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**The Tax Benefits of Direct Indexing:
Not a One-Size-Fits-All Formula**

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KEY FINDINGS

- On average, across different market environments, the tax benefits of direct-indexing strategies decay rather quickly over time.
- Without additional capital contributions, only investors with systematic short-term capital gains from other sources can enjoy the long-run tax benefits of direct-indexing strategies. For investors with only long-term capital gains from other sources, the tax benefit is reduced to zero after approximately five years since inception.
- Investors, even those with long-term capital gains, can significantly increase the tax benefits they derive from direct-indexing strategies by regular capital contributions to the strategy or by combining the strategy with a charitable giving program.

ABSTRACT

Direct-indexing strategies realize tax benefits by harvesting losses on individual stock positions. Some investors might benefit from this powerful tool for growing after-tax wealth significantly more than others. An important determinant of the tax benefits of direct-indexing strategies is the tax rates applicable to gains from other investments. We argue that high-net-worth investors with allocations to hedge funds and derivatives are the most likely investors to have systematic short-term capital gains and, therefore, derive the highest tax benefits from direct-indexing strategies. We use a long history of U.S. stock returns to estimate the level of tax benefits offered by direct-indexing strategies under different tax rate assumptions. We show that investors, even those without short-term capital gains in their portfolios, can significantly increase the tax benefits of direct indexing by regular capital contributions and charitable giving of appreciated stocks. A character-deferral decomposition of the tax benefits helps explain what drives this result.

With lower expected investment returns and a possibility of higher tax rates on the horizon, tax-loss-harvesting strategies that (at least in theory) allow investors to keep more of their pretax return are rapidly growing in popularity. A quick internet search shows that most private wealth investment platforms now offer loss-harvesting strategies in one form or another. It is, therefore, pertinent and timely for investors and their advisors to deepen their understanding of such strategies.

An investment approach that uses individual stocks to track performance of a stock index is often referred to as direct indexing. A direct-indexing strategy provides an investor with such advantages as customization of the index it tracks and

opportunity for tax-loss harvesting.¹ We focus on the latter advantage of direct indexing and construct a direct-indexing strategy as a passive long-only strategy with a loss-harvesting overlay.

The value added by loss harvesting is often referred to as tax alpha or tax benefit. In this study, we explore which investors could benefit the most from loss harvesting offered by direct indexing, in the short run and in the long run, and how much tax benefits of direct indexing, especially the long-run tax benefits, can be increased by capital contributions or by combining a direct-indexing mandate with a charitable giving program.

RELATIONSHIP TO PRIOR LITERATURE

We make the following contributions to the literature. First, past studies, predominantly using Monte-Carlo-simulated returns, have shown that the effectiveness of loss-harvesting strategies increases with stock-specific volatility² and declines with the level of market return³ and time since inception.⁴ We use almost half a century of historical returns to test these effects with real data. We run 45 strategy simulations, starting in January of every year from 1975 to 2019 and ending in December of 2019, that allow us to model tax benefits as a function of time since inception and market environment variables—cross-sectional dispersion of stock returns and the level of market return. By using historical stock returns, we continue the recent line of research that uses real market data to evaluate the tax benefits of loss-harvesting strategies.⁵

Second, compared to prior literature, we introduce several methodological changes to make our study as practically applicable as possible. First, in contrast to Chaudhuri et al. (2020) and similar to Israel and Moskowitz (2012), we use optimized portfolio construction where tax benefits are maximized subject to a tracking error constraint.⁶ Moreover, similar to Goldberg et al. (2019), we extend the Israel and Moskowitz optimization approach to include a transaction cost penalty. This way, loss harvesting is limited not only by the tracking error constraint but also by the trade-off between the tax benefits and the transaction costs of loss-harvesting trades. Maximization of tax benefits, which accounts for tracking error and transaction costs, is typical of financial industry implementation and thus our results accurately simulate an experience of an actual investor in a direct-indexing strategy.

Third, several studies have shown that capital contributions increase the level of tax benefit of loss-harvesting strategies; capital redemptions decrease it.⁷ We add another technique to such capital flow experiments—charitable giving of appreciated positions. Each month, we remove for charitable giving the most appreciated long-term positions totaling 1% of the strategy portfolio value and substitute them with 1% of

¹ See, for example, Lake (2019).

² See Stein and Narasimhan (1999), Berkin and Ye (2003), and Khang et al. (2021).

³ See Stein and Narasimhan (1999), Berkin and Ye (2003), Bouchev et al. (2015), Bouchev et al. (2016), and Khang et al. (2021).

⁴ See Stein and Narasimhan (1999), Arnott et al. (2001), Berkin and Ye (2003), Stein et al. (2008), Bouchev et al. (2015).

⁵ See Israel and Moskowitz (2012), Goldberg et al. (2019), Chaudhuri et al. (2020), and Khang et al. (2021).

⁶ Goldberg et al. (2019) also use optimized portfolio construction but introduce tracking error as a penalty, rather than a constraint.

⁷ See Berkin and Ye (2003), Chaudhuri et al. (2020), and Khang et al. (2021). Tax externalities resulting from inflows and outflows were initially analyzed in Dickson et al. (2000), albeit these authors have not considered loss-harvesting strategies.

newly contributed capital.⁸ To our knowledge, many investors utilize variants of this approach, and not considering it might significantly underestimate the tax benefits offered by direct indexing in practice.

Fourth, higher capital gains tax rates have been shown to increase the benefits of loss harvesting.⁹ Furthermore, in a recent article, Khang et al. (2021) show that, in addition to tax rates, the ability of an investor to utilize harvested losses as tax offsets determines the level of the investor's tax benefits. In fact, they find that applicable tax rates and the ability to utilize losses collectively drive about 60% of all the variation in tax benefits. We use a character-deferral decomposition proposed in Sosner et al. (2019) to explain the sources of tax benefits under different tax rate and loss utilization assumptions.

Finally, some studies report post-liquidation tax benefits.¹⁰ Post-liquidation tax benefits might understate the tax benefits experienced by investors in practice. Investors recognize that advantages of direct-indexing strategies accrue over long horizons. In fact, many investors are reluctant to redeem—and realize substantial built-in gains—even after tax benefits of the strategy are substantially reduced over time. Therefore, rather than showing post-liquidation tax benefits, we apply an effective tax rate to unrealized capital gains. The effective tax rate aims to represent the present value of discounted expected tax costs of realizing built-in gains in the future. The value of the effective tax rate is informed by the formula originally proposed in Poterba (1999).¹¹

WHICH INVESTORS ARE MOST LIKELY TO HAVE SHORT-TERM GAINS?

There are three main sources of short-term capital gains: liquidating assets with a holding period shorter than one year; receiving short-term capital gain allocations from pass-through investment vehicles, for example, hedge funds; and holding derivative contracts whose profits are taxed as 60% long-term capital gain and 40% short-term capital gain on a marked-to-market basis—the latter are known as Section 1256 contracts.¹² Importantly, regulated investment companies, such as mutual funds and ETFs, distribute their realized short-term capital gains not as short-term capital gains but, rather, as ordinary dividends reported in the line “total ordinary dividends” of the form 1099-DIV.

⁸Under the IRC Section 170, charitable contribution of a long-term position, that is, a position held for longer than 12 months, provides a deduction at fair market value, whereas charitable contribution of a short-term position only provides a deduction at the smaller of cost and fair market value—any deduction for the built-in capital gain is disallowed.

⁹See Arnott et al. (2001), Berkin and Ye (2003), and Chaudhuri et al. (2020).

¹⁰See Arnott et al. (2001), Berkin and Ye (2003), Goldberg et al. (2019), and Chaudhuri et al. (2020).

¹¹Whereas, as we show in the following, estimation of effective tax rate on unrealized gains relies on assumptions about probabilities of future events, calculation of post-liquidation tax benefits also requires making assumptions about a holding period and an absence of any offsetting losses at the time of liquidation.

¹²Additionally, short-term capital gains might arise in various complex scenarios. These include liquidating short positions in physical assets, such as stocks, irrespective of the length of the holding period, elimination of the holding period due to tax straddles, election to treat profits on foreign currency contracts, which are by default ordinary, as 60% long-term capital gain and 40% short-term capital gain, etc.

In analysis not reported here for the sake of brevity, we examined the data from the IRS¹³ and the Survey of Consumer Finances (SCF)¹⁴ to understand what happens with the three aforementioned sources of short-term capital gains in practice.

First, the SCF data show that, except for the most active traders within the highest-net-worth-percentiles (possibly advised by professional money managers), trading activity in stocks is too infrequent to generate significant short-term gains: A vast majority of investors seem to prudently stay away from high-volume trading activity. Indeed, high-volume trading is not an approach we recommend to nonprofessional small investors. In a seminal article, Barber and Odean (2000) write: “Individual investors who hold common stocks directly pay a tremendous performance penalty for active trading. Of 66,465 households with accounts at a large discount broker during 1991 to 1996, those that trade most earn an annual return of 11.4 percent, while the market returns 17.9 percent.”¹⁵ The IRS data also show no evidence that trading stocks and mutual funds generates short-term gains. In fact, the opposite is true—such trading results in short-term losses.

Second, the IRS data show that short-term gains predominantly come from “pass-through gains” (for example, gains allocated by hedge funds) and “futures contracts.” Not surprisingly, the SCF data show that hedge funds are held only by investors in the highest-net-worth percentiles. For these investors, hedge funds constitute a small but non-negligible allocation that has also increased in the past decade. Generally, to invest in pass-through entities an investor must be a “qualified purchaser” under the Investment Company Act of 1940, which for an individual means no less than \$5 million in investments. As for futures contracts, we conjecture that systematic profits from trading futures are also more likely to occur in portfolios of high-net-worth investors advised by professional investment managers than in portfolios of retail investors.

In sum, we expect systematic short-term capital gains to be largely limited to a subset of high-net-worth investors with allocations to complex investments such as hedge funds and derivatives. Thus, we argue that, whereas these particular high-net-worth investors can use short-term losses harvested by direct-indexing strategies to offset *short-term gains*, all other investors—high-net-worth investors without allocations to hedge funds or derivatives, as well as retail investors—are more likely to end up using these short-term losses to offset *long-term gains*.¹⁶

¹³The IRS data are from the IRS webpage “SOI Tax Stats—Sales of Capital Assets Reported on Individual Tax Returns” available at www.irs.gov. We used the set of files titled “Short-term and Long-term Capital Gains and Losses. Classified by: Asset Type,” which is the very first set of files on the aforementioned “SOI Tax Stats” webpage.

¹⁴The SCF is a triennial cross-sectional survey of US families sponsored by the Federal Reserve Board and the Department of the Treasury. Since 1992, the data have been collected by the NORC at the University of Chicago. As of this writing, the most recent available survey was conducted in 2019. The Federal Reserve website at www.federalreserve.gov provides a detailed description of the survey methods and procedures. To access the SCF data, we used Survey Documentation and Analysis (SDA) query tools created by the University of California, Berkeley, and available through the SDA website at sda.berkeley.edu. The SDA also provides a Codebook for the fields collected by the SCF and a Net Worth Flowchart that allowed us to link all the assets and liabilities of a family into a coherent picture of net worth.

¹⁵In a follow up chapter in the *Handbook of Economics and Finance*, Barber and Odean (2013) survey a large literature that demonstrates that a do-it-yourself approach to investing is detrimental for individual investors who face significant information asymmetry and transaction costs and are influenced by an array of behavioral biases.

¹⁶In addition, in contrast to high-net-worth investors, for whom tax-deferred accounts represent only a small portion of their investment portfolio, retail investors can shield a significant portion of their tax-inefficient investments in tax-deferred accounts. In fact, retail investors should optimally locate tax-inefficient assets in tax-deferred accounts and tax-efficient assets in taxable accounts (see, for example, Shoven and Sialm 2003, and Dammon et al. 2004).

Before we proceed, for the sake of completeness, we point out that high-net-worth investors might also have access to loss-harvesting strategies that utilize leverage and shorting (see, for example, Sialm and Sosner 2018). Prior research shows that such strategies might be able to realize higher tax benefits than long-only direct-indexing strategies.¹⁷ Sosner et al. (2019) caution that these higher tax benefits come with a number of caveats: a potential risk of underperformance relative to a benchmark, additional financing costs, and a greater difficulty to access through a separately managed account.

DECOMPOSING TAX BENEFITS REALIZED BY LOSS HARVESTING

Sosner et al. (2019) show that tax benefits resulting from loss harvesting can be decomposed into three components: the current period character component, the current period deferral component, and the expected tax liability of unrealized gains. In the following, we use this decomposition to analyze the tax benefits of direct indexing. Because we do not break any new ground on the decomposition, we relegate its description to Appendix A.

SIMULATION METHODOLOGY

Direct-Indexing Strategy Simulation

Our methodology closely follows construction of the *tax-managed passive-indexed* strategy in Sosner et al. (2019), hereafter SKP.¹⁸ Similar to SKP, we rebalance the strategy at a monthly frequency, implement tax-aware rebalancing, and limit the tracking error to the benchmark at 1%.¹⁹

We also have several important differences. First, SKP use the Russell 1000 Index as a benchmark and perform their strategy simulation over a 30-year period from 1988 to 2017. We use the S&P 500 Index, which allowed us to extend the simulation period back to 1975.²⁰ We also extend the simulation forward to the end of 2019. As a result, our 45-year sample period is 50% longer than that of SKP.

Second, whereas SKP only simulate one history of pretax returns and tax benefits over their full 30-year sample period, we simulate 45 such histories, starting in January of each year from 1975 to 2019, all ending in December of 2019. This allows us to address path-dependence in the strategy's tax benefits. We average the tax benefits across these simulations using regression analysis, as explained below.

Third, SKP only consider a scenario where there are no additional contributions or redemptions of capital after the initial investment. We model tax benefits under two additional scenarios: monthly inflows and monthly charitable giving. In the monthly inflow scenario, every month the investor contributes 1% of the total strategy portfolio

¹⁷ See Berkin and Luck (2010), Sosner et al. (2017, 2019), Sosner et al. (2020), and Sosner and Krasner (2021) for further discussion of tax-aware long-short strategies.

¹⁸ Such a strategy was originally described two decades earlier in Stein and Narasimhan (1999).

¹⁹ To our knowledge, for direct-indexing portfolios seeded with cash (as opposed to with appreciated stocks), like the ones modeled here, a very low tracking error of 1%, or even lower, is typical. For portfolios seeded with appreciated stock, direct indexing providers offer what is called a transition analysis, based on which an investor can select a specific trade-off between tracking error and realized transition gain. Therefore, investors reluctant to realize substantial built-in gains upon transition to a direct-indexing portfolio may choose portfolios with tracking error higher than 1%.

²⁰ In analysis not reported here for the sake of brevity, we find that the results remain qualitatively similar for other large-capitalization and all-capitalization indices.

value in cash which is immediately invested in portfolio positions. In the monthly charitable giving scenario, every month the investor removes 1% of the portfolio's most appreciated long-term capital gain positions to donate to charity (1% is measured as the value of the donated positions as a percentage of the portfolio value) and substitutes them with an equal amount of cash that is immediately invested in the portfolio.²¹

Finally, SKP use the tax rates of 20% and 35% on long-term and short-term realized capital gains, respectively, and 10% effective tax rate on unrealized gains. They also assume that short-term capital losses realized by the loss-harvesting strategy are instantly and fully utilized to offset short-term capital gains from other strategies. Here, we are comparing three alternative tax scenarios. In scenario one, we use the top bracket 2020 federal tax rates of 23.8% and 40.8% applicable to long-term and short-term capital gains, respectively, and assume, like SKP did, that all the short-term losses realized by the direct indexing strategy are instantly and fully utilized to offset short-term gains. In scenario two, we use the same 2020 tax rates, but assume that the realized short-term losses are utilized to offset only long-term gains. Scenario three is a hypothetical scenario that we view as a plausible and realistic stress test for the tax benefits of the direct indexing strategy: All the realized gains, long-term and short-term, are taxed at a uniformly high rate, and the step-up in the cost basis is disallowed. These were exactly the provisions of the proposed, and now largely defunct, Biden Tax Plan. Under this scenario, the tax rate applicable to all the gains and income is 43.4%.²² Appendix B provides further details on tax rate assumptions. We change the effective tax rate applicable to unrealized gains depending on the scenario, as explained in Appendix B. Without charitable giving, this rate is 10% under the 2020 tax regime and 25% under the Biden Tax Plan. With charitable giving, this rate is 5% under the 2020 tax regime and 10% under the Biden Tax Plan. The effective tax rate is higher under the Biden Tax Plan because capital gains are both taxed at a higher rate and are harder to avoid due to the elimination of the step-up in cost basis upon death.

We observe that the direct-indexing strategy portfolios in our simulations always hold all the stocks in the S&P 500 Index. This is not all that surprising, given the low tracking error of the strategy. However, it is worth noting that even when a given stock position is sold to harvest losses, it is sold only partially—some portion of it remains in the portfolio to help maintain the low tracking error. Appendix B provides further details on strategy simulation and tax rate assumptions.

Benchmark Index Simulation

We assume that the benchmark is a passive ETF indexed to the S&P 500 Index. We view an index ETF as an appropriate benchmark because it provides an easily accessible, low-cost, and highly tax-efficient market exposure. We further assume that the ETF distributes dividend income but does not generate any distributable capital gains. All distributed dividend income is treated as qualified dividend income taxed at either the 2020 tax rate of 23.8% or the Biden Tax Plan tax rate of 43.4%, depending on the specific scenario.

Each of the 45 strategy simulations previously described has a corresponding ETF benchmark simulation that starts on the same day as the strategy simulation. Investment in the ETF benchmark is modeled as holding or trading shares of the ETF, not the underlying stocks. For example, in the charitable giving scenario, 1% of the most appreciated ETF shares held for a period of longer than one year are gifted, not 1% of the most appreciated stocks in the ETF's portfolio.

Investment process in the shares of the ETF is always kept identical to the investment process in the direct-indexing strategy: If the strategy is simulated with

1% monthly inflow or, alternatively, 1% charitable giving, so is the ETF benchmark. Also, as in the direct-indexing strategy simulations, we apply an effective tax rate to unrealized gains imbedded in the shares of the ETF. The levels of effective tax rate across different scenarios are the same as described in the previous subsection for the direct-indexing strategy.

DETERMINANTS OF THE TAX BENEFITS OF A DIRECT-INDEXING STRATEGY

A Regression Model of Tax Benefits

Prior literature shows that tax benefits of passively indexed long-only loss-harvesting strategies, similar to the direct-indexing strategy considered here, decline with time²³ and the level of market return²⁴ and increase with stock-specific volatility.²⁵ We set up a regression model that allows us to test the following three hypotheses using our strategy simulation data.

Hypothesis 1: The level of tax benefit increases with cross-sectional dispersion of stock returns. Explanation: The greater the cross-sectional dispersion of stock returns, the greater the likelihood that some stocks will experience losses, even in rising markets.

Hypothesis 2: The level of tax benefit decreases with the level of market return. Explanation: Positive market returns increase gains, whereas negative market returns decrease gains and potentially create losses which could be harvested.²⁶

Hypothesis 3: The level of tax benefit declines with time since inception. Explanation: Due to the equity risk premium, an average stock appreciates over time, thus accumulating built-in gains, which in turn reduce the opportunities for loss harvesting. The loss-harvesting process itself further accelerates the accumulation of built-in gains as tax lots that are at a loss are being systematically sold while tax lots that are at a gain are being systematically retained in the portfolio.

To obtain tax benefit data for the regression model, we run 45 strategy simulations with 45 alternative start dates separated by one year and measure tax benefits for each calendar year of each simulation. This produces 45 annual data points for the first-year tax benefit, 44 annual data points for the second-year tax benefit, and so on—1,035 annual data points in total.

The regression model is designed as follows

$$T_{h,j}^S - T_{h,j}^B = \beta_1 \text{LOG_DISP}_{h,j} + \beta_2 \text{MRET}_{h,j} + \beta_3 \text{MRET_L1}_{h,j} + \gamma_1 X1_{h,j} + \gamma_2 X2_{h,j} + \dots + \gamma_{10} X10_PLUS_{h,j} + \xi_{h,j} \quad (1)$$

The subscript $h = 1, 2, \dots, 45$ denotes the strategy simulation. For example, $h = 1$ corresponds to the first 45-year-long simulation, which starts in January 1975 and ends in December 2019; $h = 45$ corresponds to the last one-year-long simulation, which starts in January 2019 and also ends in December 2019. The subscript j denotes the year of the strategy simulation. For example, for the first 45-year simula-

²³ See, for example, Stein and Narasimhan (1999), Arnott et al. (2001), Berkin and Ye (2003), Stein et al. (2008), Bouchey et al. (2015), and Bouchey et al. (2016).

²⁴ See, for example, Stein and Narasimhan (1999), Berkin and Ye (2003), Israel and Moskowitz (2012), Bouchey et al. (2015), and Bouchey et al. (2016).

²⁵ See, for example, Stein and Narasimhan (1999) and Berkin and Ye (2003).

²⁶ Sialm and Sosner (2018) show that this effect does not hold for long-short strategies, for which tax benefits increase, rather than decline, with the level of market return. Sialm and Sosner also explain the reason for this inverted relationship.

tion, denoted by $h = 1$, j ranges from 1 to 45, whereas for the last one-year simulation, denoted by $h = 45$, j only assumes a value of 1. The superscript S stands for strategy and B—for the benchmark, such that $T_{h,j}^S$ and $T_{h,j}^B$ denote the tax results of the strategy and the benchmark, respectively. The difference $T_{h,j}^S - T_{h,j}^B$ represents the active tax benefit of the direct-indexing strategy in excess of the index ETF benchmark tax.

As for the explanatory variables, $LOG_DISP_{h,j}$ is a natural logarithm of cross-sectional return dispersion and $MRET_{h,j}$ and $MRET_L1_{h,j}$ are the current and previous year's total returns of the S&P 500 Index, respectively. We apply a logarithmic transformation to the cross-sectional return dispersion because of the high positive skewness of the dispersion variable. To compute this variable, for each month of the year, we compute the cross-sectional standard deviation of monthly returns of the S&P 500 Index constituents, apply the logarithmic transformation, and average the transformed dispersion across the 12 months of the year.

The next 10 explanatory variables, $X1_{h,j}$ to $X10_PLUS_{h,j}$, are indicator (or dummy) variables. For example, $X1_{h,j}$ assumes the value of 1 for the *first* year of every simulation and 0 otherwise. Similarly, $X2_{h,j}$ equals 1 for the *second* year of every simulation and 0 otherwise. And so on until $X9_{h,j}$, which equals 1 for the *ninth* year of every simulation and 0 otherwise. The last variable $X10_PLUS_{h,j}$ assumes the value of 1 for years 10 *and later* of every simulation and 0 otherwise. Given the setup of our simulations, we have 45 data points to estimate the coefficient γ_1 , 44 data points to estimate γ_2 , down to 37 data points to estimate γ_9 . Finally, we have 666 data points to estimate the last coefficient γ_{10} because we pool the data for years 10 and later.

We want the indicator variable coefficients γ_1 to γ_{10} to show exactly the average tax benefit for a corresponding year since inception (years 10 and later in the case of γ_{10}). For this reason, we demean the market environment variables LOG_DISP , $MRET$, and $MRET_L1$ within each indicator variable group. The coefficients of the market variables β_1 to β_3 measure deviations from average in the annual active tax benefits due to variation in market conditions.²⁷

Estimation Results

In Exhibit 1, we report the estimation results of the regression model in Equation 1: the regression coefficient estimates, their t -statistics in square brackets, and the adjusted R -squared for each regression. The t -statistics are computed using White (1980) standard errors.

The first three columns of Exhibit 1 show the results for a direct-indexing investment without additional capital contributions, the middle three columns show the results for an investor who systematically contributes capital to the direct-indexing strategy (1% of the value of the strategy portfolio every month), and the last three columns show the results for an investor who combines the direct-indexing strategy with a charitable giving program (every month, donate the most appreciated long-term positions totaling 1% of the value of the strategy portfolio and replace them with newly acquired positions).

Our regression results confirm the hypotheses that stock-specific volatility (proxied by cross-sectional dispersion of stock returns), market return, and time since inception have statistically significant effects on the level of tax benefits in all scenarios we

²⁷ Whereas additional precision in the effects of market conditions can be obtained by interacting the market environment variables with the year of simulation indicator variables, we sacrifice this extra precision for the sake of parsimony of the model. Regressions omitted here for the interest of brevity show that the three market environment variables have a significantly stronger effect on the level of tax benefit in the early years since inception, especially in the first year, but converge to the long-run average estimates, obtained by estimating Equation 1, after approximately five years since inception.

EXHIBIT 1**Annual Active Tax Benefit, All Values Are in Basis Points**

	No Flow			1% Inflow			1% Charitable Giving		
	Offset STCG	Offset LTCG	Biden Tax Plan	Offset STCG	Offset LTCG	Biden Tax Plan	Offset STCG	Offset LTCG	Biden Tax Plan
LOG_DISP	113.9 [11.8]	25.2 [4.5]	25.2 [2.9]	227.7 [21.1]	93.9 [15.2]	119.8 [12.7]	290.6 [19.9]	141.8 [15.5]	248.6 [15.2]
MRET	-281.8 [-13.9]	-163.0 [-15.6]	-235.7 [-15.5]	-459.6 [-21.4]	-245.9 [-22.3]	-350.7 [-22.1]	-631.8 [-23.4]	-380.2 [-23.6]	-680.7 [-23.7]
MRET_L1	-178.2 [-16.1]	-115.4 [-18.1]	-156.3 [-16.2]	-223.4 [-20.3]	-128.3 [-20.7]	-177.1 [-18.4]	-344.2 [-22.2]	-211.9 [-22.1]	-376.8 [-22.0]
X1	339.1 [11.5]	155.3 [11.7]	214.5 [11.6]	360.6 [13.6]	164.3 [13.8]	224.5 [13.1]	443.4 [16.3]	241.8 [17.4]	433.9 [17.5]
X2	114.0 [7.1]	50.8 [6.2]	62.4 [5.4]	145.2 [11.5]	64.7 [10.4]	80.2 [9.1]	211.1 [14.7]	124.9 [16.0]	225.8 [16.3]
X3	66.5 [6.3]	25.7 [4.3]	28.3 [3.3]	109.4 [12.1]	47.3 [10.0]	59.6 [8.5]	175.3 [15.8]	104.7 [16.0]	190.2 [16.3]
X4	52.2 [5.5]	18.9 [3.4]	23.6 [2.8]	96.0 [10.7]	39.3 [7.8]	49.5 [6.5]	157.1 [14.6]	93.1 [14.4]	169.3 [14.7]
X5	36.8 [4.9]	8.0 [1.7]	6.9 [0.9]	86.5 [10.3]	32.8 [6.6]	40.5 [5.1]	145.4 [12.7]	85.1 [12.3]	154.8 [12.5]
X6	30.1 [4.9]	4.8 [1.2]	4.3 [0.7]	84.0 [10.9]	31.8 [7.2]	40.2 [5.7]	143.2 [13.6]	82.4 [12.7]	149.8 [12.9]
X7	24.1 [3.9]	-0.5 [-0.1]	-5.0 [-0.8]	80.3 [10.2]	28.7 [6.1]	35.2 [4.6]	138.4 [12.5]	77.8 [11.2]	141.3 [11.3]
X8	21.8 [3.6]	-2.4 [-0.6]	-7.9 [-1.1]	79.7 [9.3]	27.8 [5.5]	32.7 [4.0]	134.4 [12.2]	74.8 [10.9]	135.5 [11.0]
X9	19.8 [3.1]	-3.1 [-0.8]	-7.8 [-1.1]	76.1 [9.1]	25.7 [5.3]	29.9 [3.8]	128.1 [11.0]	71.1 [9.7]	129.1 [9.8]
X10_PLUS	18.2 [13.4]	-4.3 [-4.8]	-8.7 [-5.8]	79.9 [41.1]	27.4 [24.3]	31.9 [17.8]	140.8 [49.6]	80.4 [44.6]	146.2 [45.2]
R ² ADJ	0.75	0.69	0.65	0.87	0.83	0.79	0.89	0.88	0.88
N OBS	1,035	1,035	1,035	1,035	1,035	1,035	1,035	1,035	1,035

considered. As indicated by *t*-statistics meaningfully higher than 2.0 and lower than -2.0, the regression coefficients are highly statistically significant in all cases, except for year-since-inception indicator variables for years five to nine in the no-flow scenario under the offset long-term gains and the Biden Tax Plan tax rate assumptions. Furthermore, the adjusted *R*-squared, which is shown below the coefficient estimates in the exhibit, range from 0.65 to 0.89. This demonstrates that our regression model provides a highly accurate fit of annual active tax benefits of the direct-indexing strategy.

Economic Significance of the Effects of Market Environment Variables on Tax Benefits

In Exhibit 1, we saw that the effects of the market environment variables on the level of tax benefits are highly statistically significant. In this section, we explore their economic significance.

EXHIBIT 2

Percentile Values of Explanatory Variables

Dependent Variable	Percentile					
	P10	P25	P75	P90	P75-P25	P90-P10
LOG_DISP	-0.23	-0.14	0.10	0.42	0.24	0.65
MRET	-0.20	-0.09	0.14	0.20	0.23	0.40
MRET_L1	-0.19	-0.08	0.11	0.20	0.20	0.39

Exhibit 2 shows the 10th, 25th, 75th, and 90th percentiles of the market environment explanatory variables.²⁸ Using these percentile values and the estimated regression coefficients in Exhibit 1, we calculate how much variation in the market environment variables affects the level of tax benefit predicted by the regression model. We summarize these calculations in Exhibit 3.

In Exhibit 3, each panel corresponds to an explanatory variable. Within the panels, the results are broken down first by the flow scenario and then by the tax rate assumption. The bars show how much the predicted tax benefit changes when the explanatory variable varies between its 25th and 75th and its 10th and 90th percentiles, respectively.

We can see that in most cases variation in market environment variables has an economically significant impact on tax benefits. For example, in the no-flow scenario, under the short-term gain offset assumption, the average level of tax benefit is 41 bps (see Exhibit 1); however, if the cross-sectional dispersion increases from its 10th to its 90th percentile, the level of the tax benefit is expected to increase by as much as 74 bps. Similarly, an increase in the current (past) market return from its 10th to its 90th percentile predicts a decrease of 113 bps (69 bps) in the tax benefit.

Notably, in scenarios where the levels of tax benefits are higher, the susceptibility of tax benefit to variation in the market environment variables is also higher. Compare, for example, the “No Flow/Offset Long-Term Capital Gains” scenario to the “1% Charitable Giving/Offset Short-Term Capital Gains” scenario. For the former, the average tax benefits vary from 155.3 bps in year 1 to 50.8 bps in year 2 to -4.3 bps in years 10 and later. For the latter, the average tax benefit is as high as 443.4 in year 1, 211.1 bps in year 2, and 140.8 bps in years 10 and later. At the same time, the response to 10th- to 90th-percentile increase in the cross-sectional dispersion, market return, and past market return is, respectively, 16, -65, and -45 bps for the former and 189, -254, and -134 bps for the latter.²⁹

Time Decay of Tax Benefits

Exhibit 4 helps visualize the rate of decay in tax benefits over time. It plots the estimated regression coefficients of variables X1 to X10_PLUS reported in Exhibit 1. Annual active tax benefits are reported in basis points. The charts show tax benefits computed under three alternative capital flow scenarios: no capital flows, 1% monthly inflow, and 1% monthly charitable giving. This corresponds to the three sets

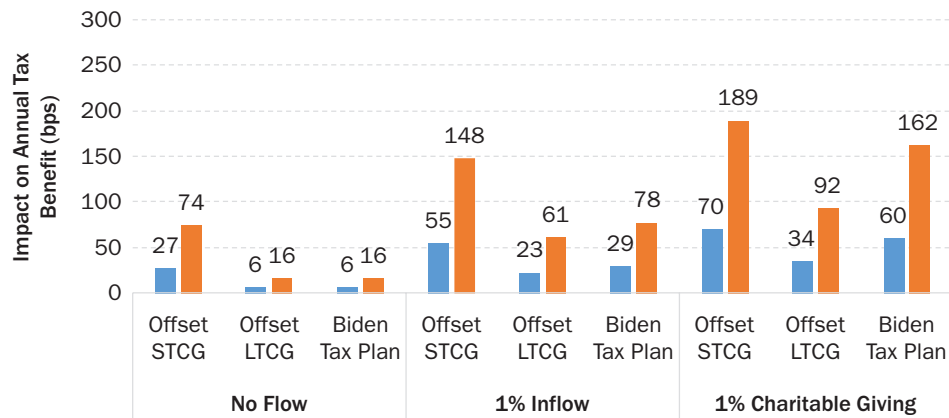
²⁸Recall that the explanatory variables in the regression are transformed to be deviations from the average, such that, for example, -0.20 and 0.20 market return are not the 10th and the 90th percentiles of the market return, but rather the 10th and the 90th percentiles of the *deviation* from average market return.

²⁹Although, in this study, we do not explore the effects of shorting on tax benefits, Sialm and Sosner (2018) show that, for actively managed strategies, the negative effect of market return on the level of tax benefit is attenuated by shorting.

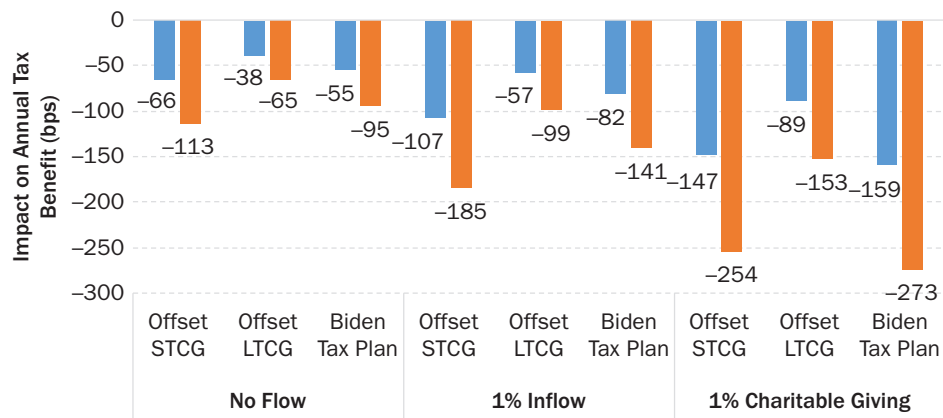
EXHIBIT 3

Impact of Variation in Explanatory Variables on Active Annual Tax Benefits

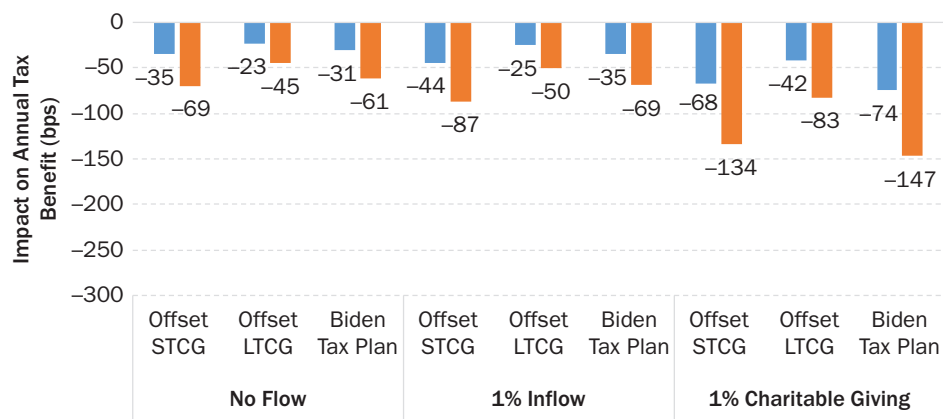
Panel A: Log of Cross-Sectional Return Dispersion



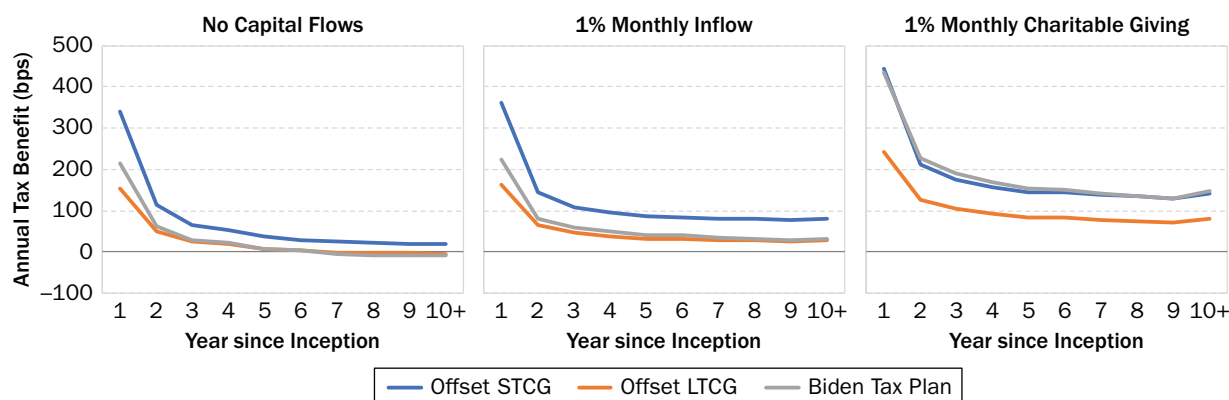
Panel B: Current Year Market Return



Panel C: Last Year Market Return



■ P75-P25 ■ P90-P10

EXHIBIT 4**Time Evolution of Active Tax Benefit under Different Capital Flow and Tax-Regime Assumptions**

of columns in Exhibit 1. The “Offset STCG,” “Offset LTCG,” and “Biden Tax Plan” lines correspond to our three alternative tax rate assumptions.

Three clear patterns emerge from Exhibit 4. First, in all cases, the tax benefits of the direct-indexing strategy decay rather quickly with time. The left-most chart shows that without additional capital contributions, under the 2020 tax rates, for investors with only long-term capital gains from other investments or under the proposed Biden Tax Plan uniform tax rates on long-term and short-term capital gains for high-income investors, the active tax benefit is reduced to 0 after about five years. Under the 2020 tax rates, for investors with large amounts of short-term capital gains from other investments, the active tax benefit declines to a long-run level of about 20 bps a year.

Second, investors seeking to increase the long-run tax benefit of the direct-indexing strategy can do so by systematically contributing capital to the strategy or by combining the strategy with a charitable giving program. The middle chart shows that monthly capital contributions of 1% of the strategy portfolio value over the term of the investment improve the long-run tax benefits for those investors who, under the 2020 tax rates, can use the strategy losses to offset short-term gains from 20 bps to 80 bps a year. For investors who cannot benefit from the capital gains tax rate differential, such as investors with only long-term gains or investors who are subject to the Biden Tax Plan uniform tax rates for high earners, the long-run active tax benefit increases from approximately 0 to approximately 30 bps a year.

Finally, by combining the direct-indexing strategy with a systematic charitable giving program, investors can meaningfully increase the long-run tax benefit of the strategy under all tax rate assumptions. The right-most chart shows that even an investor with only long-term gains from other investments, under the 2020 tax rates, achieves the long-run active tax benefit of about 80 bps, whereas the other two tax rate assumptions—large amount of short-term gains from other investments and the proposed Biden Tax Plan—show long-run active tax benefits as high as approximately 140 bps.

Which Investors Can Benefit the Most from Direct-Indexing Strategies in the Long Run?

Exhibit 4 shows that most investors with capital gains in their portfolios, long term or short term, will enjoy substantial tax benefits from a direct-indexing investment in its early years. Unfortunately, investors without short-term gains from other invest-

ments might see those benefits decline to zero. However, not everything is lost at that point. An investor with only long-term gains may increase the tax benefits of her direct-indexing strategy through systematic capital contributions and may increase them even further by combining the strategy with a charitable giving program. In the latter case, the long-run sustainable tax benefits can be as high as 80 bps a year.

Investors with large amounts of short-term gains from other investments—typically, high-net-worth investors with allocations to hedge funds and derivatives—not only obtain very high tax benefits in the short run but can continue to enjoy sustainable long-run tax benefits. Whereas, under this short-term gain offset assumption, without additional capital contributions to the direct-indexing investment the long-run tax benefit is only about 20 bps, it can be quadrupled by capital contributions and then almost doubled again to 140 bps by combining the direct-indexing investment with a charitable giving program.

Our results show the power of direct indexing, particularly for charitably inclined high-net-worth investors. Importantly, our results in Exhibit 4 show that for this category of investors, neither the long-run nor the short-run tax benefits are adversely affected by the Biden Tax Plan.

Sources of Tax Benefits over Time

A decomposition of the total active tax benefit into its components (see Appendix A for details of the decomposition) helps us dig deeper into causes of time decay of the tax benefits of direct-indexing strategies. We show the decomposition in Exhibit 5. Panels A and B plot, respectively, the character and the deferral components of the current-period preliquidation active tax benefit. Panel C plots the present value of the discounted future liquidation tax costs (in excess of the tax costs of liquidating an index ETF benchmark).

Panel A shows that the character component of the tax benefit, which results from direct-indexing strategies realizing capital gains and income as long-term gains and qualified dividend income and capital losses as short-term losses. Note that such a benefit exists only when two conditions are met: There is a difference between short-term and long-term capital gains tax rates and short-term capital losses can offset short-term capital gains from other investments. Hence, Panel A shows only one tax scenario—the 2020 tax rates with short-term gains from other investments.

The character benefit of the direct-indexing strategy starts out at approximately 50 bps in the first year since inception in all three capital flow scenarios.³⁰ It is small compared to the approximately 350 bp to 450 bp total active tax benefit for the corresponding tax rate assumptions (that is, the 2020 tax rates and short-term gains from other investments) shown in Exhibit 4. After the first year, the character benefit declines rather quickly to approximately 15 bps for the no capital flows scenario and to approximately 30 bps for the 1% monthly inflow and 1% monthly charitable giving scenarios. This level of character benefit, albeit low, persists in the long run.³¹ Notably, in the no-capital-flow scenario, the 15 bp long-run character benefit accounts for most of the about 20 bp long-run active tax benefit, such that although the character benefit is small, it might be the only benefit available in the

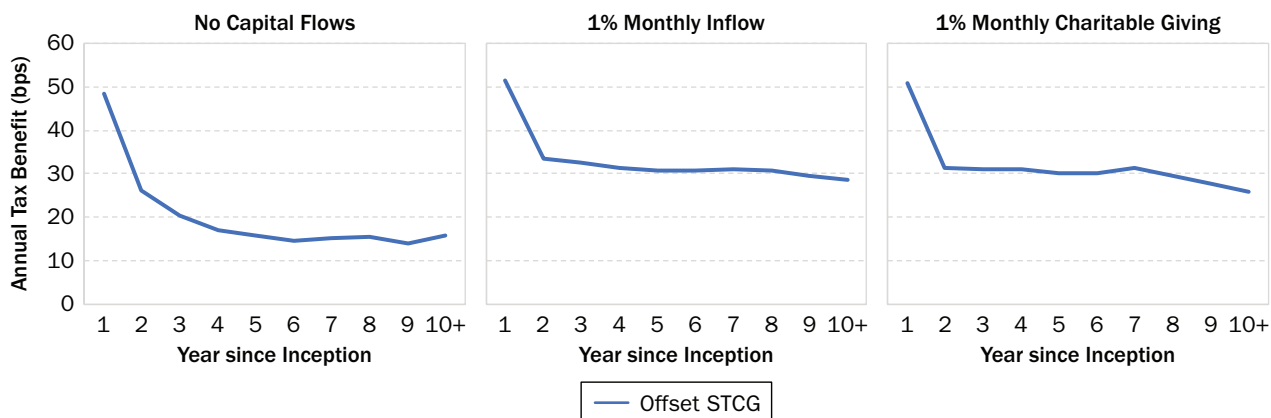
³⁰Note that there is a character benefit in the first year, despite not having long-term gains. This character benefit results from a matching amount of qualified dividend income and short-term capital loss, multiplied by the difference in applicable rates. See Appendix A for further explanation of this calculation.

³¹Sosner et al. (2019) show that character benefits are substantially higher for strategies that utilize leverage and shorting.

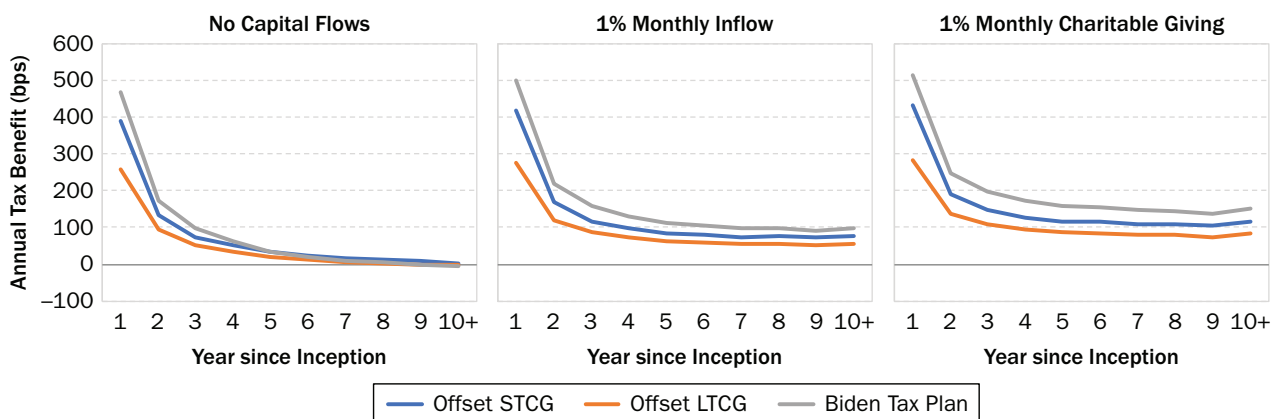
EXHIBIT 5

Time Evolution of the Components of the Active Tax Benefit with and without Capital Contributions

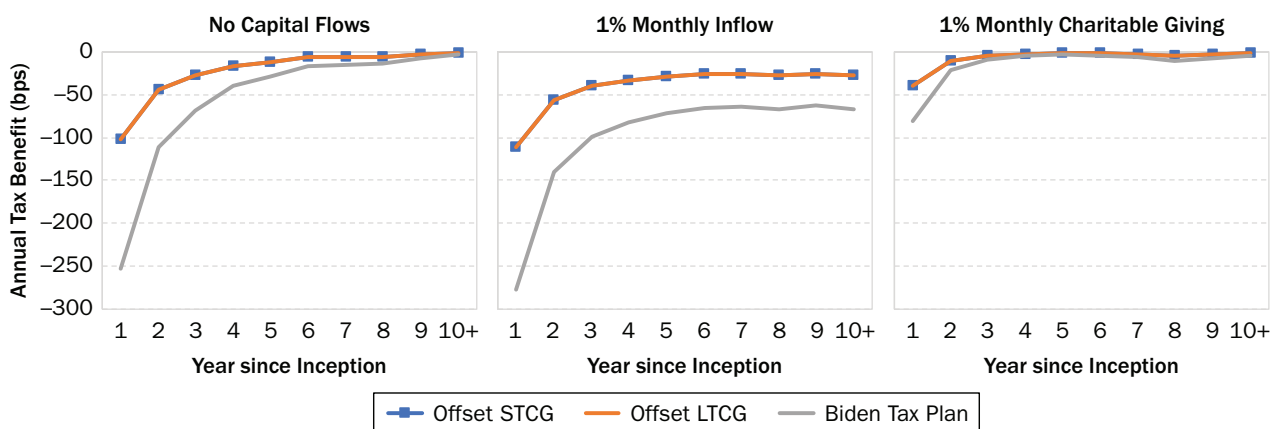
Panel A: The Character Component



Panel B: The Deferral Component



Panel C: The Unrealized Gains Component



long run to an investor who does not plan to systematically contribute capital to the direct-indexing strategy.³²

³²Stein et al. (2008) and Goldberg et al. (2021) show that the tax benefits of a loss-harvesting strategy can be increased by gain management. Both studies show that a systematic realization of long-term gains with the goal of resetting the cost bases and holding periods of portfolio positions

Panel B plots the deferral component of the active tax benefit, which results from the direct-indexing strategy realizing a net capital loss. The deferral component of the tax benefit is a net loss credited with the tax rate applicable to the gain this loss can offset. For example, under the 2020 tax rates, in the case of the short-term capital gain offset, the net loss is multiplied by the short-term capital gains tax rate of 40.8%; in the case of the long-term capital gain offset, the net loss is multiplied by the long-term capital gains tax rate of 23.8%, and under the proposed Biden Tax Plan, the net loss is multiplied by the tax rate of 43.4%.

The direct-indexing strategy realizes large net losses in early years, and higher tax rates applicable to these losses result in higher deferral benefits. However, without additional capital contributions, the deferral benefits eventually run out as the highly appreciated strategy portfolio loses its ability to realize a net loss.

The middle chart in Panel B shows that adding new capital to the strategy substantially increases its ability to realize a net loss, and, therefore, a deferral benefit, in the long run. The right-most chart shows that, if, in addition to adding new capital, the investor can also remove the most appreciated positions in the portfolio through charitable giving, the ability to realize a net loss, and thereby create a deferral benefit, is enhanced further.

Finally, Panel C shows the component of the active tax benefit that accounts for the present value of the expected future liquidation cost of unrealized gains. Note that because we measure an active tax benefit of the direct-indexing strategy, this liquidation cost is computed as the difference between the discounted future liquidation costs of the strategy and the index ETF benchmark. The left-most chart shows that the present value of the cost of unrealized gains is significantly higher under the Biden Tax Plan than under the 2020 tax rates. This is for two reasons. First, under the Biden Tax Plan, the long-term liquidation gains are taxed at 43.4%, rather than at 23.8% under the 2020 tax rates. Second, the Biden Tax Plan eliminates the step-up in the cost basis upon death, which under the 2020 tax rules results in elimination of unrealized gains accumulated prior to death.

The middle chart in Panel C shows the flipside of the deferral benefit in Panel B: A higher future liquidation tax cost that results from the ability to realize a greater net loss in the present. This cost is again substantially higher under the Biden Tax Plan where, compared to the 2020 tax rules, the statutory tax rates on long-term capital gains are higher and the step-up in the cost basis is eliminated.

Finally, the right-most chart in Panel C shows the real benefit of combining a direct-indexing strategy with a charitable giving program. Despite realizing a high net loss, which results in a high deferral benefit shown in the right-most chart in Panel B, the unrealized gain, and thus its cost, is minimal. This is because removing appreciated positions for charitable giving, to a large extent, eliminates the unrealized gain (measured in excess of an index ETF benchmark).

enhances future loss-harvesting opportunities. Note that this technique does not increase net losses realized by the strategy (what we define as the deferral component of the tax benefit) but might increase the benefit from realizing capital gains as long term and capital losses as short term (what we define as the character component of the tax benefit). Indeed, Stein et al. (2008) show that the additional tax benefit resulting from gain management would decline sharply if the tax rate applicable to long-term gains were to increase. This is consistent with the equations in Appendix A, where the magnitude of the character benefit depends on the gap between the short-term and long-term capital gain tax rates.

CONCLUSION

We study the tax benefits of a direct-indexing strategy with a focus on what type of investors could benefit the most from the strategy and how the tax benefits offered by the strategy could be increased.

First, we argue that high-net-worth investors, and more specifically those high-net-worth investors with allocations to hedge funds and derivatives, are most likely to have the systematic short-term capital gains that are necessary to derive the highest tax benefits from a direct-indexing strategy.³³

We then show that, although market environment variables—cross-sectional dispersion of stock returns and the level of market return—have statistically and economically significant effects on the level of tax benefits, on average, across different market environments, the tax benefits of direct-indexing strategies decay rather quickly over time. In fact, only investors with systematic short-term gains in their portfolios—likely, high-net-worth investors with allocations to hedge funds and derivatives—can enjoy the long-run tax benefits of direct-indexing strategies. For these investors, the long-run tax benefit comes in the form of the character benefit resulting from the difference between short-term and long-term capital gain tax rates. When investors only have long-term gains or when long-term gains are taxed at the same rate as short-term gains, the character benefit disappears, and the long-run tax benefit is reduced to zero.

However, investors can increase the tax benefits they derive from direct indexing even when they cannot benefit from the difference between the long-term and short-term capital gain tax rates.³⁴ Systematic contributions of capital to a direct-indexing strategy enhance opportunities to realize net losses and thereby increase the deferral benefit. Moreover, combining the strategy with a charitable giving program results in removing the most appreciated positions from the strategy portfolio, and thus in addition to enhancing net losses (that is, the deferral benefit), also reduces the expected future tax liability of unrealized gains. Increasing the deferral benefit is valuable for investors who cannot benefit from the difference in tax rates, like those investors with only long-term gains from other investments or high-income investors under the now defunct Biden Tax Plan. Reducing unrealized gains in the portfolio through charitable disposition of appreciated stocks is particularly valuable under the Biden Tax Plan, which not only proposed to tax long-term gains at the ordinary income tax rate for investors with income in excess of \$1 million but also sought to eliminate the step-up in cost basis upon death for gains in excess of \$1 million.

³³ Notably, high-net-worth investors, and in particular those investors who already allocate to hedge funds, might also have access to loss-harvesting strategies that utilize leverage and shorting that, as prior research shows, realize substantially higher tax benefits than long-only loss-harvesting strategies, like the direct-indexing strategy considered here. These investors are also more likely to tolerate the risks and costs associated with long-short investing.

³⁴ Stein et al. (2008) and Goldberg et al. (2021) show that the tax benefits of a loss-harvesting strategy can be increased by gain management. Although a strategic gain realization described in these two studies might be a powerful technique for enhancing tax benefits, it will likely be implemented by the strategy manager rather than by an investor and will add benefits only to those direct-indexing investors who can take advantage of the difference between long-term and short-term capital gain tax rates. As a result, the focus of these studies is different from ours.

APPENDIX A

DECOMPOSING THE TAX BENEFIT OF A DIRECT-INDEXING STRATEGY

Short-Term Capital Losses Offset Short-Term Capital Gains from Other Investments

First, following Sosner et al. (2019), we define the after-tax return $r_{AT,i}$ of a strategy and a benchmark, as

$$r_{AT,s} = r_{PT,s} - (g_{L,s} + q_s)t_L - (g_{S,s} + i_s)t_H - u_s t_E \quad (A1)$$

and

$$r_{AT,b} = r_{PT,b} - (g_{L,b} + q_b)t_L - (g_{S,b} + i_b)t_H - u_b t_E, \quad (A1')$$

where the subscripts s and b stand for the direct-indexing strategy and the passive benchmark, respectively, $r_{PT,i}$ is the pretax return, $g_{L,i}$ and $g_{S,i}$ are the long-term and short-term capital gains (or losses), respectively, q_i is the qualified dividend income, i_i is the ordinary income (or loss), u_i defined as $u_i \equiv r_{PT,i} - (g_{L,i} + q_i + g_{S,i} + i_i)$ is a one-period incremental unrealized gain, t_L and t_H are the lower and the higher tax rates, respectively, and t_E is the effective tax rate applicable to unrealized gains that reflects the present value of future tax liabilities created by unrealized gains. See Appendix B for further discussion and estimation of the effective tax rate on unrealized gains.

Further, following Sosner et al. (2019), we decompose the current-period tax result of the direct-indexing strategy and the benchmark, that is, $-(g_{L,i} + q_i)t_L - (g_{S,i} + i_i)t_H$, into character and deferral components denoted by C_i and D_i , respectively, as follows:

1. If $g_{L,i} + q_i$ and $g_{S,i} + i_i$ do not have a different sign (which includes 0 for either one or both sums), that is, both are a gain or a loss, or either or both are 0, then the current-period tax result is only a deferral benefit (or liability)

$$D_i = \underbrace{-(g_{L,i} + q_i)t_L - (g_{S,i} + i_i)t_H}_{\text{deferral}} \quad (A2)$$

2. If $g_{L,i} + q_i$ and $g_{S,i} + i_i$ have a different sign, that is, one is a gain and the other is a loss, and $|g_{S,i} + i_i| \geq |g_{L,i} + q_i|$, the character benefit is calculated on the $g_{L,i} + q_i$ amount and the remaining excess short-term loss (or gain) gives rise to the deferral benefit (or liability)

$$C_i + D_i = \underbrace{(g_{L,i} + q_i)(t_H - t_L)}_{\text{character}} + \underbrace{(-1)(g_{L,i} + q_i + g_{S,i} + i_i)t_H}_{\text{deferral}} \quad (A3)$$

3. If $g_{L,i} + q_i$ and $g_{S,i} + i_i$ have a different sign, that is, one is a gain and the other is a loss, and $|g_{L,i} + q_i| > |g_{S,i} + i_i|$, the character benefit is calculated on the $g_{S,i} + i_i$ amount and the remaining excess long-term loss (or gain) gives rise to the deferral benefit (or liability)

$$C_i + D_i = \underbrace{(-1)(g_{S,i} + i_i)(t_H - t_L)}_{\text{character}} + \underbrace{(-1)(g_{L,i} + q_i + g_{S,i} + i_i)t_L}_{\text{deferral}} \quad (A4)$$

Finally, we can define the unrealized gain contribution to the total tax as

$$U_i = u_i t_E. \quad (A5)$$

Now, using Equations A2 to A5, we can define the decomposition of the active tax of the direct-indexing strategy in excess of the benchmark tax as

$$T_s - T_b = (C_s - C_b) + (D_s - D_b) - (U_s - U_b). \quad (\text{A6})$$

Short-Term Capital Losses Offset Long-Term Capital Gains from Other Investments

Suppose that an investor uses short-term capital losses of the direct-indexing strategy to offset long-term gains from other investments. In terms of the decomposition in the previous subsection, this translates into substituting the rate t_H applicable to the short-term capital result for a rate \tilde{t} , which depends on the sign of the tax result as follows

$$\tilde{t} = \begin{cases} t_H & \text{if } g_{S,i} \geq 0 \\ t_L & \text{if } g_{S,i} < 0 \end{cases}. \quad (\text{A7})$$

Substituting the conditional rate in Equation A7 for the rate t_H in Equations B2 through B6 yields the decomposition in the absence of short-term capital gains from other investments.

In the case where a direct-indexing strategy s harvests net short-term capital losses, that is, $g_{S,s} < 0$, Equation A7 reduces to $\tilde{t} = t_L$.

Further, assuming that the direct-indexing strategy realizes only qualified dividend income and no ordinary dividends, that is, $i_s = 0$, Equations A2, A3, and A4 all reduce to

$$D_i = \underbrace{(-1)(g_{L,i} + q_i + g_{S,i})}_{\text{deferral}} t_L, \quad (\text{A8})$$

meaning that the direct-indexing strategy does not yield any character benefit, only a deferral benefit, which is further attenuated due to the fact that only lower-taxed long-term capital gains from other strategies are being offset. Equation A6 then simplifies to only two terms

$$T_s - T_b = (D_s - D_b) - (U_s - U_b). \quad (\text{A9})$$

That is, the active tax of the direct-indexing strategy only consists of a deferral benefit and an expected cost of unrealized gains in excess of a passive benchmark.

APPENDIX B

EMPIRICAL METHODOLOGY³⁵

Active Tax Management

Stein and Narasimhan (1999) made a distinction between active alpha and active tax management. According to Stein and Narasimhan, a manager who is active with respect to security selection but ignores the tax consequences of trading is “passive with respect to tax.” Active tax management seeks to improve after-tax returns via acceleration of capital losses and deferral of capital gains, a technique otherwise known as *loss harvesting*. A manager who is passive with respect to security selection, for example, seeking only to match an index, might thus still be active with respect to tax. This is exactly the type of strategy we model in this study—active from a tax perspective but passive from a security selection perspective.

Direct-Indexing Strategy Construction

Using covariance matrix and transaction cost estimates described further in this appendix and the S&P 500 Index constituent universe, we constructed the direct-indexing strategy portfolios, updating them every month-end. We run 45 separate strategy simulations starting in January of every year from 1975 to 2019 and all ending in December 2019. Our longest simulation, starting in January 1975, thus lasts for 45 years, or 540 months, and our shortest simulation, starting in January 2019, lasts for only 12 months. The portfolio weights of the individual securities are all positive and sum to 100%. The portfolio beta relative to the S&P 500 Index is constrained to be close to 1.0.

Loss harvesting is directly incorporated into portfolio construction by making it the objective of portfolio optimization:

$$\begin{aligned} & \max_{w_1 \dots w_N} -\gamma T - c \\ & \text{s.t.} \\ & \sum_i \sum_j w_i w_j \sigma_{ij} \leq TE^2 \\ & \sum_i (b_i + w_i) = 1 \\ & 0.99 \leq \sum_i (b_i + w_i) \beta_i \leq 1.01, \end{aligned}$$

where w_i corresponds to the active portfolio weight of security i , $\gamma = 0.5$ is the tax aversion coefficient, T is the tax cost of rebalancing the portfolio in the current period, c is transaction costs described in detail in the following, σ_{ij} is the covariance between the returns of securities i and j derived from an MSCI Barra risk model, TE is the target tracking error of 1% annually, b_i is the S&P 500 benchmark weight of security i , and β_i is the beta of security i with respect to the S&P 500 Index predicted by the MSCI Barra risk model. Both the covariance and the beta estimates are point-in-time forward-looking estimates. In addition, we lag these estimates by one month to ensure that the risk model data had been released before the portfolio construction date.

The first term in the objective function rewards the realization of losses and penalizes the realization of gains. Short-term losses are rewarded more than long-term losses, and short-term gains are penalized more than long-term gains. Also, the higher the tax aversion coefficient, the higher the importance of reducing tax costs (or increasing tax benefits) as compared to transaction costs. More specifically, the tax cost of rebalancing a portfolio is defined as follows:

$$T = t_L g_L + t_H g_S,$$

where t_L and t_H are the lower tax rate on long-term capital gains and the higher tax rate on short-term capital gains, respectively, and g_L and g_S are the net long-term and net short-term capital gains aggregated from individual tax lots, respectively.

Although dividend taxes are not explicitly incorporated into the optimization, they are included in the reported after-tax returns. Unrealized gains are also not included in the optimization problem, which implies that at the portfolio construction stage, we assume that the tax rate applicable to unrealized gains is zero. However, following Sosner et al. (2019), when calculating tax benefits, we apply a tax rate to unrealized gains at a level explained further in this appendix. This tax rate estimate is not included in portfolio optimization because, as we show shortly, it is highly dependent on numerous assumptions.

Several studies have documented that the choice of accounting method for tax lot selection has a nontrivial effect on after-tax returns (Dickson et al. 2000; Berkin and Ye 2003; Israel and Moskowitz 2012). Because the effects of tax lot accounting are not central to our conclusions and have been analyzed elsewhere, we use the FIFO (highest in, first out) tax lot accounting method throughout this article.

Tax Rate Assumptions

We modeled three alternative tax-rate scenarios: the 2020 tax regime with unlimited short-term gains from other investments, the 2020 tax regime with only long-term gains from other investments, and the tax regime originally proposed by the Biden administration on the campaign trail and later codified in the American Families Plan proposal on April 28, 2021 (we refer to it as the Biden Tax Plan). Note that none of the Biden Tax Plan proposals made it into the final text of the Build Back Better Act drafted in late 2021.

Under the 2020 tax rate regime, the tax rates on short-term and long-term capital gains were assumed to be 40.8% and 23.8%, respectively. These rates include the highest federal tax bracket rates of 37% and 20%, respectively, and the net investment income tax of 3.8%.³⁶ All dividends are assumed to be qualified dividend income (QDI) and are thus taxed at a 23.8% rate, which is consistent with strategies that have relatively long holding periods.³⁷ Under this tax regime, we use two alternative assumptions: (1) losses of the loss-harvesting strategy offset only short-term gains, and thus are credited with a tax rate of 40.8%, and (2) losses of the loss-harvesting strategy offset only long-term gains, and thus are credited with a tax rate of 23.8%.

Under the Biden Tax Plan, for taxpayers with incomes greater than \$1,000,000, all capital gains, short-term and long-term, would be taxed at the ordinary income tax rate. The same high tax rate would also apply to QDI. In addition, the ordinary income federal tax rate for the highest federal tax bracket would be increased from 37% in 2020 to its pre-2018 level of 39.6%. We thus assume that, under the Biden Tax Plan, all the gains and dividends are taxed at a uniform rate of 43.4%, which includes the highest bracket federal tax rate of 39.6% and the net investment income tax of 3.8%. The Biden Tax Plan also proposed to eliminate the step-up in the cost basis at death. We discuss the consequences of this provision for our analysis in the next subsection.

Because the portfolios are rebalanced monthly, we assume that the trades are not subject to the wash-sale rule, which defers capital losses for tax purposes if the investor reestablishes a position disposed of at a loss within a period beginning 30 days before and ending 30 days after the date of the disposition, excluding the day of disposition.³⁸

³⁶ IRC §§ 1222 and 1223 define the holding periods for the determination of long-term and short-term capital gains and losses, and IRC § 1 provides the applicable tax rates for short-term and long-term gains. As of 2020, under IRC § 1, the top-bracket tax rates for long-term and short-term capital gains were 20% and 37%, respectively. In addition to this base rate, under IRC § 1411, a 3.8% Medicare surtax is imposed on net investment income for modified adjusted gross income (MAGI) levels above \$200,000 for individuals, \$250,000 for couples filing jointly, and \$125,000 for spouses filing separately. Note that many states impose additional taxes on capital gains, which are not included in these rates. Throughout our study, we assume that the strategies invest in physical equities and not in equity swaps. For physical equities, gains and losses are generally taxed at the time of realization (IRC § 1001), thus allowing for the evolution of holding periods from short term to long term by holding a position for longer than 12 months (IRC § 1223).

³⁷ Under IRC § 1(h)(11), qualified dividend income is defined as dividends on a share of stock held for longer than 60 days during the 121-day period beginning 60 days before and ending 60 days after the ex-dividend date and is taxed at the long-term capital gains rate. The definition of qualified dividend income is adjusted in the case of extraordinary dividends and when a stock is preferred rather than common.

³⁸ The wash-sale rule is governed by IRC § 1091. In our strategy simulations, the wash-sale rule could be violated in months shorter than 31 days or in months whose month-end occurs on a weekend. Although the wash-sale rule can be explicitly incorporated as a constraint into the optimization problem, we do not use this functionality in our study to simplify the rebalancing process in our simulations.

Effective Tax Rate on Unrealized Gains

Following Poterba (1999), we define the effective tax rate on unrealized capital gains as expected present value of future tax liabilities. Let r be the appropriate annual after-tax discount rate, t_L be the tax rate applicable to long-term capital gains, p be the probability of liquidating the capital gain position in a given year, λ be the probability that the liquidation does not generate a taxable capital gain (as, for example, would be the case if the investor were to opportunistically liquidate the capital gain position at the time when there are offsetting realized losses from other positions), q be the probability of death in each year, and d be the probability of contributing the position to charity. Note, that for the purpose of this calculation, the probability of death only matters to the extent that death allows a step-up in the cost basis when assets are passing through the estate.

The probability that the position is in the portfolio after h periods is given by $((1-d)(1-q)(1-p))^h$. The probability that in any given period the position is liquidated via a taxable liquidation is $(1-d)p(1-\lambda)$. Therefore, the expected present value of future tax liabilities can be described by the following equation:

$$t_E = \sum_{h=1}^{\infty} t_L (1-d)p(1-\lambda) \left(\frac{(1-d)(1-q)(1-p)}{1+r} \right)^h.$$

Solving this infinite geometric series, we obtain

$$t_E = t_L (1-d)p(1-\lambda) \frac{(1-d)(1-q)(1-p)}{1+r - (1-d)(1-q)(1-p)}. \quad (B1)$$

We can now use Equation B1 to estimate the effective tax rate applicable to unrealized capital gains.

Exhibit B1 shows parameter assumptions and the effective unrealized gain tax rates under alternative scenarios explored in the study. Our assumptions are broadly consistent with the ones used in Poterba (1999) with two important modifications. First, under the Biden Tax Plan, there is no step-up in the cost basis upon death, which effectively translates into $q = 0$, that is, the death event does not lead to elimination of built-in capital gains. Second, under the charitable giving scenario, we assume that the position is equally likely to be liquidated or donated and that the sum of the probability of liquidation and the probability of donation amounts to the probability of liquidation under the no charitable giving scenario.

For simplicity, we round the effective tax rates resulting from substituting the assumed parameter values in Equation B1. These rounded effective tax rates, shown in the last row of Exhibit B1, are used throughout the study.

Covariance Matrix

Similar to Sialm and Sosner (2018) and Sosner et al. (2019), we use covariance matrices from MSCI Barra, which applies a multifactor approach to covariance matrix estimation. The MSCI Barra USE3L risk model provides a covariance matrix of all stocks traded on US exchanges. The model uses 52 industries and 13 risk factors—including volatility, size, value, momentum, and leverage—to capture the common variation in stock returns. The model is updated monthly using information about stock returns and fundamentals available at month-end. As indicated by the release date of the model handbook (Barra 1998), the model's factor structure was chosen before February 1998.

Similar to the Fama–MacBeth (1973) procedure, the model first computes factor loadings using past data and then estimates cross-sectional regressions of stock-level returns on those factor loadings. The regression coefficients estimated in each period are factor returns for that period, and the regression residuals are stock-specific returns for that period. Time-series factor returns up to that period are then used to compute

EXHIBIT B1**Parameter Assumptions and the Effective Unrealized Gain Tax Rates**

	2020 Tax Rates, No Charitable Giving	Biden Tax Plan, No Charitable Giving	2020 Tax Rates, With Charitable Giving	Biden Tax Plan, With Charitable Giving
r	0.03	0.03	0.03	0.03
t_L	23.8%	43.4%	23.8%	43.4%
p	0.10	0.10	0.05	0.05
λ	0.25	0.25	0.25	0.25
q	0.02	0.02	0.02	0.02
d	0.00	0.00	0.05	0.05
t_E	10.6%	22.5%	5.2%	10.9%
Rounded t_E	10%	25%	5%	10%

a forward-looking forecast of the factor covariance matrix. Stock-specific returns up to that period are used to compute forward-looking stock-specific volatility forecasts. More details about the model estimation are available in the model handbook (Barra 1998).

Management Fees and Transaction Costs

All the results in the study are reported gross of management fees. We use a simple transaction costs model informed by the academic research such as Almgren et al. (2005). Transaction costs per dollar traded in basis points are modeled as

$$\text{transaction costs}_{i,t} = 15 + 0.075 \times \text{VIX}_t + 2.5 \times \text{srisk}_{i,t} \times \sqrt{\frac{T\$_{i,t}}{\text{DTV}\$_{i,t}}},$$

where VIX_t is the most recent VIX index level known on the date of the trade, $\text{srisk}_{i,t}$ is the specific volatility of stock i as estimated by the MSCI Barra USE3L model,³⁹ and $T\$_{i,t}$ and $\text{DTV}\$_{i,t}$ are the dollar trade size and dollar daily trading volume of stock i , respectively.

We use Frazzini et al.'s (2015) results to confirm our model assumptions. Frazzini et al. estimate that the average market impact cost for a large institutional investor following quantitative strategies in the large-capitalization developed markets universe was less than 20 bps of the trading value over the period from 1998 to 2013. These market impact costs correspond to average trade sizes of around half a million and amounting to around 1% of the average daily trading volume. If we substitute 20 for VIX—the average VIX level from January 1, 1986, to August 31, 2021, 20 for specific risk—the average MSCI Barra stock-specific risk for large-capitalization stocks in percentage points, and 1% for the trade as a fraction of DTV, we obtain a 21.5 bp transaction cost, on average. For a few trades that represent a high fraction of DTV, for example, 5%, the cost becomes 27.7 bps.

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³⁹ MSCI Barra stock specific volatilities are computed using stock returns residual to Barra model factors.

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