes less than three years on average to realize a 100% CNCL, and after four years fewer than 10% of the vintages fail to achieve a 100% CNCL. RC 250/150 reaches a 100%

[10] Note that the 5% CNCL is the average of nine different historical simulations. Recent investors in direct indexing strategies might have experienced different CNCL outcomes. For example, in our last simulation vintage from 2012 to 2021, the average annual index return was approximately 16% and the year-10 CNCL was 13%, that is, at the 90th percentile of year-10 CNCL level. The obvious advantage of historical simulations is that they provide evidence above and beyond the most recent history.

EXHIBIT 2

Direct Indexing Strategy: Performance Statistics

	Years 1 to 5	Years 6 to 10
Gross of Cost Pre-Tax Alpha		
Average	0.03%	0.08%
10th Percentile	–0.59%	–0.87%
90th Percentile	0.61%	0.78%
Net of Cost Pre-Tax Alpha		
Average	–0.12%	0.03%
10th Percentile	–0.73%	–0.94%
90th Percentile	0.45%	0.69%
Tax Benefit		
Average	0.69%	0.09%
10th Percentile	0.40%	–0.02%
90th Percentile	1.18%	0.27%
Net of Cost After-Tax Alpha		
Average	0.57%	0.12%
10th Percentile	–0.02%	–0.73%
90th Percentile	1.14%	0.87%
Realized Tracking Error		
Average	1.21%	1.13%
10th Percentile	0.98%	0.89%
90th Percentile	1.52%	1.35%
Costs		
Average Turnover	155%	55%
Transaction Costs	0.16%	0.06%

CNCL in less than two years and only about 10% of the vintages fail to realize a 100% CNCL after two years. These results show that the speed with which 100% CNCL is achieved by an RC strategy depends on the level of leverage and TE that an investor is prepared to tolerate—economic risk and tax benefits go hand in hand. Since the TE of our RC strategies is informed by an alpha model, as we will see shortly below, additional leverage and TE also allow the investor to enjoy higher levels of pre-tax alpha.

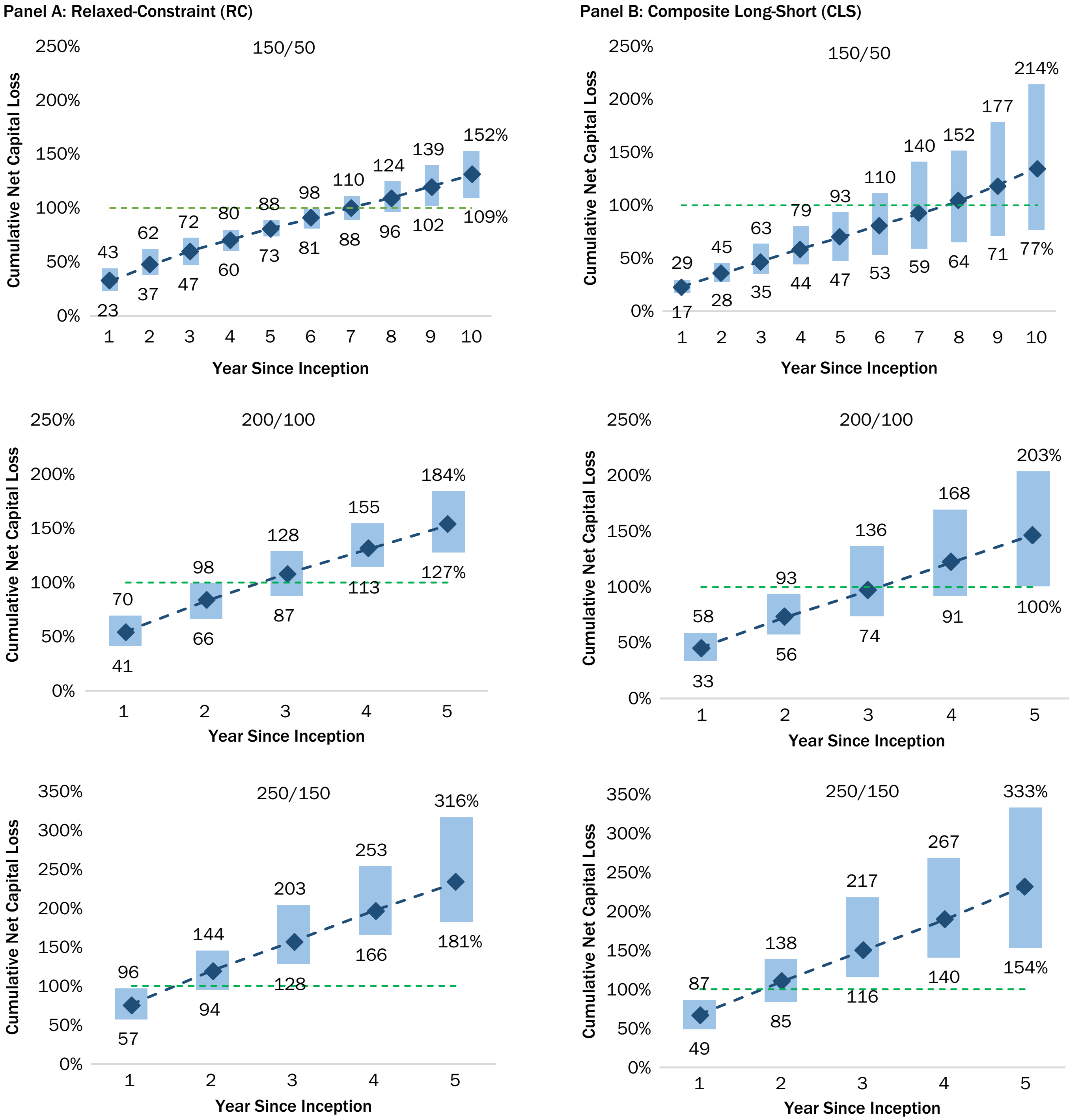
The results for the CLS strategies are shown in Exhibit 3, Panel B. The average level of CNCLs realized by CLS is similar to RC: CLS 150/50 achieves a CNCL of 100% in about eight years, CLS 200/100 in about three, and CLS 250/150 in less than two. However, compared to RC, CLS strategies exhibit a considerably higher dispersion of CNCL outcomes in later years of the simulation, as indicated by the 10th and 90th percentiles. While we leave a detailed examination of the sources of the greater CNCL variability of the CLS strategies for further research, our preliminary analysis indicates that this greater variability is, at least in part, related to market returns. For example, in falling markets, the index fund component of a CLS strategy might still be at a cumulative unrealized gain, whereas a portion of individual stocks representing beta-one market exposure of an RC strategy can be at a loss, thus, providing the RC strategy with greater opportunities for realizing net losses in comparison to CLS. On the other hand, in rising markets, CLS strategies benefit, from a tax perspective, from separation of alpha and beta described in Liberman et al. (2020) and realize lower gains than corresponding RC strategies.

It is worth pointing out that the mechanism for net capital loss realization is very different between the direct indexing strategy and the factor-based strategies. The direct indexing strategy seeks to replicate passive index holdings, which, from an economic perspective, requires minimal trading. To realize losses, the strategy generates turnover based primarily on tax considerations. In contrast, our RC and CLS strategies have a high level of alpha-model-induced turnover. For these strategies, tax-aware optimization helps achieve a desired level of alpha-model exposure without sacrificing tax efficiency. In fact, Sialm and Sosner (2018) show that for factor-based strategies tax efficiency comes not from accelerating losses but rather from slowing down the realization of gains.

Both the RC and CLS strategies generate a significant pre-tax alpha net of transaction and financing costs. Exhibit 4 shows pre-tax and after-tax performance statistics for the RC and CLS strategies in Panels A and B, respectively. All the results are shown in excess of the benchmark index. We divide our ten-year simulation period into two five-year subperiods. Not surprisingly, both pre-tax alpha and tax benefits increase with leverage and TE. For example, for the RC strategy in Panel A, the average annual net-of-costs pre-tax alpha in years 1 to 5 increases from 73 bps for RC 150/50 to 1.99% for RC 200/100 to 3.46% for RC 250/150, and the average annual tax benefit increases from 2.08% to 3.66% to 5.24%, respectively. In years 6 to 10, the average annual net-of-costs pre-tax alpha increases from 50 bps

EXHIBIT 3

Long-Short Strategies: Cumulative Net Capital Loss Realization



NOTE: The chart shows the mean and the 10th and 90th percentile.

EXHIBIT 4

Long-Short Strategies: Performance Statistics

	150/50, 2% Tracking Error		200/100, 4% Tracking Error		250/150, 6% Tracking Error	
	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10
Panel A: Relaxed-Constraint (RC)						
Gross of Cost Pre-Tax Alpha						
Average	1.78%	1.39%	3.90%	2.93%	6.20%	4.73%
10th Percentile	0.26%	0.29%	0.89%	0.34%	1.17%	0.76%
90th Percentile	3.37%	3.04%	6.95%	6.85%	10.96%	10.54%
Net of Cost Pre-Tax Alpha						
Average	0.73%	0.50%	1.99%	1.26%	3.46%	2.26%
10th Percentile	–0.87%	–0.62%	–1.05%	–1.32%	–1.60%	–1.61%
90th Percentile	2.35%	2.08%	5.03%	5.03%	8.19%	7.86%
Tax Benefit						
Average	2.08%	0.72%	3.66%	1.43%	5.24%	2.25%
10th Percentile	1.67%	0.37%	3.14%	0.91%	4.52%	1.45%
90th Percentile	2.58%	1.03%	4.14%	2.24%	5.89%	3.46%
Net of Cost After-Tax Alpha						
Average	2.82%	1.22%	5.65%	2.69%	8.70%	4.51%
10th Percentile	1.62%	–0.07%	2.92%	–0.20%	3.95%	0.22%
90th Percentile	4.16%	3.06%	8.48%	6.94%	13.02%	11.38%
Realized Tracking Error						
Average	2.34%	2.39%	4.70%	4.73%	6.89%	7.15%
10th Percentile	1.97%	1.97%	3.98%	3.95%	5.69%	5.92%
90th Percentile	2.83%	2.73%	5.53%	5.50%	8.26%	8.35%
Costs						
Average Turnover	548%	387%	911%	678%	1243%	976%
Transaction Costs	0.55%	0.39%	0.91%	0.68%	1.24%	0.98%
Average Short-Side Leverage	50%	50%	100%	100%	150%	150%
Financing Costs	0.50%	0.50%	1.00%	1.00%	1.50%	1.50%
Panel B: Composite Long-Short (CLS)						
Gross of Cost Pre-Tax Alpha						
Average	1.96%	1.28%	3.97%	2.53%	5.97%	3.84%
10th Percentile	0.58%	0.17%	1.04%	0.51%	1.46%	0.46%
90th Percentile	3.60%	3.15%	7.14%	6.30%	10.56%	9.33%
Net of Cost Pre-Tax Alpha						
Average	1.11%	0.50%	2.25%	0.98%	3.41%	1.50%
10th Percentile	–0.25%	–0.59%	–0.60%	–0.94%	–1.08%	–1.72%
90th Percentile	2.72%	2.31%	5.37%	4.61%	7.90%	6.80%
Tax Benefit						
Average	1.57%	0.76%	3.25%	1.66%	4.98%	2.55%
10th Percentile	1.09%	0.20%	2.63%	0.93%	4.22%	1.56%
90th Percentile	2.03%	1.53%	3.96%	2.87%	5.93%	4.23%
Net of Cost After-Tax Alpha						
Average	2.68%	1.26%	5.50%	2.63%	8.40%	4.06%
10th Percentile	1.15%	–0.10%	2.97%	0.31%	4.64%	0.14%
90th Percentile	4.43%	3.39%	8.69%	7.08%	13.01%	10.55%

(continued)

EXHIBIT 4

Long-Short Strategies: Performance Statistics *(continued)*

	150/50, 2% Tracking Error		200/100, 4% Tracking Error		250/150, 6% Tracking Error	
	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10
Realized Tracking Error						
Average	2.24%	2.27%	4.50%	4.58%	6.73%	6.90%
10th Percentile	1.85%	1.88%	3.79%	3.81%	5.65%	5.94%
90th Percentile	2.59%	2.66%	5.34%	5.41%	8.02%	8.24%
Costs						
Average Turnover	356%	273%	714%	551%	1062%	832%
Transaction Costs	0.36%	0.27%	0.71%	0.55%	1.06%	0.83%
Average Short-Side Leverage	50%	50%	100%	100%	150%	150%
Financing Costs	0.50%	0.50%	1.00%	1.00%	1.50%	1.50%

for RC 150/50 to 1.26% for RC 200/100 to 2.26% for RC 250/150, and the average annual tax benefit increases from 72 bps to 1.43% to 2.25%, respectively.

The results for the CLS strategy shown in Panel B are broadly consistent with those of the RC strategy. Similar to RC, CLS generates a meaningful net-of-costs pre-tax alpha and a level of tax benefits well in excess of those achieved by direct indexing (shown in Exhibit 2).

For all the strategies we modeled, there is a noticeable reduction in the pre-tax alpha in years 6 to 10 in comparison to years 1 to 5. Similar to what we have seen for the direct indexing strategy, portfolio turnover also declines. This reduction in pre-tax alpha and turnover is a natural outcome of tax-aware implementation. It is clear from Exhibit 3 that the strategy portfolios become more appreciated over time. As built-in gains in portfolio positions increase, tax-aware optimization needs to "work harder" to maintain the strategy's tax efficiency, and as a result, the efficiency of the alpha model implementation is reduced.

Dynamic Strategies: Modulating the Leverage and Tracking Error

Financing a levered portfolio adds to the costs of running the strategy, while TE, even though informed by an alpha model, increases the risk of underperforming the benchmark. As a result, investors might look to manage the strategy's leverage and TE dynamically based on their time-varying perception of the trade-off between the potential for alpha and risks and costs required for achieving it.

For example, an investor might want to test out the strategy at a low level of leverage and TE, and only scale them up after gaining sufficient confidence that the risk of significantly underperforming the benchmark is low while the potential for outperforming the benchmark is high. We expect that such risk-on trades should be relatively easy to execute tax efficiently. For example, Sosner and Krasner (2021) show that leverage and TE of a diversified low-basis portfolio can be increased instantly without incurring any tax burden. In experiments not reported here for the sake of brevity, we replicated their findings on increasing the leverage and TE without a tax cost for low-basis RC and CLS portfolios.

Alternatively, an investor might want to reduce the leverage and TE upon becoming more skeptical about the level of the strategy's alpha or less certain about the diversification it provides to the overall investment portfolio. Intuitively, scaling back leverage and TE of a low-basis portfolio without incurring substantial tax costs should be difficult because appreciated positions would need to be pared down. In this article, we quantify what "difficult" means in the context of our RC and CLS portfolios. To stress-test our ability to scale back high-leverage high-TE strategies, we initiate leverage and TE reduction when the strategy achieves a CNCL of 100%.

As we have seen in Exhibit 3, 150/50 strategies achieve a 100% CNCL after almost a decade. At the same time, 200/100 and 250/150 strategies reach the 100% CNCL level within the first two to three years. Since we work with ten-year simulation periods, we view the latter two strategies as more suitable candidates for our scaling-back experiments than the 150/50 strategies. For brevity, we will refer to the process of reducing leverage and TE simply as "de-risking."

We consider two approaches to de-risking: scheduled and optimized. Irrespective of the approach, de-risking always starts in the first January rebalance following the year when an average CNCL reaches 100%.

Scheduled de-risking reduces leverage and TE linearly over the course of two years by using progressively tighter constraints. Both leverage and TE are ratcheted down in every monthly rebalance toward a specified endpoint that is achieved at the end of the two-year de-risking period. We model scheduled de-risking with three alternative endpoints. The first endpoint is our 1% TE direct indexing strategy, in the case of RC, and a zero-TE index fund, in the case of CLS. In both cases, the endpoint has neither leverage nor pre-tax alpha. The second endpoint is 150/50 with our value-momentum-quality alpha model and 2% TE. The third endpoint is the same as the second one but with a TE of 3% instead of 2%. As we will see shortly, the latter two endpoints allow us to avoid a large recognition of capital gains upon de-risking.

In contrast, *optimized* de-risking does not set a fixed period or objective for reducing leverage and TE. Rather than forcing them down through constraints, it applies a penalty to both leverage and TE. After the de-risking begins, the strategy is permanently in a state of maximizing the exposure to alpha model and minimizing leverage and TE while trying to prevent the recognition of net capital gains.

These two approaches to de-risking have their respective advantages and shortcomings. Scheduled de-risking might miss opportunities to de-risk more actively in periods when the strategy portfolio has relatively small unrealized gains. At the same time, it could be de-risking too aggressively in periods when the portfolio has relatively large unrealized gains. In contrast, optimized de-risking will slow down or accelerate by appropriately considering the balance of unrealized gains and losses embedded in portfolio positions. On the other hand, the optimized de-risking relies on calibration of leverage and TE penalty coefficients within a specific sample period (see Appendix A for details). A simple rules-based scheduled de-risking will not suffer from such a potential overfitting problem and will allow us to evaluate the opportunities for leverage and TE reduction without fine-tuning optimization parameters to our sample. As a result, we believe that scheduled and optimized de-risking results in combination provide a good indication of how much leverage and TE could be reduced in a tax-efficient manner.

The next two subsections examine de-risking for 200/100 and 250/150 strategies, respectively. In each case, we consider an RC and a corresponding CLS strategy. As a brief preview, we find that in all scenarios where a scheduled de-risking fully eliminates long and short extensions (that is, achieves a direct indexing target for RC or an index fund target for CLS), the gain realized upon de-risking substantially offsets the previously accrued CNCL. On the other hand, if the investor is willing to tolerate a small amount of leverage and TE at the end of the de-risking process, the CNCL can be mostly preserved.

Furthermore, along the initial leverage-TE dimension, 200/100 at 4% TE compared to 250/150 at 6% TE, the de-risking results are similar within each implementation approach, RC or CLS. On the other hand, along the strategy implementation dimension, RC compared to CLS, we find that the de-risking process is more tax-efficient for RC than for CLS for both levels of initial leverage and TE, 200/100 at 4% TE and 250/150 at 6% TE, particularly when it comes to optimized de-risking and scheduled de-risking to an index portfolio (direct indexing in the case of RC and an index fund in the case of CLS). This difference between RC and CLS is not surprising: Over time, the strategy portfolio accumulates bult-in gains, and having additional 100% of long exposure via individual stocks offers the RC strategies greater opportunities for "storing" those gains in portfolio positions as leverage is being reduced.

De-Risking 200/100 Strategies

Let's begin with scheduled de-risking of the 200/100 4% TE strategies. For both RC and CLS, we run the strategy at its initial leverage and TE for three years, linearly de-risk it over the next two years and continue running the scaled-down strategy for the remaining five years of the simulation. As we mentioned above, we explore three levels of de-risking: from the most aggressive de-risking where the target is a long-only direct indexing strategy at 1% TE (or an index fund for CLS) to the least aggressive de-risking where the target is a 150/50 strategy at 3% TE. Exhibit 5 shows the evolution of CNCL under these alternative de-risking targets. Panels A and B summarize the results for RC and CLS strategies, respectively.

The top chart in Exhibit 5, Panel A, shows that the RC 200/100 4% TE strategy reaches a 100% CNCL in about three years, on average. We begin reducing the leverage and TE linearly at the beginning of year 4 such that at the end of year 4 the leverage is 150/50 and the TE is 2.5% and at the end of year 5 both leverage and TE are at their target levels—zero leverage (that is, a long-only portfolio) and 1% TE. Since the target strategy is passive direct indexing, we also ratchet down the exposure to the alpha model, such that the alpha model is completely phased out by the end of year 5 when de-risking is complete. Note that in our de-risking exhibits, the blue area signifies a pre-de-risking stage, the yellow area signifies a de-risking stage, and the green area signifies a post-de-risking stage.

We can see that transition to RC 150/50 2.5% TE after one year (that is, half-way to the target 1% TE of direct indexing) is achieved highly tax efficiently—the level of CNCL barely changes by the end of year 4. In other words, in the beginning of year 4, the 200/100 4% TE portfolio has enough positions with either built-in losses or sufficiently small built-in gains to de-risk significantly over the next twelve months without realizing a net gain. The problem arises with the next stage of de-risking. Over the course of year 5, reducing the extensions from 50/50 to zero and TE from 2.5% to 1.0% triggers so much gain realization that the average CNCL is reduced from above 100% to less than 20%, meaning that the amount of net gain realization is greater than 80% of the capital initially contributed to the strategy.

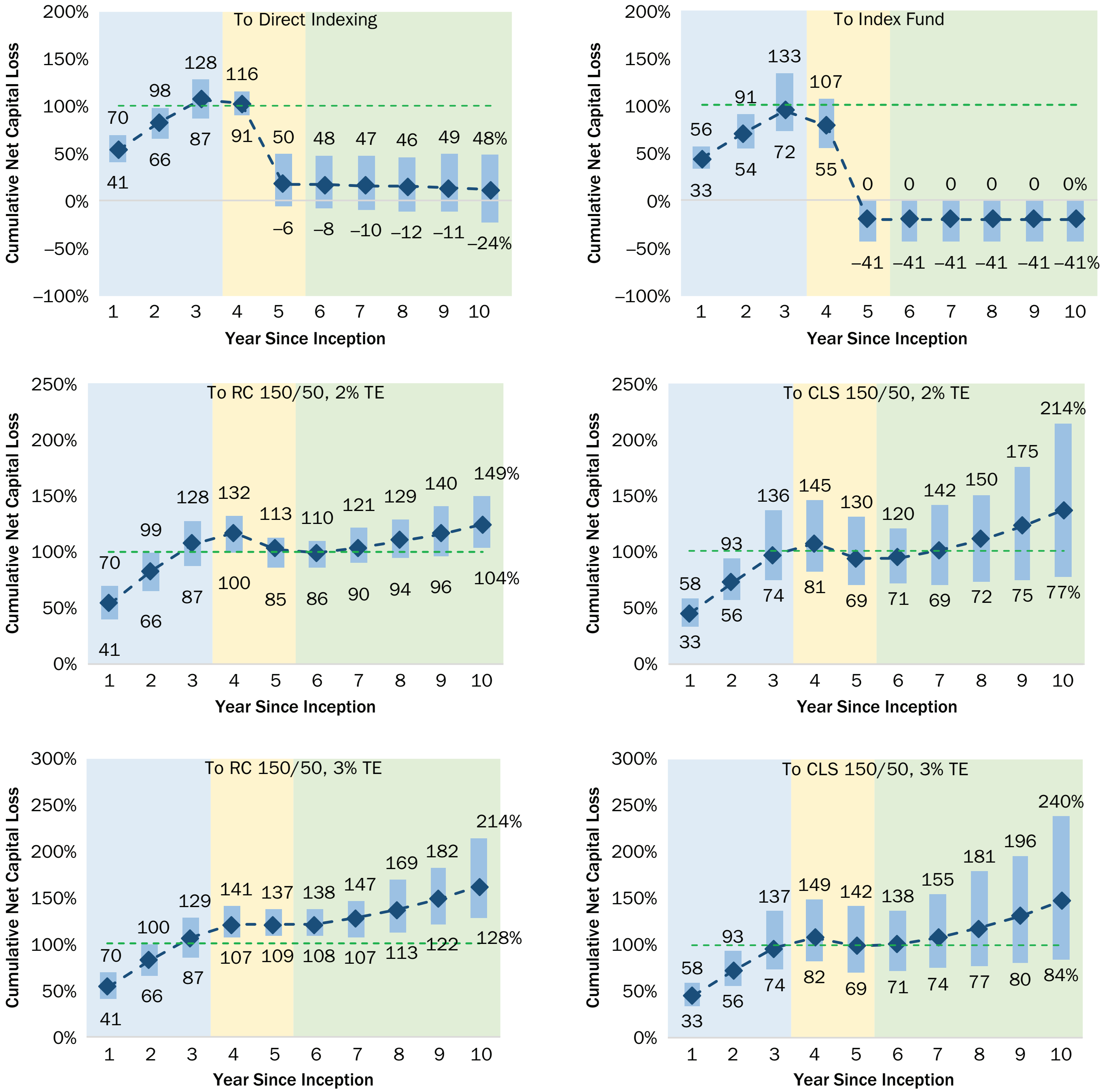
The results of the first year of de-risking, that is, year 4, give us hope that some level of reduction of leverage and TE can be achieved without reducing the CNCL below 100%. At the same time, the results for year 5 provide stark evidence that, without undoing most of the CNCL accumulated in years 1, 2, and 3, transitioning to direct indexing is not possible, at least within a short period of time. We will come back to the point of time horizon for reducing leverage and TE when we discuss optimized de-risking.

The middle chart in Exhibit 5, Panel A, shows the de-risking from RC 200/100 4% TE to the target of RC 150/50 2% TE at the end of year 5. Since the de-risking process is linear, at the end of year 4, the strategy is at 175/75 3% TE. As opposed to transition to direct indexing, in this case, we do not ratchet down the exposure to the alpha model because the target of de-risking is still an alpha-oriented process, just managed at lower leverage and TE. The results generally look promising: On average, the CNCL remains close to, or above, 100%. Moreover, the CNCL continues to increase in later years, which means that the strategy continues to realize net capital losses after reaching its target leverage and TE.

EXHIBIT 5

Scheduled De-Risking of 200/100 Strategies

Panel A: Relaxed-Constraint (RC) **Panel B: Composite Long-Short (CLS)**



NOTE: The chart shows the mean and the 10th and 90th percentile.

The main concern with de-risking to RC 150/50 2% TE is that in years 5, 6, and 7, that is, the second year of de-risking and the first two years after de-risking is complete, a substantial portion of our simulated histories end up with CNCL meaningfully below 100%. In other words, net capital gains are realized in those histories during de-risking and immediately after de-risking. Increasing the target TE from 2%

to 3%, that is, targeting RC 150/50 at 3% TE solves this issue. The bottom chart in Exhibit 5, Panel A, shows that, in this case, even the 10th percentile of CNCL remains comfortably above 100%.

Thus far, our results have shown that RC strategies can realize substantial amounts of CNCL rather quickly and then can be de-risked to lower levels of leverage and TE without substantially reducing CNCL. De-risking to direct indexing without reversing most of the CNCL seems impossible, but the leverage can be reduced to 150/50 and TE can be reduced to a 2% to 3% range quite quickly and tax efficiently. We will revisit these conclusions shortly when we analyze optimized de-risking.

De-risking results for the CLS strategy are shown in Exhibit 5, Panel B. We can draw three conclusions from a side-by-side comparison of CLS with RC. First, as we can see in the top chart, de-risking to zero-leverage is substantially more punitive for CLS than for RC. In fact, the 90th percentile CNCL outcomes realizes all the deferred gains and thereby reverses all the CNCL accrued in the first three years. Moreover, on average, de-risking the CLS strategy to just holding an index fund position realizes a net cumulative gain (that is, a negative CNCL). This is because, thanks to the alpha model, the long-short component of the CLS strategy generates pre-tax profits all of which are realized when the long-short component of CLS is liquidated. Second, de-risking CLS to 150/50 2% TE and 3% TE (the middle and bottom charts in Exhibit 5, Panel B) looks similar to de-risking RC. This is because the long-short component of the CLS strategy is efficient at “storing” unrealized gains in the remaining positions. Third, de-risking CLS to one of the 150/50 strategies results in a larger dispersion of outcomes compared to RC.

We now turn to optimized de-risking. Calibration of relative preferences toward alpha model exposure, risk aversion, leverage aversion, and gain realization aversion during the de-risking process can be performed in infinitely many ways. Therefore, the reader should view our optimized de-risking not as a definitive strategy but as an example of what can be realistically achieved by an investor seeking to reduce leverage and TE without realizing net capital gains in the process. In every monthly optimization starting from January of year 4, we constrain a net gain realization to zero and penalize leverage and TE. The leverage is always balanced, that is, the size of long extension is always equal to the size of the short extension by construction. The alpha model is retained as an integral part of the process. The results of our optimized de-risking experiment are summarized in Exhibit 6.

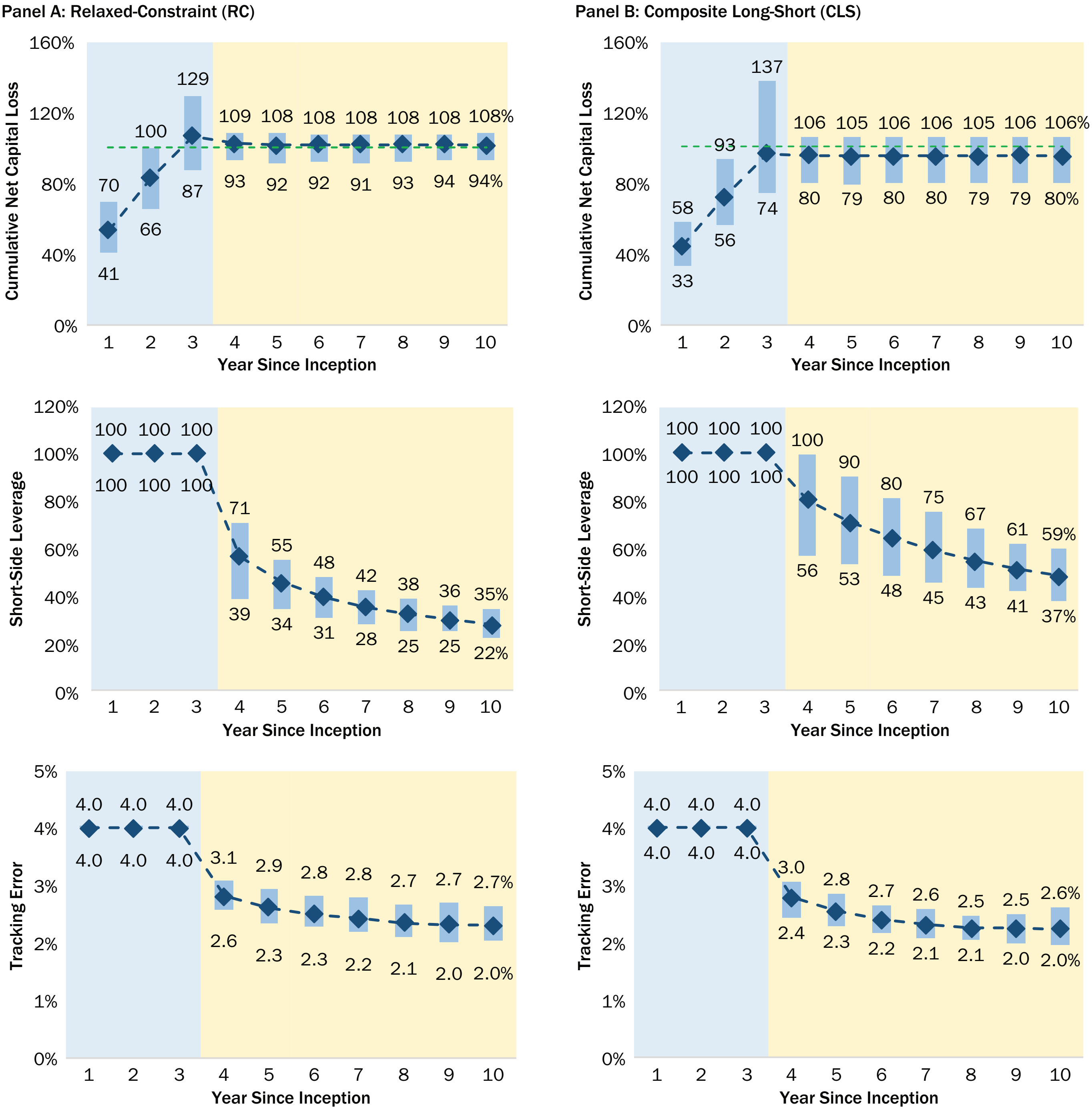
Exhibit 6, Panel A, shows the results for the RC strategy. The top chart shows that the CNCL generally remains at the level achieved before the de-risking has begun. The largest drop in leverage occurs in the first year of de-risking—from 200/100 to, on average, 157/57. In the second year, the average level of leverage drops further to 146/46, and then continues to decline to 128/28 by the end of our simulation period. The TE drops from 4% to 2.8% by the end of the first year of de-risking and then gradually declines to 2.3% in year 10. These results are consistent with scheduled de-risking, where we saw that the leverage can be reduced to 150/50 and TE to 2 to 3% range. However, the excess CNCL that we saw in the scheduled de-risking in the last five years of the simulation is now used up to reduce leverage below 150/50 and TE toward the lower bound of the 2 to 3% range.

Exhibit 6, Panel B, shows the results for the CLS strategy. Similar to RC, the top chart shows that the reduction in leverage and TE generally occurs without realizing a net capital gain. The middle chart shows that the reduction in leverage for CLS is slower than for RC, and that the dispersion around the average level is higher. The bottom chart shows that TE reduction is quite similar to that of RC.

The faster leverage reduction for RC in comparison with CLS can be attributed to its additional long-equities exposure via individual stocks. The RC strategies effectively

EXHIBIT 6

Optimized De-Risking of 200/100 Strategies



NOTE: The chart shows the mean and the 10th and 90th percentile.

"store" built-in gains in their larger long portfolio positions as the leverage is being reduced. This allows them to cut leverage quicky and tax efficiently.

Finally, Exhibit 7 shows the pre-tax and after-tax returns of the RC and CLS strategies. All the results are shown in excess of the benchmark index. We report the results separately for years 1 to 5 and for years 6 to 10. Recall that in the first five

EXHIBIT 7

De-Risking of 200/100 Strategies: Performance Statistics

Panel A: Relaxed-Constraint (RC)

	To Direct Indexing		To RC 150/50, 2% TE		To RC 150/50, 3% TE		To RC Optimized	
	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10
Gross of Cost Pre-Tax Alpha								
Average	3.12%	0.19%	3.33%	1.34%	3.65%	2.29%	3.47%	1.86%
10th Percentile	0.35%	–0.54%	0.60%	0.02%	0.75%	0.44%	0.59%	0.53%
90th Percentile	5.35%	0.95%	6.11%	3.38%	6.55%	5.50%	6.82%	4.44%
Net of Cost Pre-Tax Alpha								
Average	1.58%	0.13%	1.62%	0.48%	1.95%	1.47%	1.88%	1.28%
10th Percentile	–1.23%	–0.59%	–1.13%	–0.87%	–0.98%	–0.35%	–1.00%	–0.12%
90th Percentile	3.84%	0.90%	4.35%	2.46%	4.91%	4.61%	5.23%	3.89%
Tax Benefit								
Average	0.61%	–0.03%	2.80%	0.31%	3.07%	0.30%	2.68%	–0.07%
10th Percentile	–0.61%	–0.23%	2.07%	–0.03%	2.53%	–0.09%	2.13%	–0.35%
90th Percentile	1.49%	0.20%	3.32%	0.66%	3.57%	0.72%	3.29%	0.22%
Net of Cost After-Tax Alpha								
Average	2.19%	0.10%	4.42%	0.79%	5.02%	1.77%	4.56%	1.21%
10th Percentile	–0.06%	–0.49%	2.14%	–0.68%	2.67%	–0.17%	1.82%	–0.62%
90th Percentile	4.01%	0.79%	6.66%	2.83%	7.57%	5.09%	7.25%	3.99%
Realized Tracking Error								
Average	4.21%	1.01%	4.33%	2.41%	4.47%	3.41%	4.26%	2.73%
10th Percentile	3.37%	0.93%	3.60%	2.05%	3.71%	2.84%	3.49%	2.36%
90th Percentile	5.29%	1.10%	5.37%	2.75%	5.47%	3.91%	5.37%	3.16%
Costs								
Average Turnover	755%	58%	816%	356%	801%	324%	757%	226%
Transaction Costs	0.76%	0.06%	0.82%	0.36%	0.80%	0.32%	0.76%	0.23%
Average Short-Side Leverage	79%	0%	90%	50%	90%	50%	84%	35%
Financing Costs	0.79%	0.00%	0.90%	0.50%	0.90%	0.50%	0.84%	0.35%

Panel B: Composite Long-Short (CLS)

	To Index Fund		To CLS 150/50, 2% TE		To CLS 150/50, 3% TE		To CLS Optimized	
	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10
Gross of Cost Pre-Tax Alpha								
Average	2.92%	0.00%	3.39%	1.20%	3.68%	1.93%	3.33%	1.48%
10th Percentile	0.29%	0.00%	0.72%	0.05%	0.86%	–0.11%	0.61%	–0.21%
90th Percentile	5.32%	0.00%	6.27%	3.45%	6.54%	5.58%	5.73%	3.77%
Net of Cost Pre-Tax Alpha								
Average	1.55%	0.00%	1.86%	0.46%	2.15%	1.18%	1.77%	0.62%
10th Percentile	–1.04%	0.00%	–0.74%	–0.67%	–0.60%	–0.80%	–1.05%	–1.06%
90th Percentile	4.02%	0.00%	4.72%	2.65%	4.96%	4.76%	4.21%	3.05%
Tax Benefit								
Average	–0.53%	–0.01%	2.34%	0.41%	2.39%	0.30%	2.54%	0.16%
10th Percentile	–2.89%	–0.34%	1.67%	–0.32%	1.76%	–0.29%	2.01%	–0.05%
90th Percentile	0.89%	0.31%	3.12%	1.25%	3.17%	1.05%	3.05%	0.38%

(continued)

EXHIBIT 7

De-Risking of 200/100 Strategies: Performance Statistics *(continued)*

	To Index Fund		To CLS 150/50, 2% TE		To CLS 150/50, 3% TE		To CLS Optimized	
	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10	Years 1 to 5	Years 6 to 10
Net of Cost After-Tax Alpha								
Average	1.02%	–0.01%	4.20%	0.87%	4.55%	1.48%	4.31%	0.78%
10th Percentile	–1.77%	–0.34%	1.79%	–0.51%	1.97%	–0.63%	1.91%	–1.12%
90th Percentile	3.41%	0.31%	6.56%	3.44%	7.46%	5.40%	6.40%	3.18%
Realized Tracking Error								
Average	3.87%	0.00%	4.14%	2.22%	4.29%	3.27%	4.10%	2.73%
10th Percentile	3.11%	0.00%	3.37%	1.87%	3.59%	2.82%	3.26%	2.24%
90th Percentile	4.64%	0.00%	4.88%	2.57%	5.05%	3.92%	5.06%	3.39%
Costs								
Average Turnover	582%	2%	628%	240%	625%	249%	641%	285%
Transaction Costs	0.58%	0.00%	0.63%	0.24%	0.63%	0.25%	0.64%	0.29%
Average Short-Side Leverage	79%	0%	90%	50%	90%	50%	93%	57%
Financing Costs	0.79%	0.00%	0.90%	0.50%	0.90%	0.50%	0.93%	0.57%

years the strategy is managed with high leverage and TE for the first three years and then de-risked. In the next five years, the strategy is either de-risked to the target level under scheduled de-risking or continues to be gradually de-risked under optimized de-risking. Panels A and B show the data for the RC and CLS strategies, respectively.

In the first five years, both RC and CLS realize an average annual net-of-costs pre-tax alpha roughly between 1.5% and 2.2% (at the bottom of each table we show the calculation of transaction and financing costs that enter the calculation of net-of-costs alpha). It is lower than the level of alpha without de-risking shown in the previous section but is consistent with the fact that the strategies spend two out of five years in de-risking. In years 6 to 10, pre-tax alpha of the direct indexing strategy remaining after de-risking the RC strategy is approximately 0. In a parallel case for the CLS strategy, pre-tax alpha is exactly 0 because the long-short strategy is liquidated, and the investor only holds the benchmark index fund. When strategies, whether RC or CLS, are de-risked to 150/50 or are de-risked optimally, they retain their ability to generate net-of-costs pre-tax alpha in years 6 to 10. In the optimized de-risking case, on average, the net-of-costs pre-tax alpha is lower for the CLS than for RC strategy: 0.62% for the former compared to 1.28% for the latter. This difference in net-of-costs alpha is driven in part by the higher costs of the CLS strategy, particularly, the financing costs (due to CLS holding on to a higher leverage), and in part by a lower gross alpha of the CLS strategy. Overall, we find that differences in net-of-costs pre-tax alpha between RC and CLS are not large.

The differences in tax benefits between RC and CLS are also quite small. The largest difference in tax benefits can be observed in years 1 to 5 for de-risking to zero leverage (to direct indexing for RC and to index fund for CLS): The average tax benefit is 61 bps for RC and –53 bps for CLS. This is consistent with the larger de-risking gain realization of the CLS strategy in years 4 and 5 that we saw in Exhibit 6.[11] In the rest of the de-risking scenarios, RC also achieves a somewhat higher tax benefit

[11] The non-zero tax benefit for the index fund position in Panel B (–34 bps at the 10th percentile and 31 bps at the 90th percentile) results from benchmark rebalancing. For consistency, for all the strategies, the benchmark is the index portfolio implemented with individual stocks. Therefore, although holding the index fund position in years 6 to 10 does not realize any gain or loss, rebalancing of the

than CLS in the first five-year subperiod. These differences are due to RC being a more tax-efficient strategy than CLS both before de-risking (see Exhibits 3 and 4) and during de-risking (see Exhibits 5 and 6).

De-Risking 250/150 Strategies

We now repeat the de-risking experiments with 250/150 6% TE strategies. Both RC and CLS strategies reach the 100% CNCL level in less than two years. As a result, rather than initiating de-risking after three years, as we have done for the 200/100 strategies, for 250/150, we begin de-risking after two. As for the 200/100 strategies, we consider scheduled de-risking (where leverage and TE are reduced linearly over a two-year period) and optimized de-risking. In the case of scheduled de-risking, the de-risked strategy runs for the remainder of the ten-year period—from year 5 to year 10, six years in total. In the case of optimized de-risking, the de-risking process continues until the end of the simulation period in year 10.

Exhibits 8, 9, and 10 show the same information as Exhibits 5, 6, and 7 but now for the 250/150 6% TE RC and CLS strategies. In Exhibit 10, consistent with the de-risking stages, we split the simulation period into two subperiods: years 1 to 4 and 5 to 10. All the results look consistent with de-risking the 200/100 strategies. The optimized de-risking shown in Exhibit 9 ends up with virtually the same levels of leverage and TE as those we have seen for optimized de-risking of the 200/100 strategy in Exhibit 6.

Comparing Exhibits 10 and 7, where we summarize the pre-tax returns, tax benefits, and after-tax returns, we can see that, in the early years of strategy simulations, both pre-tax returns and tax benefits of the 250/150 strategies are significantly higher than those of the 200/100 strategies. This is consistent with the higher leverage and TE of the 250/150 strategy which provides a relatively higher exposure to both pre-tax alpha and potential loss-realization opportunities. The only exception to this pattern is the tax benefit in years 1 to 4 when the long-short strategies are de-risked to zero leverage. In this case, the 250/150 strategies realize greater de-risking gains than their 200/100 counterparts, thus, resulting in 250/150 having a smaller tax benefit in the RC case and a greater tax liability in the CLS case. Even then, the net-of-costs after-tax return is still higher for the 250/150 strategy thanks to its higher pre-tax return. After de-risking is complete (or mostly complete, as in the case of optimized de-risking), in years 5 to 10, the results for the de-risked 250/150 strategies are virtually identical to those for the 200/100 strategies in years 6 to 10.

In conclusion, the choice between 250/150 and 200/100 dynamically de-risked strategies boils down to whether the investor is comfortable with tolerating additional leverage and TE in the early years of the strategy in order to achieve the advantages that the 250/150 yields in terms of pre-tax alpha, loss-realization potential (measured by CNCL), and tax benefits. In the end, however, “All roads lead to Rome:” Whether the strategy starts out as 200/100 at 4% TE or 250/150 at 6% TE, is implemented as RC or CLS, is de-risked on a fixed schedule or optimally, if an investor wants to avoid realizing net capital gains, she will have to accept a leverage of 130/30 to 150/50 and a TE of 2% to 3% as the end point of the de-risking process. Reducing leverage and TE further is likely to result in recognition of net capital gains. De-risking all the way to an index portfolio, depending on the specific scenario, is likely to undo most, if not all, of the previously accrued CNCL.

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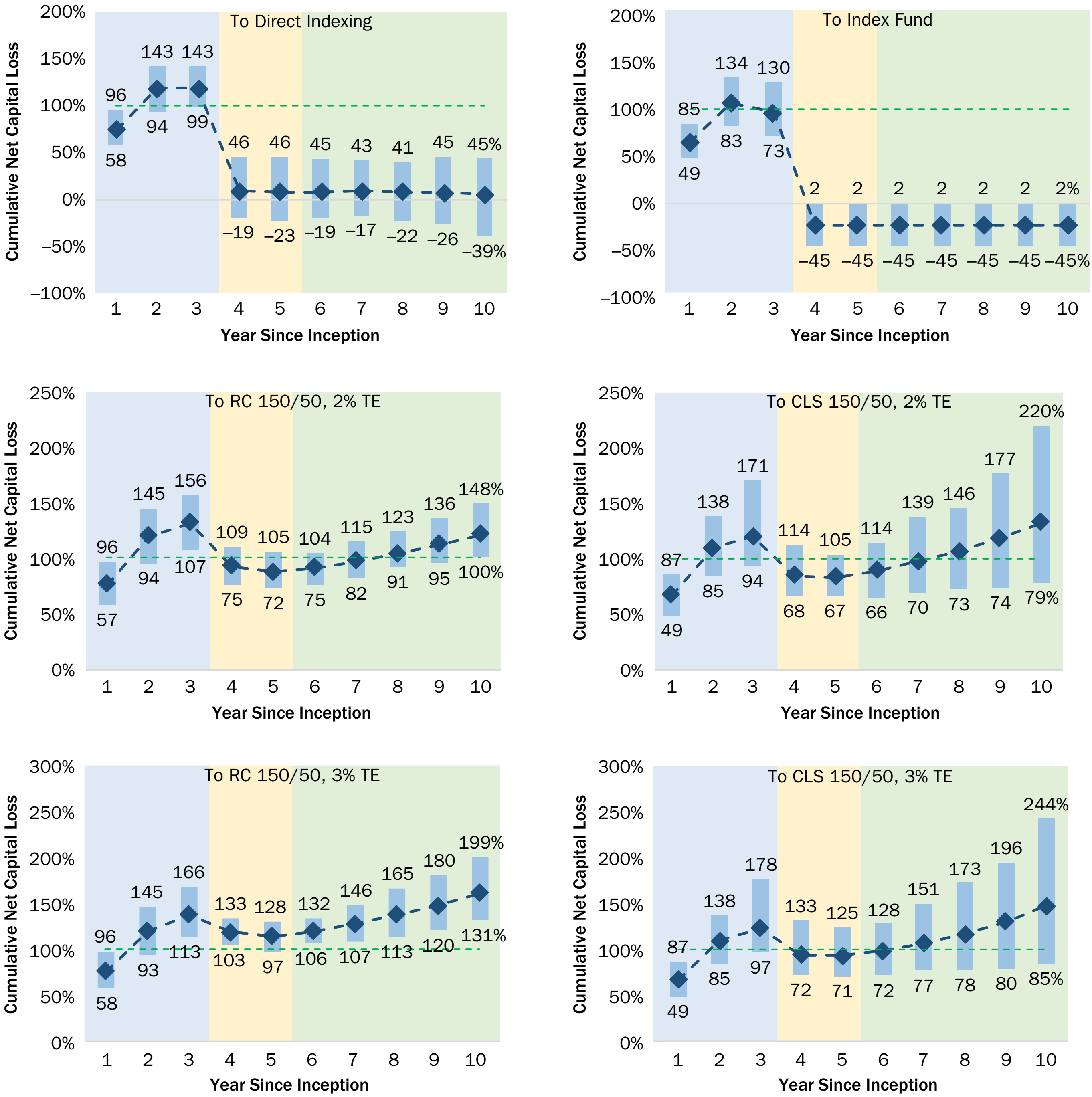
benchmark index portfolio results in realized gains or losses, which in turn lead to *excess* losses or gains for the index fund.

EXHIBIT 8

Scheduled De-Risking of 250/150 Strategies

Panel A: Relaxed-Constraint (RC)

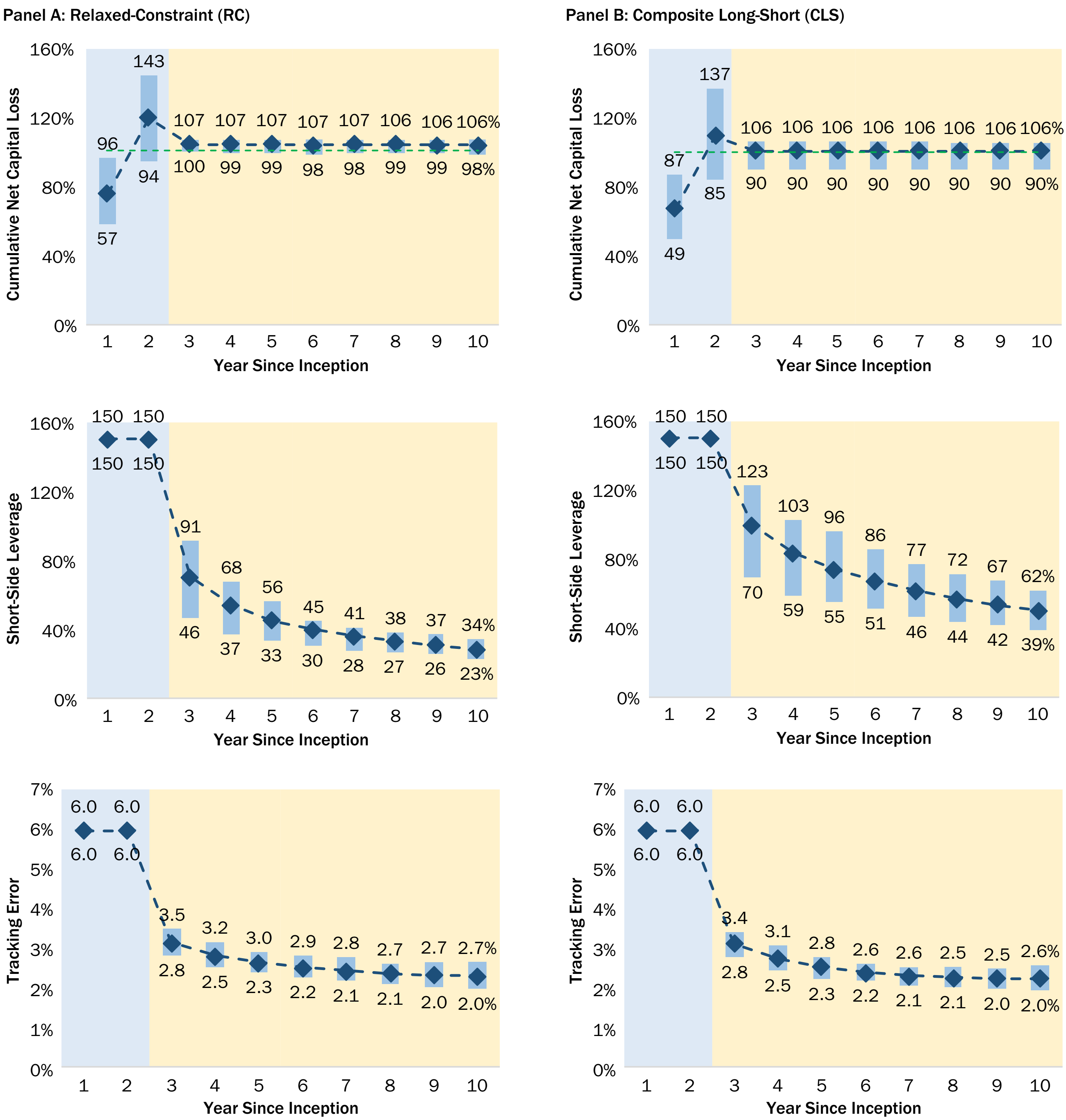
Panel B: Composite Long-Short (CLS)



NOTE: The chart shows the mean and the 10th and 90th percentile.

EXHIBIT 9

Optimized De-Risking of 250/150 Strategies



NOTE: The chart shows the mean and the 10th and 90th percentile.

EXHIBIT 10

De-Risking of 250/150 Strategies: Performance Statistics

Panel A: Relaxed-Constraint (RC)

	To Direct Indexing		To RC 150/50, 2% TE		To RC 150/50, 3% TE		To RC Optimized	
	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10
Gross of Cost Pre-Tax Alpha								
Average	4.46%	0.16%	4.69%	1.39%	5.09%	2.21%	4.76%	1.98%
10th Percentile	0.47%	–0.43%	0.06%	–0.07%	0.63%	0.62%	1.26%	0.52%
90th Percentile	7.86%	0.97%	8.39%	2.78%	9.22%	4.44%	8.32%	4.18%
Net of Cost Pre-Tax Alpha								
Average	2.34%	0.09%	2.38%	0.52%	2.80%	1.39%	2.66%	1.37%
10th Percentile	–1.76%	–0.49%	–2.33%	–0.95%	–1.72%	–0.19%	–0.89%	–0.11%
90th Percentile	5.78%	0.90%	6.12%	1.87%	7.03%	3.56%	6.23%	3.63%
Tax Benefit								
Average	0.30%	0.02%	3.39%	0.32%	4.02%	0.21%	3.34%	–0.09%
10th Percentile	–1.58%	–0.21%	2.45%	–0.20%	3.17%	–0.29%	2.68%	–0.37%
90th Percentile	1.53%	0.25%	4.22%	0.74%	4.71%	0.72%	4.11%	0.20%
Net of Cost After-Tax Alpha								
Average	2.64%	0.11%	5.77%	0.85%	6.82%	1.60%	6.00%	1.28%
10th Percentile	–1.12%	–0.47%	1.68%	–0.65%	2.70%	–0.15%	1.83%	–0.40%
90th Percentile	5.89%	0.78%	9.38%	2.28%	10.65%	4.18%	9.44%	3.54%
Realized Tracking Error								
Average	5.90%	1.05%	6.01%	2.38%	6.25%	3.51%	5.74%	2.85%
10th Percentile	4.40%	0.93%	4.44%	2.10%	4.54%	3.09%	4.41%	2.46%
90th Percentile	7.67%	1.17%	7.75%	2.67%	7.99%	3.85%	7.35%	3.27%
Costs								
Average Turnover	1015%	64%	1077%	363%	1056%	324%	981%	237%
Transaction Costs	1.01%	0.06%	1.08%	0.36%	1.06%	0.32%	0.98%	0.24%
Average Short-Side Leverage	111%	0%	123%	50%	124%	50%	112%	37%
Financing Costs	1.11%	0.00%	1.23%	0.50%	1.24%	0.50%	1.12%	0.37%

Panel B: Composite Long-Short (CLS)

	To Index Fund		To CLS 150/50, 2% TE		To CLS 150/50, 3% TE		To CLS Optimized	
	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10
Gross of Cost Pre-Tax Alpha								
Average	4.31%	0.00%	4.89%	1.26%	5.30%	1.94%	4.65%	1.61%
10th Percentile	–0.23%	0.00%	0.53%	0.34%	0.73%	0.41%	0.61%	0.29%
90th Percentile	7.83%	0.00%	8.41%	3.08%	9.05%	5.04%	7.76%	3.48%
Net of Cost Pre-Tax Alpha								
Average	2.34%	0.00%	2.75%	0.52%	3.15%	1.20%	2.47%	0.68%
10th Percentile	–2.22%	0.00%	–1.68%	–0.39%	–1.32%	–0.34%	–1.67%	–0.78%
90th Percentile	5.98%	0.00%	6.25%	2.27%	6.85%	4.22%	5.66%	2.69%
Tax Benefit								
Average	–1.05%	–0.01%	2.98%	0.33%	3.15%	0.20%	3.51%	0.17%
10th Percentile	–5.01%	–0.34%	1.82%	–0.39%	2.09%	–0.50%	2.84%	–0.01%
90th Percentile	1.07%	0.28%	3.96%	1.08%	4.14%	0.95%	4.14%	0.35%

(continued)

EXHIBIT 10

De-Risking of 250/150 Strategies: Performance Statistics *(continued)*

	To Index Fund		To CLS 150/50, 2% TE		To CLS 150/50, 3% TE		To CLS Optimized	
	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10	Years 1 to 4	Years 5 to 10
Net of Cost After-Tax Alpha								
Average	1.30%	–0.01%	5.73%	0.86%	6.30%	1.40%	5.98%	0.85%
10th Percentile	–4.30%	–0.34%	2.06%	–0.54%	2.53%	–0.60%	2.19%	–0.86%
90th Percentile	5.13%	0.28%	9.68%	3.19%	10.49%	5.07%	8.81%	2.85%
Realized Tracking Error								
Average	5.58%	0.00%	5.90%	2.23%	6.08%	3.29%	5.67%	2.84%
10th Percentile	4.27%	0.00%	4.31%	1.97%	4.62%	2.89%	4.23%	2.43%
90th Percentile	7.11%	0.00%	7.29%	2.53%	7.46%	3.82%	7.20%	3.42%
Costs								
Average Turnover	857%	2%	909%	238%	902%	243%	911%	300%
Transaction Costs	0.86%	0.00%	0.91%	0.24%	0.90%	0.24%	0.91%	0.30%
Average Short-Side Leverage	111%	0%	123%	50%	124%	50%	126%	63%
Financing Costs	1.11%	0.00%	1.23%	0.50%	1.24%	0.50%	1.26%	0.63%

TAX-AWARE STRATEGIES: RISKS AND COSTS

Tax-aware strategies can be a valuable tax management tool. Crucially, as we have seen in this article, tax benefits derived from these strategies increase with the magnitude of deviation from a passive benchmark measured by a greater leverage and a higher TE. As a result, when evaluating tax-aware strategies, investors should take into consideration not only their tax benefits but also their risks and costs.

First, even direct indexing strategies need to deviate from benchmark weights to achieve their loss-harvesting benefits.[12] These deviations carry the risk of underperformance relative to the benchmark. In addition, compared to a passive benchmark, direct indexing strategies will have higher transaction costs, which result from the loss-harvesting trades, and possibly higher management fees due to added complexity. Leverage and shorting can further increase risks and costs of tax-aware strategies. TE of strategies that utilize leverage and shorting might be higher than that of direct indexing, leading to a higher risk of underperforming the benchmark. These strategies might have greater opportunities to realize capital losses than direct indexing but, as a result, will have a greater turnover and, thus, higher transactions costs. Furthermore, compared to a direct indexing portfolio or a passive index fund, leverage leads to financing costs and higher complexity of managing long-short strategies results in higher management fees.

Second, tax benefits of the tax-aware strategies are uncertain, and, as a result, an investor can experience lower than expected tax benefits. While, as we have seen, this is particularly true for direct indexing strategies, tax-aware long-short strategies also realize variable levels of tax benefits.

Finally, an investor might decide to reduce exposure to the manager due to losing confidence in the manager's ability to produce attractive after-tax returns.

[12] In the words of Stein and Narasimhan (1999, emphasis in original), these strategies are "*active with respect to tax management* by seeking an after-tax excess return that derives from the management of taxes rather than from security selection."

In such cases, the investor could be forced to realize a substantial built-in gain thereby undoing part of the previously accrued tax benefits.

In this article, we tried to tackle these risks and costs in several ways. First, to address the risk of underperforming the benchmark, we showed the range of both pre-tax and tax outcomes from historical simulations. Second, we estimated the levels of transaction and financing costs and included them in our calculations of after-tax returns. Third, by modeling dynamic strategies, we showed the potential for and the limits of tax-efficient de-risking. Finally, we penalized current unrealized gains with an effective tax rate of 10% to account for the present value of expected cost of recognizing them in the future.

Even with these caveats, our results show that significant benefits can be derived from tax-aware strategies, especially, long-short tax-aware strategies. Nevertheless, investors considering such strategies should carefully evaluate their confidence in the manager's stock-picking ability, expected value added of the strategies to their overall investment program, and their tolerance toward risks and costs of these strategies.

CONCLUSION

The appeal of direct indexing for taxable investors has led to its widespread adoption by the wealth management industry. One shortcoming of direct indexing strategies is that they are quite limited in the amount of net capital losses that they can realize. In this article, we show that factor-based long-short strategies can achieve substantially higher cumulative net capital loss realizations than traditional direct indexing strategies.

Moreover, compared to long-only strategies, long-short strategies are characterized by a higher implementation quality of quantitative alpha models (see Clarke et al. 2004, and Jacobs and Levy 2006) and by a more efficient tradeoff between alpha and tax awareness (see Sialm and Sosner 2018). Indeed, in line with this prior research, we find that our simulated factor-based long-short strategies deliver high levels of pre-tax alpha. As a result, by going *beyond direct indexing*, investors can simultaneously enjoy a substantial pre-tax alpha and access a significantly higher loss-realization potential.

Importantly, leverage and tracking error of tax-aware long-short strategies can be modulated over time without realizing net capital gains. This gives rise to an approach that we call *dynamic direct long-short investing*. Under this approach, an investor does not need to be permanently stuck with an undesirable level of alpha-model exposure. Rather this exposure can be scaled up or down by varying the levels of leverage and tracking error.

Until recently, investment solutions that incorporate leverage and shorting have been only available for ultra-affluent investors with institutional-size accounts. However, as the private wealth management continues to evolve and innovate, such solutions might be just around the corner even for accounts as small as a few million dollars. As such, investors and their advisors should begin to familiarize themselves with both the capabilities offered by tax-aware long-short investing and the potential risks and costs that accompany it.

ACKNOWLEDGMENTS

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APPENDIX A

EMPIRICAL METHODOLOGY

Alpha Model

We model quantitative strategies that combine value, momentum, and quality signals. Value is measured by book-to-market ratio (see, e.g., Fama and French 1992). Following Asness and Frazzini (2013) and Asness et al. (2015), we scale the book value of a company by its most recent market capitalization. Twelve-month momentum effects in equities have been first documented in Jegadeesh and Titman (1993) and Asness (1994). Following Asness (1994) and Asness et al. (2015), momentum is measured as the total return over the preceding 12 months, excluding the most recent month. Quality is measured using gross profitability, more specifically, gross profits over assets (see Novy-Marx 2013). Also, Asness et al. (2019) use gross profits over assets as one of their many measures of quality. Value, momentum, and quality signals are equal-weighted as we explain shortly below.

Every month, we construct a factor-based model portfolio $\tilde{\nu}$ as follows. We first convert the value, momentum, and quality raw signals into three market-neutral factor portfolios using the following sequence of steps: First, for each signal, stocks are ranked within each industry according to their signal scores. Second, these within-industry ranks are demeaned (by subtracting the average rank within an industry) and standardized (by dividing by the standard deviation of the ranks within an industry) to create an industry-neutral portfolio. Finally, market neutrality of the portfolio is ensured by regressing out the market beta using an OLS regression.[13] The market-neutral factor portfolios are denoted by ν_{VAL}, ν_{MOM}, and ν_{QUAL}, respectively.

The value, momentum, and quality factor portfolios are then scaled by their respective forecast volatility:

$$\tilde{\nu}_{VAL} = \frac{1}{\sigma_{VAL}}\nu_{VAL},\ \tilde{\nu}_{MOM} = \frac{1}{\sigma_{MOM}}\nu_{MOM},\ \tilde{\nu}_{QAUL} = \frac{1}{\sigma_{QUAL}}\nu_{QUAL}$$

To compute forecast volatility, we utilize a covariance matrix produced by the MSCI Barra USE3L risk model. We use the model covariance matrix, Σ, lagged by one month,[14] to compute σ_{VAL}, σ_{MOM}, and σ_{QUAL} as follows:

$$\sigma_{VAL} = \sqrt{\nu_{VAL}\Sigma\nu_{VAL}},\ \sigma_{MOM} = \sqrt{\nu_{MOM}\Sigma\nu_{MOM}},\ \sigma_{QUAL} = \sqrt{\nu_{QUAL}\Sigma\nu_{QUAL}}$$

This yields three factor portfolios with unit predicted volatility.[15]

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[13] For example, let $\boldsymbol{w}$ be the vector of portfolio weights before beta-adjustment and β be the vector of market betas for every stock in the portfolio. A vector of residuals ν from the regression $\boldsymbol{w} = a\beta + \nu$, by construction, will have a beta of 0 and, thus, be a market-neutral portfolio. We found this beta-adjustment step to be important because, while our portfolios sorted on either value or momentum do not exhibit a systematic positive or negative beta, our quality-sorted portfolio is characterized by a high and persistently negative beta. Therefore, without beta-adjustment, the former two portfolios are approximately market-neutral, while the latter is systematically short the market.

[14] Lagging the covariance matrix by a month, ensures that the covariance matrix used for the volatility forecasts is available at the time of factor portfolio formation. This is because it takes a few days after the month-end for the risk model to be released.

[15] This is easy to see. For each factor *i*: $\sqrt{\tilde{\nu}_i\,\Sigma\tilde{\nu}_i} = \sqrt{\frac{1}{\sigma_i}\nu_i \sum \frac{1}{\sigma_i}\nu_i} = \frac{1}{\sigma_i}\sigma_i = 1.$

The model portfolio is constructed as an equal-weighted average of the factor portfolios:

$$\nu = \frac{1}{3}\tilde{\nu}_{VAL} + \frac{1}{3}\tilde{\nu}_{MOM} + \frac{1}{3}\tilde{\nu}_{QUAL}$$

Since all three factor portfolios have the same unit predicted volatility, the model portfolio effectively allocates equal risk to each of the three factors. Similar to factor portfolios, the model portfolio is scaled by its forecast volatility:

$$\tilde{\nu} = \frac{1}{\sqrt{\nu\ \Sigma\nu}}\nu$$

Following the methodology originally proposed in Jones et al. (2007), every month we convert the model portfolio ν into a vector of stock-level alphas by multiplying it by the stock-level covariance matrix Σ (from MSCI Barra's USE3L risk model), lagged by one month:

$$\alpha = \Sigma\tilde{\nu}$$

Scaling of the alpha model portfolio by its volatility $\sqrt{\nu\ \Sigma\nu}$ leads to a convenient result that the predicted information ratio of an active portfolio is also its predicted correlation with the model portfolio, which in turn can be viewed as a measure of implementation efficiency of the active portfolio (see Israel et al. 2019, Appendix B, for further discussion):

$$IR_w = \frac{\boldsymbol{w}\ \alpha}{\sqrt{\boldsymbol{w}\ \Sigma\boldsymbol{w}}} = \frac{\boldsymbol{w}\ \Sigma\tilde{\nu}}{\sqrt{\boldsymbol{w}\ \Sigma\boldsymbol{w}}} = \frac{\boldsymbol{w}\ \Sigma\nu}{\sqrt{\boldsymbol{w}\ \Sigma\boldsymbol{w}}\sqrt{\nu\ \Sigma\nu}}$$

MANAGEMENT FEE, TRANSACTION, LEVERAGE, AND TAX COST ASSUMPTIONS

All the results in this article are reported gross of management fees.

For transaction costs in portfolio optimization, we use a simple model informed by the research in Almgren et al. (2005). Transaction costs per dollar traded in basis points are modeled as

$$c_{i,t} = 5 + 0.075 \times VIX_t + 2.5 \times srisk_{i,t} \times \sqrt{\frac{T\$_{i,t}}{DTV\$_{i,t}}}$$

where VIX_t is the most recent VIX index level known on the date of the trade, $srisk_{i,t}$ is the specific volatility of stock i as estimated by MSCI Barra USE3L model lagged by one month in percentage points (for example, for 50% volatility, the value substituted into the model will be 50), and $T\$_{i,t}$ and $DTV\$_{i,t}$ are the dollar trade size and dollar daily trading volume of stock i, respectively. This model yields an average cost of approximately 10 bps per dollar traded, which is a reasonable estimate for the US large capitalization stocks and is similar to the 12-bps cost used in Goldberg et al. (2022b).

For the cost of financing the long-short leverage, following Sorensen et al. (2007) and Sialm and Sosner (2018), we use a conservative assumption of 100 bps per unit of one-sided leverage per year. For example, for a 150/50 relaxed-constraint portfolio this implies an annual cost of 50 bps (that is, 0.5 times 100 bps).

The tax cost of rebalancing a portfolio is defined as

$$T = t_{LT}g_{LT} + t_{ST}g_{ST}$$

where t_{LT} and t_{ST} are the long-term and short-term capital gains tax rates, respectively, and g_{LT} and g_{ST} are the net long-term and short-term realized capital gains aggregated across all the traded individual tax lots, respectively. Tax cost defined this way rewards the realization of losses and penalizes the realization of gains. Moreover, due to the difference in tax rates, the realization of net short-term losses is rewarded more than the realization of net long-term losses, while the realization of net short-term gains is penalized more than that of net long-term gains. Taxes on dividends and deductions associated with in-lieu dividends on short positions are not incorporated into the optimization problem.[16] However, they are included in the reported tax benefits and after-tax returns. As a lot-relief method, we use the HIFO (highest in, first out).

Portfolio Construction

Base Case. For the direct indexing strategy, the optimization problem is defined as follows:

$$\max_{w_1 \dots w_N} -T - C$$

$$s.t.$$

$$\sum_i \sum_j w_i w_j \sigma_{ij} \leq TE^2$$

$$\sum_i (b_i + w_i) = 1$$

$$0.98 \leq \sum_i (b_i + w_i)\beta_i \leq 1.02$$

where w_i corresponds to the active portfolio weight of security i, T is the tax cost of rebalancing the portfolio in the current period, C is the aggregate transaction cost, $\sigma_{i,j}$ is the covariance between the returns of securities i and j derived from MSCI Barra's risk model, TE is the target tracking error of 1% annually, b_i is the benchmark weight of security i, and β_i corresponds to the beta of security i with respect to the Russell 1000 index predicted by MSCI Barra USE3L risk model. Both the covariance and the beta estimates are point-in-time forward-looking estimates. In addition, we lag these estimates by one month to ensure that the risk model data is released before the portfolio construction date.

The optimization problem for the relaxed-constraint strategies is defined as follows:

$$\max_{w_1 \dots w_N} \sum_i w_i \alpha_i - T - C$$

$$s.t.$$

$$\sum_i \sum_j w_i w_j \sigma_{ij} \leq TE^2$$

[16] Israel et al. (2019) show that taking taxation of dividends into account reduces implementation efficiency of quantitative multi-style strategies, thereby lowering their expected pre-tax returns, and also detracts from the ability to manage the realization of capital gains and losses.

$$\sum_i (b_i + w_i) = 1$$

$$\sum_i |b_i + w_i| = 1 + 2L$$

$$0.98 \le \sum_i (b_i + w_i)\beta_i \le 1.02$$

where α_i corresponds to the alpha of security i as explained above and L is the target leverage, that is, L is 0.5, 1.0, and 1.5 for the 150/50, 200/100, and 250/150 strategies, respectively. The remaining notation is as defined above. The tracking error TE is 2%, 4%, and 6% for the 150/50, 200/100, and 250/150 strategies, respectively.

The optimization problem for the long-short component of the composite strategies is defined as follows:

$$\max_{w_1 \ldots w_N} \sum_i w_i \alpha_i - T - C$$

$$s.t.$$

$$\sum_i \sum_j w_i w_j \sigma_{ij} \le R^2$$

$$\sum_i w_i = 0$$

$$\sum_i |w_i| = 2L$$

$$-0.02 \le \sum_i w_i \beta_i \le 0.02$$

where R is the target risk level of 2%, 4%, and 6% for the 50/50, 100/100, and 150/150 long-short component, respectively. The rebalancing also ensures that the relative weights of the index fund and long-short components are such that the beta of the overall strategy remains close to 1. Specifically, if since the last rebalance the long-short component realizes a positive return, its relative weight is reduced, and vice versa.

Scheduled De-Risking

For the sake of brevity, we only show how scheduled de-risking is defined for the relaxed-constraint strategies. De-risking of the composite long-short strategies follows the same process with a small modification in the case of de-risking to zero leverage: Since for the composite strategy at the end of the de-risking period all the long-short portfolio positions are fully liquidated (and the investor only holds an index fund) the target TE is 0% rather than 1% as in de-risking to direct indexing. We introduce the following parameters into the optimization problem: δ is the alpha attractiveness, γ is the tax aversion, τ is the TE scaling, and λ is the leverage scaling.

$$\max_{w_1 \ldots w_N} \delta \sum_i w_i \alpha_i - \gamma T - C$$

$$s.t.$$

$$\sum_i \sum_j w_i w_j \sigma_{ij} \leq (\tau TE)^2$$

$$\sum_i (b_i + w_i) = 1$$

$$\sum_i \left| b_i + w_i \right| = 1 + 2\lambda L$$

$$0.98 \leq \sum_i (b_i + w_i)\beta_i \leq 1.02$$

For example, for the 200/100 4% TE strategy, in transition to direct indexing, δ is linearly reduced from 1 to 0, τ is linearly reduced from 1 to ¼, λ is linearly reduced from 1 to 0, and γ changes from 1 to 0.5 at the end of the de-risking period. For the same initial strategy, in transition to 150/50 2% TE, δ remains at 1 for the entire simulation period, τ is linearly reduced from 1 to ½, λ is linearly reduced from 1 to ½, and γ changes from 1 to 0.5 at the end of the de-risking period.

Optimized De-Risking

Again, for the sake of brevity, we only show how optimized de-risking is defined for the relaxed-constraint strategies. De-risking of the composite long-short strategies follows the same process. We introduce the following parameters into the optimization problem: ρ is the active variance aversion and θ is leverage scaling.

$$\max_{w_1 \ldots w_N} \sum_i w_i \alpha_i - T - C - \rho TE^2 - \theta L$$

$$s.t.$$

$$\sum_i (b_i + w_i) = 1$$

$$0.98 \leq \sum_i (b_i + w_i)\beta_i \leq 1.02$$

$$\sum_{s=h}^{t} (g_{ST,s} + g_{LT,s}) \leq 0$$

The penalty coefficients ρ and θ are calibrated to produce a reasonable tradeoff between pre-tax returns, TE reduction, and leverage reduction. The last constraint ensures that no net gains are realized in the process of de-risking: A cumulative net gain from the start of the de-risking process on date h until the current rebalance on date t is constrained to be less or equal zero, that is, no cumulative net gain is permitted.

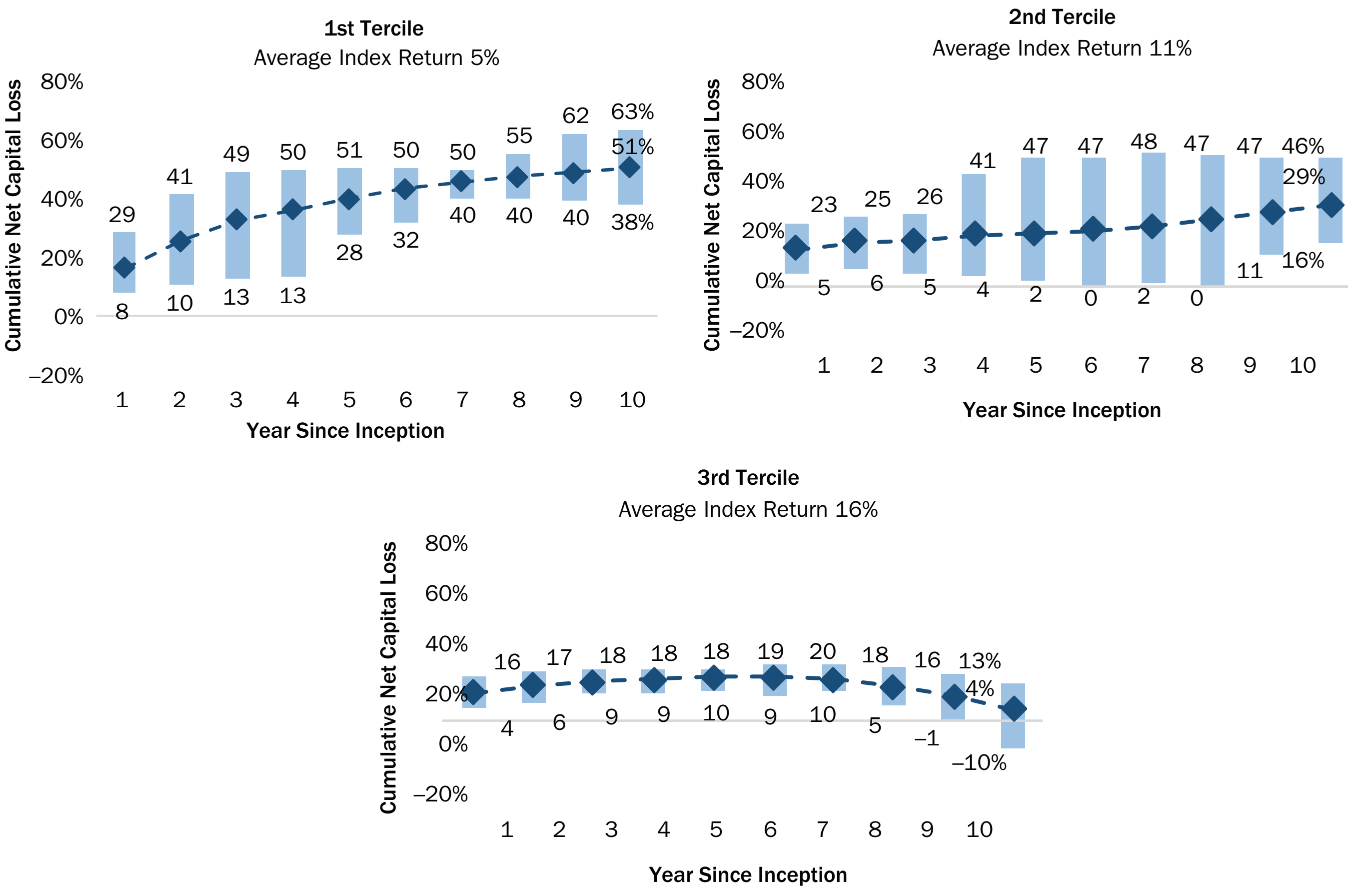
APPENDIX B

CNCL OF DIRECT INDEXING STRATEGY BY INDEX RETURN

Exhibit B1 shows the distribution of CNCL of the direct indexing strategy by the Russell 1000 index return tercile. The 27 historical simulations are divided into three equally sized groups containing 9 distinct simulations each. The average annualized index return is 5%, 11%, and 16% for the bottom, middle, and top index return terciles, respectively.

EXHIBIT B1

Direct Indexing Strategy: CNCL Realization by Index Return Tercile



NOTE: The chart shows the mean and the 10th and 90th percentile.

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