

The Journal of **Alternative** *Investments*

Summer 2017 – Volume 20 Number 1 | www.iiJAI.com

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Investors have become increasingly focused on how to harvest returns efficiently. A big part of that process involves understanding the systematic sources of risk and reward in their portfolios. *Risk-based investing* generally views a portfolio as a collection of return-generating processes or factors. The most straightforward of these processes is investing in asset classes, such as stocks and bonds (asset class premiums). Such risk taking has been rewarded globally over the long term and has historically represented the biggest driver of returns for investors. However, asset class premiums represent just one dimension of returns. A largely independent, separate source comes from style premiums, which are a set of systematic sources of returns that are well researched, are geographically pervasive, and have been shown to be persistent. There is a logical, economic rationale for why they provide a long-term source of return (and are likely to continue to do so) (Asness [2015]). Finally, they can be applied across multiple asset classes.¹

The common feature of risk-based investing is its emphasis on improved risk diversification, which can be achieved by identifying the sources of returns that are underrepresented in a portfolio. Investors who understand the risk sources to which their portfolios are exposed (and the magnitude of these exposures) may be better suited to evaluate existing and potential managers.

Without an understanding of portfolio risk factor exposures, how else would investors be able to tell if their value manager, for example, is actually providing significant value exposure, is truly delivering alpha and not some other factor exposure, or even whether a new manager would be additive to their existing portfolio?

These are important questions for investors to answer, but quantifying them may be difficult. There are many ways to measure and interpret the results of factor analysis. Some investors may use a holdings-based approach, whereas others use returns-based regression analysis.² There are also many variations in portfolio construction and factor portfolio design. Even a single factor, such as value, has variations that an investor should consider—it can be applied as a tilt to a long-only equity portfolio,³ or it can be applied in a purer form through long-short strategies; it can be based on multiple measures of value, a single measure such as book-to-price (B/P); or it can span multiple asset classes or geographies. Simply put, even two factors that aim to capture the same economic phenomenon can differ significantly in their construction, and these differences can matter.

In this article, we discuss some of the difficulties associated with measuring and interpreting factor exposures. We look at a regression-based approach and explore some common pitfalls of regression analysis;

we also describe the differences associated with academic versus practitioner factors and outline various choices that can affect the results. We hope that after reading this article investors will be better able to measure portfolio factor exposures, understand the results of factor models, and, ultimately, determine whether their portfolios are accessing the sources of return they want in a diversified manner.

A BRIEF HISTORY OF FACTORS

Asset pricing models generally dictate that risk factors command a risk premium. Modern portfolio theory quantifies the relationship between risk and expected return, distinguishing between two types of risks: idiosyncratic risk (that which can be diversified away) and systematic risk (such as market risk that cannot be diversified away). The capital asset pricing model (CAPM) provides a framework to evaluate the risk premium of systematic market risk.⁴ In the CAPM single-factor world, we can use linear regression analysis to decompose returns into two components: alpha and beta. *Alpha* is the portion of returns that cannot be explained by exposure to the market, and *beta* is the portion of returns that can be attributed to the market.⁵ Studies have shown, however, that single-factor models may not adequately explain the relationship between risk and expected return and that there are other risk factors at play. For example, under the framework of Fama and French [1992, 1993], the returns to a portfolio could be better explained by looking not only at how the overall equity market performed but also at the performance of size and value factors (i.e., the relative performance between small- and large-cap stocks and between cheap and expensive stocks). Adding these two factors (value and size) to the market created a multifactor model for asset pricing. Academics have continued to explore other risk factors, such as momentum (Jegadeesh and Titman [1993]; Asness [1994]) and low beta or low risk (Black [1972]; Black, Jensen, and Scholes [1972]; Frazzini and Pedersen [2014]), and have shown that these factors have been effective in explaining long-run average returns.

In general, style premiums have been most widely studied in equity markets, with some classic examples being the work of Fama and French already referenced. For each style, they use single, simple, and fairly standard definitions, described in Exhibit 1.⁶

EXHIBIT 1

Common Academic Factor Definitions

HML	High Minus Low: a long–short measure of value that goes long stocks with high book-to-market values and short stocks with low book-to-market values
UMD	Up Minus Down: a long–short measure of momentum that goes long stocks with high returns over the past 12 months (skipping the most recent month) and short stocks with low returns over the same period
SMB	Small Minus Big: a long–short measure of size that goes long small-market-cap stocks and short large-market-cap stocks

ASSESSING FACTOR EXPOSURES IN A PORTFOLIO

Using these well-known academic factors, we can analyze an illustrative portfolio's factor exposures. But before we do, we should emphasize that the factors studied here are not a definitive or exhaustive list of factors. We should also emphasize that different design choices in factor construction can result in very different measured portfolio exposures. Indeed, the fact that you can still get large differences based on specific design choices is much of our point; we will revisit these design choices later in the article.

A common approach to measuring factor exposures is linear regression analysis, which describes the relationship between a dependent variable (portfolio returns) and explanatory variables (risk factors). Static (full sample) regression analysis provides information on average exposures over a given period but will not provide any insight into whether a manager varies factor exposures over time. To understand how factor exposures vary over time, one can look at dynamic (rolling-window) regression betas, ideally using at least 36 months of data.⁷

Regression analysis can be done with one risk factor or with as many as the portfolio aims to capture. If the portfolio captures multiple styles, then multiple factors should be used. If the portfolio is a global multiasset style portfolio, then the relevant factors should cover multiple assets in a global context. Ideally, the factors used should be similar to those implemented in the portfolio, or one should at least account for those differences in assessing the results. For example, long-only portfolios are more constrained in harvesting style premiums because underweights are capped at their respective benchmark weights. In contrast, long–short factors

EXHIBIT 2

Hypothetical Performance Statistics (January 1980–December 2014)

	Portfolio	Market	Value (HML)	Momentum (UMD)	Size (SMB)
Annual Excess Returns	13.5%	7.8%	3.6%	7.3%	1.6%
Volatility	17.8%	15.6%	10.5%	15.8%	10.6%
Correlation with Portfolio		0.84	−0.06	−0.08	0.53

Notes: All returns are arithmetic averages. Returns are in excess of cash. The portfolio is a hypothetical simple 50/50 value and momentum long-only, small-cap equity portfolio, gross of fees and transaction costs and excess of cash. The portfolio is rebalanced monthly. The academic explanatory variables are the contemporaneous monthly Fama–French factors for the market (MKT-RF), value (HML), momentum (UMD), and size (SMB). The market is the value-weight return of all Center for Research in Security Prices (CRSP) firms. Hypothetical data have inherent limitations, some of which are discussed herein.

Sources: AQR, Ken French Data Library.

(and portfolios) are purer in that they are unconstrained. These differences should be accounted for when performing and interpreting factor analysis.⁸

For this analysis, we examine a hypothetical long-only equity portfolio that aims to capture returns from value, momentum, and size. Specifically, the portfolio is constructed with 50/50 weight on simple measures of value (B/P, using current prices⁹) and momentum (price return over the last 12 months) within the small-cap universe.¹⁰ In practice, an investor may not know the portfolio exposures in advance, but because our goal is to illustrate how to best apply the analysis, we will proceed as if we do.

We start with a simple one-factor model and then add the additional factors that the portfolio aims to capture. We analyze style exposures using academic factors (over practitioner factors) for simplicity and illustrative purposes. The performance characteristics of the portfolio and factors used are shown in Exhibit 2, which shows that the portfolio returned an annual 13.5% in excess of cash, on average, from 1980 to 2014.

It is important to note that we are analyzing a very long history, which may not be available in practice. In general it is important to include as many observations as possible, with a minimum of 36 months being a general rule of thumb.

We can use these returns and betas from regression analysis to decompose portfolio excess of cash returns

$(R_i - R_f)$.¹¹ The first regression model we look at is the CAPM with the market as the only factor¹²:

$$(R_i - R_f) = \alpha + \beta_{\text{MKT}}(R_{\text{MKT}} - R_f) + \epsilon \quad (1)$$

Or roughly,

$$\begin{aligned} &\text{Portfolio returns in excess of cash} \\ &= \text{Alpha} + \text{Beta} \times \text{Market risk premium}^{13} \end{aligned}$$

The results in Exhibit 3 show that the portfolio had a market beta of 0.96 (based on Model 1 in Panel A). This means—not surprisingly, because the portfolio is long-only—that the portfolio had meaningful exposure to the market. We also know (from Exhibit 2) that the equity market has done well over this period, delivering 7.8% excess of cash returns. As a result, we can see (in Panel B of Exhibit 3) that the portfolio’s positive exposure to the market contributed 7.4% to overall returns¹⁴ and that 6.1% was “alpha” in excess of market exposure.

The same framework can be applied for multiple risk factors. Our first multivariate regression adds the value factor:

$$\begin{aligned} (R_i - R_f) &= \alpha + \beta_{\text{MKT}}(R_{\text{MKT}} - R_f) \\ &+ \beta_{\text{HML}}(R_{\text{HML}}) + \epsilon \end{aligned} \quad (2)$$

The results under Model 2 show that the portfolio had positive exposure to value (with a beta of 0.43), which means that the portfolio, on average, bought cheap stocks.¹⁵ Because value is a historically rewarded long-run source of returns, having positive exposure benefited the portfolio, with value contributing 1.6% to portfolio returns (HML beta $\times$ HML risk premium = $0.43 \times 3.6\%$).

Next we add the momentum factor in Model 3:

$$\begin{aligned} (R_i - R_f) &= \alpha + \beta_{\text{MKT}}(R_{\text{MKT}} - R_f) + \beta_{\text{HML}}(R_{\text{HML}}) \\ &+ \beta_{\text{UMD}}(R_{\text{UMD}}) + \epsilon \end{aligned} \quad (3)$$

As one would expect, we see that the momentum loading is positive (with a beta of 0.09), which means that the portfolio, on average, bought recent winners. The magnitude of this exposure, however, is smaller than expected for a portfolio that aims to capture returns from momentum investing. It seems that momentum

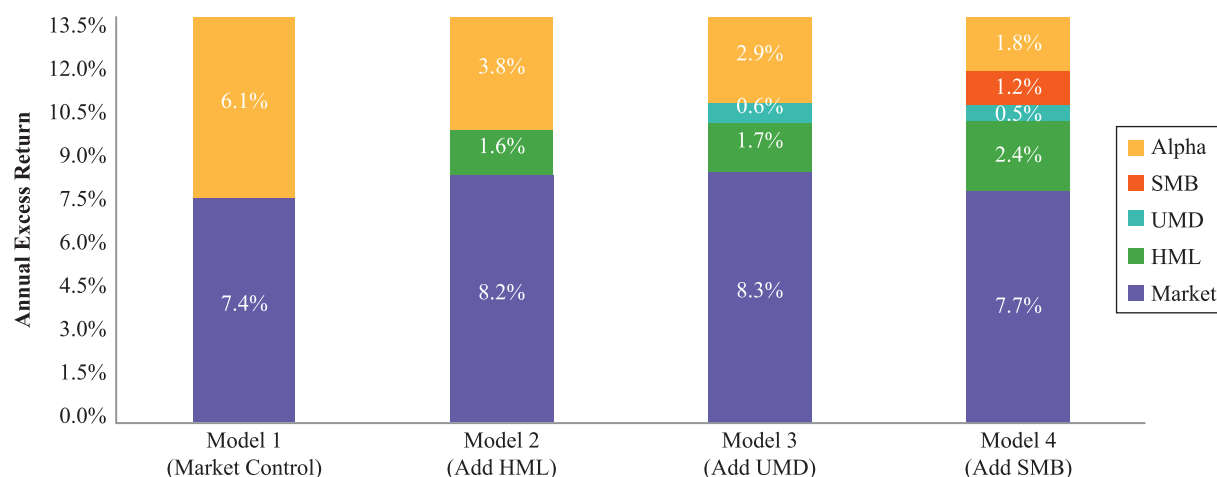
EXHIBIT 3

Decomposing Hypothetical Portfolio Returns by Factors (January 1980–December 2014)

Panel A: Regression Results

	Model 1 (Market Control)	Model 2 (Add HML)	Model 3 (Add UMD)	Model 4 (Add SMB)
Alpha (ann.)	6.1%	3.8%	2.9%	1.8%
<i>t</i> -statistic	3.6	2.5	1.9	2.2
Market Beta	0.96	1.05	1.07	0.99
<i>t</i> -statistic	31.1	35.7	36.0	61.5
HML Beta		0.43	0.46	0.65
<i>t</i> -statistic		9.8	10.3	26.4
UMD Beta			0.09	0.07
<i>t</i> -statistic			3.0	4.6
SMB Beta				0.74
<i>t</i> -statistic				32.2
R ²	0.70	0.75	0.76	0.93

Panel B: Portfolio Return Decomposition



Notes: All returns are arithmetic averages. The bar chart in Panel B uses the factor returns (from Exhibit 2) and factor betas (from Panel A) to decompose portfolio returns. Numbers may not exactly tie out because of rounding. AQR analysis is based on a hypothetical simple 50/50 value and momentum long-only, small-cap equity portfolio, gross of fees and transaction costs and excess of cash. The portfolio is rebalanced monthly. The academic explanatory variables are the contemporaneous monthly Fama–French factors for the market (MKT-RF), value (HML), momentum (UMD), and size (SMB). The market is the value-weight return of all CRSP firms. Hypothetical data have inherent limitations, some of which are discussed herein.

Sources: AQR, Ken French Data Library.

only contributed 0.6% to portfolio returns (UMD beta $\times$ UMD risk premium = $0.09 \times 7.3\%$), whereas value contributed 1.7%. This may seem odd for a portfolio that is built with a 50/50 combination of value and momentum, but we should keep in mind that we are still looking at an incomplete model—one without all the risk factors in the portfolio. Let's see what happens when we add the size variable in our next model (Model 4 in Exhibit 3):

$$(R_i - R_f) = \alpha + \beta_{\text{MKT}}(R_{\text{MKT}} - R_f) + \beta_{\text{HML}}(R_{\text{HML}}) + \beta_{\text{UMD}}(R_{\text{UMD}}) + \beta_{\text{SMB}}(R_{\text{SMB}}) + \epsilon \quad (4)$$

In our final model (which includes all the sources of return that the portfolio aims to capture), we still see a small beta on momentum, with the factor contributing 0.5% to portfolio returns over the period (UMD beta $\times$ UMD risk premium = $0.07 \times 7.3\%$).

However, this unintuitive result can be largely explained by factor design differences. Stay tuned and we will come back to this issue later in the article.¹⁶

The good news is that when it comes to the other factors in Model 4, the results are consistent with intuition. For size, we see a large positive exposure (beta of 0.74), which means the portfolio had meaningful exposure to small-cap stocks. This exposure contributed 1.2% to portfolio returns over the period. We also see that after controlling for size, value had an even larger beta, which means that it contributed 2.4% to portfolio returns.

Ultimately, in interpreting the results of regression analysis, investors should focus on the model that includes the systematic sources of returns that the portfolio aims to capture; in this case, it would be Model 4. For portfolios that capture styles in an integrated way, it is important to include multiple factors to control for the correlation between factors—in other words, to take into account how factors are related to each other. It is well known that value and momentum are negatively correlated, and portfolios formed in an integrated way can take advantage of this correlation. Focusing on how value performs as a standalone (i.e., Model 2) has little implication on how value adds to a portfolio that combines value with other factors synergistically (i.e., Model 4). One of the benefits of multifactor investing is the relatively low correlation factors have with each other, thus making the whole more efficient than the sum of its parts.

Alpha vs. Beta

Although betas are important in understanding factor exposures in a portfolio, alpha can be useful in analyzing manager “skill.”¹⁷ It is important that investors are able to tell whether a manager is actually providing alpha above and beyond their intended factor exposures, but this means that they need to be sure that they are using the correct model when analyzing factor exposures. Without the proper model, rewards for factor exposures may be misconstrued as alpha. This can lead to suboptimal investment choices, such as hiring a manager who seems to deliver alpha but really just provides simple factor tilts.

To illustrate this point, we can look at the alpha estimates in Exhibit 3.¹⁸ By looking at each model on a stepwise basis, we can see how the inclusion of additional risk factors reduces alpha significantly; in other words,

alpha has been replaced by factor exposures. When the market is the only factor (Model 1), it seems as though the portfolio has significant alpha at 6.1%, but when we add the other risk factors we see that alpha is reduced to 2.9% with value and momentum and finally to 1.8% with all four factors.¹⁹ These results have important implications: If one does not control for multiple exposures in a multifactor portfolio, then excess returns will look as if they are mostly alpha.

It is also important to note, however, that “alpha” depends on what is already in a portfolio. For *any* portfolio, adding positive expected return strategies that are uncorrelated to existing risk exposures can provide significant portfolio alpha. For the market portfolio, adding value and momentum exposures can have the same effect as adding alpha (because both represent new, more efficient sources of portfolio returns).²⁰ Along the same lines, adding momentum to a value portfolio can provide significant alpha.

The main point is this: To determine whether such a factor can be “alpha to you,” investors must first determine which factors are already present in their existing portfolio—those that are not can potentially be alpha.

FACTOR DIFFERENCES: ACADEMICS VS. PRACTITIONERS

So far, we have focused on factor analysis and how to interpret the results, but the results of factor analysis are highly influenced by how factors are formed. There are many differences between the ways factors are measured from an academic standpoint versus how they are implemented in portfolios. Investors should be aware that not all factors are the same, even those attempting to measure essentially the same economic phenomenon, and these differences can matter. We focus here on design decisions that can have a meaningful impact on the results of factor analysis.

Implementability

At a basic level, academic factors do not account for implementation costs. They are gross of fees, transaction costs, and taxes. They do not face any of the real-world frictions that implementable portfolios do. Essentially, they are not a perfect representation of how factors are implemented in practice.

Differences in implementation approaches may be reflected in factor model results. Even if a portfolio captures the factors perfectly, it could still have negative alpha in the regression, which would represent implementation differences associated with capturing the factors. For example, if one compares a portfolio that faces trading costs with a portfolio that does not, clearly the former will underperform the latter, possibly implying negative alpha. In fact, this is exactly what we see when we look at a composite of mutual funds; these results are shown in Appendix D. When looking at a live portfolio against academic factors, investors should not be surprised by negative alpha. In these cases, investors should either use practitioner factors on the right-hand-side or just focus on beta comparisons because trading costs and other implementation issues do not affect these estimates.²¹

Investment Universe

Academic factors (such as those used here) span a wide market-capitalization range and are, in fact, overly reliant on small-cap stocks or even micro-cap stocks (we will explain this in greater detail in the next section). The factors include the entire CRSP universe of approximately 5,000 stocks. Many practitioners would agree that a trading strategy that dips far below the Russell 3000 is not a very implementable one, and this is likely where most of the bottom two quintiles in the academic factors fall.

Practitioners mainly focus on large- to mid-cap universes for investability reasons. For portfolios that provide exposure to the large-cap universe, academic factors may not be an accurate representation of desired exposures. Given that academic factors span a wide range of market capitalization, factor analysis results will be greatly affected by the influence of some other part of the capitalization range—a range that is not being captured in the portfolio by design.

Factor Weighting

Generally, academic factors are formed using an intersection of size and their particular factor (value, in the case of HML).²² For the factors described in Exhibit 1, a stock's size is determined by the median market capitalization, which means a roughly equal number of stocks are considered “big” and “small.”²³ The factors are

formed by giving equal capital weight to each universe, which, given the higher risk of small stocks, likely means that an even greater risk weight and contribution comes from small stocks. Thus, betas tend to be underestimated because of stale prices for micro-cap stocks.

Practitioners generally take views on the entire universe, assigning larger positive weights to the stocks that rank more favorably on some measure and larger negative weights to the less-favorable stocks. For example, practitioners may weight stocks by accounting for the relative cheapness, or how “strong” in value each stock is. This approach assigns larger positive weights to the stocks that rank more favorably on B/P, for example. Practitioners would weight stocks in the resulting portfolio via each stock's standardized rank (i.e., signal-weighting), or they might use a blend—say 50/50—of the standardized rank and market-capitalization weight. Both weighting schemes result in increased exposure to stocks with high value ranks as compared to a simple value portfolio that weights the top 50% B/P stocks based on market cap.

Industries

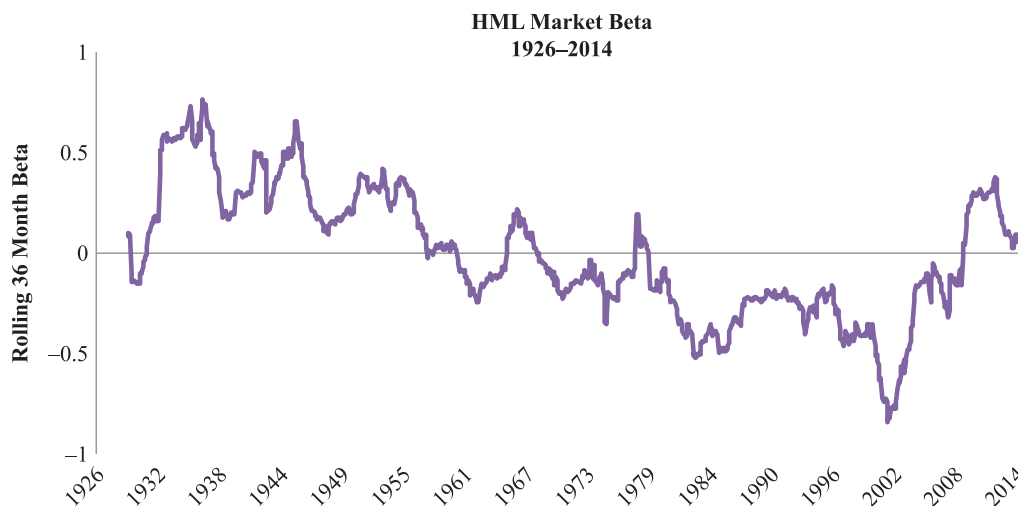
Academic factors do a simple ranking across stocks, and in doing so they implicitly take style bets within and across industries (and across countries in international portfolios), without any explicit risk controls on the relative contributions of each. In contrast, the factors implemented by practitioners may differentiate stocks within and across industries (i.e., industry views). They are designed to capture and target risk to both *independently*. This distinction can result in a more diversified portfolio, one without unintended industry concentrations.

Risk Targeting

Risk targeting is a technique that many practitioners use when constructing factors; this approach dynamically targets risk to provide more consistent realized volatility in changing market conditions. Although this technique can ensure that a portfolio stays diversified over time (so portfolio risk does not fluctuate with market volatility), it can also help when building a risk-balanced multistyle portfolio. That is, this technique can help practitioners ensure that their desired risk weights are maintained (in a similar vein

EXHIBIT 4

Varying Market Exposure of HML over Time



Notes: Analysis based on the market (MKT-RF) and HML portfolios. The market is the value-weight return of all CRSP firms.

Sources: AQR, Ken French Data Library.

to rebalancing portfolios to preserve strategic asset allocations). Practitioners can also build market (or beta) neutral long–short portfolios.

Academic factors typically do not use risk targeting because their factors are returns to a \$1 long/\$1 short portfolio, whose risk and market exposures can vary. The effect of this can be seen in Exhibit 4, which shows how HML has significant variation in market exposure over time.²⁴

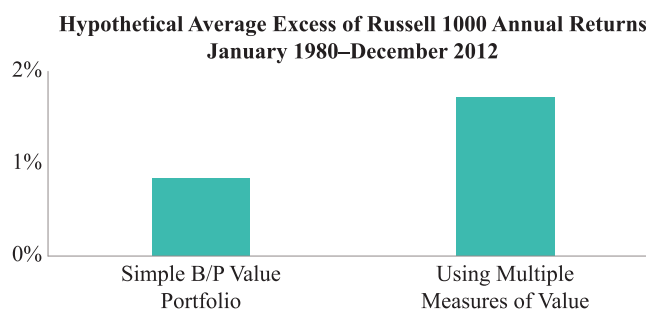
Multiple Measures of Styles

Although stocks selected using the traditional academic value measure perform well in empirical studies, there is no theory that says B/P is the best measure for value. Other measures can be used and applied simultaneously to form a more robust and reliable view of a stock's value. For example, investors can look at a variety of other reasonable fundamentals, including earnings, cash flows, and sales, all normalized by some form of price. Factors that draw on multiple measures of value can significantly improve performance, as shown in Exhibit 5 (Asness et al. [2014, 2015a] Israel and Moskowitz [2013]).

The same intuition applies for other styles. For example, momentum factors that include both earnings momentum and price momentum may be more robust portfolios.

EXHIBIT 5

Design Decisions Are Important in Portfolio Construction



Notes: B/P is defined using current price. The multiple measures of value include B/P, earnings-to-price, forecasted earnings-to-price, cash flow-to-enterprise value, and sales-to-enterprise value.

Source: Frazzini et al. [2013].

Other Factor Design Choices

Other design decisions can have a meaningful impact on returns. Looking at the case of value, Fama and French construct HML using a lagged value for price that creates a noisy combination of value and momentum. When forming their value portfolio on B/P, they use the price that existed contemporaneously with the book value, which because of financial

reporting can be lagged by 6 to 18 months. Thus, a company that looked expensive based on its book value and price from six months ago and whose stock has fallen over the past six months should look better from a valuation perspective (because the price is lower and the holding book value constant²⁵). Yet, in a traditional definition (using *lagged prices*), the stock is viewed the same way irrespective of the price move.

An alternative way of looking at the stock discussed above is to define value with the *current price*, which means the stock is now cheaper. On the other hand, if one incorporates momentum into the process, the stock does not look any better because its price has fallen over the past six months. Putting the two together, the stock looks more attractive from a value perspective but less attractive from a momentum perspective, with the net effect ending up potentially in the same place as the traditional definition of value. As a result, the traditional definition can be viewed as an incidental bet on both value and momentum; in fact, empirically the traditional definition of value ends up being approximately 80% pure value (current price) and 20% momentum (Asness and Frazzini [2013]).

To correct for this noisy combination of value and momentum, Asness and Frazzini [2013] suggested replacing the 6- to 18-month lagged price with the current price to compute valuation ratios that use more updated information. Measuring HML using current price (what it is called *HML Devil*) eliminates any incidental exposure to momentum, resulting in a better proxy for true value while still using information available at the time of investing.

This factor design choice is especially important when interpreting regression results. When regressing a portfolio of value and momentum on UMD and HML (using the traditional academic definition), it will appear that UMD has a lower loading because HML is eating up some of the UMD loading that would otherwise exist. This is exactly what we saw in Exhibit 3, where UMD had a very low loading. However, if HML is defined using current price (as is the case with HML Devil), the value loading will no longer have exposure to momentum and any momentum exposure in the portfolio will go directly into UMD, thus raising its loading. This is consistent with what we see when we make the HML Devil correction to the analysis from Exhibit 3: The UMD loading increases from 0.07 to 0.32 (these results are shown in the Appendix in Exhibit B1).

In this section, we have discussed a few factor differences that can meaningfully affect the results of factor analysis. As a result, we encourage investors to be aware of these differences when interpreting regression results.

CONCLUDING REMARKS

Market exposure has historically rewarded long-term investors, but market risk is only one exposure among several that can deliver robust long-term returns. Measuring exposure to risk factors can be a challenge: Factors can be formed multiple ways, and statistical analysis is ridden with nuances. Ultimately, investors who understand how to measure factor exposures may be better able to build truly diversified portfolios.

The following summary points are useful for investors to think about when decomposing portfolios into risk factors:

- Even a single factor, such as value, has variations that an investor should consider: There are many differences between how factors are constructed from an academic standpoint versus how they are implemented in portfolios. In conducting factor analysis, investors should ask themselves: What exactly are these factors I'm using? Are they the same as those I'm getting in my portfolio? The answers to these questions affect beta and alpha estimates. Factor loadings are highly influenced by the design and universe of factors, and alpha estimates reflect the implementation differences associated with capturing the factors. For example, if one compares a portfolio that faces trading costs versus one that does not, it is not surprising the former will underperform the latter and possibly show negative alpha. When investors want to compare alphas and betas across managers, they should be sure they are using the factors being captured in the portfolios. Ultimately, not properly accounting for factor exposures can lead to suboptimal investment choices, such as hiring an expensive manager that seems to deliver alpha but really just provides simple factor tilts.
- Many things should be considered when performing statistical analysis on portfolios. For portfolios with more than one risk factor, multivariate models are most appropriate. As discussed in Appendix A,

investors should consider t -statistics, not just betas, to assess factor exposures, especially when comparing portfolios with different volatilities.

- To determine whether a certain factor exposure can be “alpha to you,” investors must first determine which factors are already present in their existing portfolio—those that are not can potentially be alpha.

APPENDIX A

THE STATISTICS OF REGRESSION ANALYSIS

We hope these details will help investors better understand and interpret the results of regression models.

The Mechanics of Beta

Investors looking to analyze portfolio exposures often look at betas of regression results. Beta measures the sensitivity of the portfolio to a certain factor. In the case of market beta, it tells us how much a security or portfolio’s price tends to change when the market moves. From a mathematical standpoint, the beta for portfolio i is equal to its correlation with the market times the ratio of the portfolio’s volatility to the market’s volatility.²⁶

$$\beta_{i,m} = \rho_{i,m}(\sigma_i/\sigma_m)$$

or,

Factor beta

$$= \text{Factor correlation with portfolio} \times \left(\frac{\text{Portfolio volatility}}{\text{Factor volatility}} \right)$$

Because volatility varies considerably across portfolios, comparisons of betas can be misleading. Using the preceding equation, we can see that for the same level of correlation, the higher a portfolio’s volatility, the higher its beta. Let’s see why this matters.

Suppose an investor is comparing value exposure for two different portfolios: Portfolio A is a defensive equity portfolio (with lower volatility), and portfolio B is a levered equity portfolio (with higher volatility). It could be the case that portfolio B has a higher value beta, which would seem to indicate that it has higher value exposure. However, the higher beta could be a result of portfolio B’s higher volatility rather than more meaningful value exposure (assuming the same level of correlation between both portfolios and the value factor). When investors fail to account for different

levels of volatilities between portfolios, they may conclude that one portfolio is providing more value exposure than another—it does in notional terms, but that may not be the case in terms of exposure per unit of risk.

This approach can also be extended to comparisons of different factors for the same portfolio. Looking back at Exhibit 3 under Model 4, we can compare the loadings on value and momentum. One would expect similar betas on these factors because the portfolio is built to target each equally (with 50/50 weight).²⁷ Even with similar correlation with the portfolio, however, value has a meaningfully higher loading (looking at Model 4). Does this mean that value contributes more than momentum? Not necessarily, because we need to account for their differing levels of volatility. For the same level of correlation, the higher a factor’s volatility, the lower its beta. Put differently, the lower beta on UMD versus HML is partly driven by differing volatility levels²⁸—from Exhibit 2, we see that UMD had volatility of 15.8%, whereas HML had volatility of 10.5%.

Investors can make adjustments to allow for more direct beta comparisons. When comparing factors for the same portfolio, the impact of differing volatilities should be eliminated; this can be done by volatility scaling the right-hand-side factors such that they all realize the same volatility. For those looking to compare betas across portfolios (on a risk-adjusted basis), they can look at correlations, which are invariant to volatility and can be compared more directly across portfolios with different volatilities.²⁹

Portfolio Risk Decomposition

Betas from regression analysis can also be used in portfolio risk attribution. This approach is best thought of as variance decomposition and is done by using factor beta, factor volatility, portfolio volatility, and factor correlations.³⁰ For example, from Exhibits 2 and 3 we see that the market factor had an average volatility of 15.6% and a market beta of 0.96 (based on Model 1). This tells us that the market factor dominates the risk profile of the portfolio, contributing an estimated 14.9% risk to the portfolio ($\sqrt{\text{market beta}^2 \times \text{market volatility}^2} = \sqrt{0.96^2 \times 15.6\%^2}$).³¹ Given that overall portfolio risk is 17.8%, we can estimate the proportion of variance that is being driven by market exposure $\left(\frac{\text{Market variance contribution}}{\text{Portfolio variance}} \right) = \left(\frac{14.9\%^2}{17.8\%^2} \right) = 0.70$. This means that roughly 70% of portfolio variance can be attributed to the market risk factor.³² There is an interesting application of this result: 0.70 is the same as the R^2 measure for Model 1 (shown in the final row of the regression table in Exhibit 3). We will now discuss R^2 in more detail.

The R^2 Measure: Model Explanatory Power

The R^2 measure provides information on the overall explanatory power of the regression model. It tells us how much of returns are explained by factors included on the right-hand side of the equation. Generally, the higher the R^2 , the better the model explains portfolio returns. We can see from the R^2 measure at the bottom of the table in Exhibit 3 that multivariate analysis is more effective (than univariate) at explaining returns for a multifactor portfolio. In particular, we see in the final column of the table that the inclusion of additional risk factors has improved the explanatory power of the model (i.e., how much of the portfolio variance is being captured by these factors), with the R^2 improving from 0.70 to 0.93.³³

The t -statistic: A Measure of Statistical Significance

Although beta tells us whether a factor exposure is economically meaningful (and how much a factor may contribute to risk and returns), it does not tell us whether the factor exposure is statistically significant. Just because a portfolio has a high beta coefficient to a factor does not mean it is statistically different from a portfolio with a zero beta, or no factor exposure. As such, it is important to look at the t -statistic.

This measure tells us whether a particular factor exposure is statistically significant. It is a measure of how confident we are about our beta estimates.³⁴ When the t -statistic is greater than two, we can say with 95% confidence (or a 5% chance we are wrong) that the beta estimate is statistically different from zero.³⁵ In other words, we can say that a portfolio has significant exposure to a factor.

Looking back at the momentum factor, even though the portfolio may not have an economically meaningful beta (at 0.07 in Model 4), we can see that it is statistically significant (with a t -statistic greater than two). The t -statistic is an especially important measure for comparing portfolios with different volatilities.

At the end of the day, both beta and t -statistics provide valuable information when assessing factor exposures. A factor exposure that is both economically meaningful and statistically significant (reliable) can be counted on to affect a portfolio in a big way. An exposure that is only economically meaningful but not reliable *could* have a big impact, but with a high degree of uncertainty. Finally, an exposure that is small but reliable means one can expect (with greater certainty) that it will affect the portfolio, but only in a small way. Although an investor may not care much about this last application, it is still worth understanding when analyzing the regression output.

APPENDIX B

CORRECTING FOR HML DEVIL

EXHIBIT B 1

Decomposing Hypothetical Portfolio Returns by Factors (January 1980–December 2014)

Panel A: Regression Results

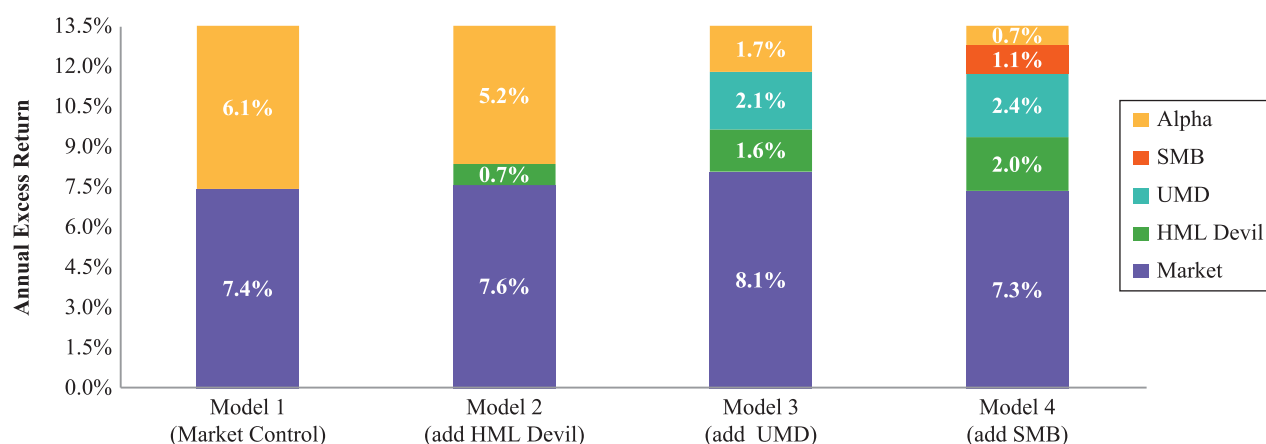
	Model 1 (Market Control)	Model 2 (Add HML Devil)	Model 3 (Add UMD)	Model 4 (Add SMB)
Alpha (ann.)	6.1%	5.2%	1.7%	0.7%
<i>t</i> -statistic	3.6	3.2	1.1	0.7
Market Beta	0.96	0.98	1.04	0.94
<i>t</i> -statistic	31.1	32.8	35.5	50.0
HML Devil Beta		0.22	0.48	0.61
<i>t</i> -statistic		5.9	9.6	19.0
UMD Beta			0.29	0.32
<i>t</i> -statistic			7.3	12.9
SMB Beta				0.68
<i>t</i> -statistic				25.1
R^2	0.70	0.72	0.75	0.90

(continued)

EXHIBIT B 1 (continued)

Decomposing Hypothetical Portfolio Returns by Factors (January 1980–December 2014)

Panel B: Return Decomposition



Notes: AQR analysis based on a hypothetical simple 50/50 value and momentum long-only, small-cap equity portfolio, gross of fees and transaction costs and excess of cash. The portfolio is rebalanced monthly. The academic explanatory variables are the contemporaneous monthly academic factors for the market (MKT-RF), value (HML Devil), momentum (UMD), and size (SMB). The portfolio returned 13.5% in excess of cash, on average, over the period, the market returned 7.8% excess of cash, HML Devil returned 3.3%, UMD returned 7.3%, and SMB returned 1.6%. The market is the value-weight return of all CRSP firms. Hypothetical data have inherent limitations, some of which are discussed herein.

APPENDIX C

ALTERNATE METHOD OF HYPOTHETICAL PORTFOLIO RISK DECOMPOSITION

For this example, we use a simple 50/50 value/momentum long/short portfolio.

Step 1: Determine the Covariance Matrix

Using assumptions on volatility and correlation³⁶ (inputs in blue), we create the covariance matrix.

Portfolio Inputs		
	Volatility	
Value (HML)	11%	
Momentum (UMD)	16%	
Correlation Matrix		
	Value (HML)	Momentum (UMD)
Value (HML)	1.0	-0.2
Momentum (UMD)	-0.2	1.0
Covariance Matrix		
	Value (HML)	Momentum (UMD)
Value (HML)	0.012	-0.003
Momentum (UMD)	-0.003	0.012

Covariance (HML, UMD)

$$\begin{aligned}
 &= \text{Correlation (HML, UMD)} \times \text{Volatility (HML)} \\
 &\quad \times \text{Volatility (UMD)} \\
 &= -0.2 \times 11\% \times 16\% \\
 &= -0.003
 \end{aligned}$$

Step 2: Determine the Variance Contribution of Each Factor

Using capital weights and the covariance matrix from step 1 (shown by the inputs in blue in the following), we can determine the variance contribution (VAR Contrib.) of each factor.

Portfolio Inputs		
	Volatility	Capital Weights
Value (HML)	11%	50%
Momentum (UMD)	16%	50%
Covariance Matrix		
	Value (HML)	Momentum (UMD)
Value (HML)	0.012	-0.003
Momentum (UMD)	-0.003	0.012
Variance		
Value (HML)	0.23%	
Momentum (UMD)	0.57%	
Portfolio	0.80%	

VAR Contrib. (HML)

$$\begin{aligned}
 &= \text{Weight (HML)}^2 \times \text{Volatility (HML)}^2 \times \text{Weight (HML)} \\
 &\quad \times \text{Weight (UMD)} \times \text{Covariance (HML, UMD)} \\
 &= 50\%^2 \times 11\%^2 + 50\% \times 50\% \times -0.003 \\
 &= 0.23\%
 \end{aligned}$$

Note that unlike volatility, portfolio variance is additive:

$$\begin{aligned}
 \text{VAR (Portfolio)} &= \text{VAR Contrib. (HML)} + \text{VAR Contrib. (UMD)} \\
 &= 0.23\% + 0.57\% \\
 &= 0.80\%
 \end{aligned}$$

Step 3: Determine the Percent Risk/Variance Contribution of Each Factor

Finally, using the variance from step 2, we can determine the percent of portfolio variance coming from each factor.

	Volatility	Capital Weights	Variance
Value (HML)	11.0%	50%	0.23%
Momentum (UMD)	16.0%	50%	0.57%
Portfolio	8.9%	100%	0.80%

	% Contribution to Variance
Value (HML)	30%
Momentum (UMD)	70%
Portfolio	100%

% Contribution to Variance (HML)

$$\begin{aligned}
 &= \frac{\text{VAR Contrib. (HML)}}{\text{VAR (Portfolio)}} \\
 &= \frac{0.23\%}{0.80\%} \\
 &\approx 30\%
 \end{aligned}$$

APPENDIX D

APPLICATIONS FOR A LIVE PORTFOLIO

In this article, we have focused on a hypothetical portfolio that aims to capture returns from value and momentum. We have done this for simplicity and illustrative purposes, but the same framework can be applied for any portfolio. So, what about a live portfolio? Should we expect the same results? In this section, we use the Morningstar style boxes to identify and analyze the universe of small-cap value managers. That is, we look at a composite of all small-cap value managers as identified by Morningstar.³⁷

The factor exposures shown here are directionally similar to those shown for the hypothetical portfolio we have analyzed in this article. As expected, we see positive and significant exposure to the market, value, and size.³⁸ An interesting result, however, comes from a comparison of alpha: We see that alpha goes from zero to negative in the final model. Although this result is different from the stylized example we examined in this article, it is consistent with our section on implementable factors. Ultimately, live portfolios face fees, transaction costs, and taxes—all of which fall out of alpha.

EXHIBIT D1

Analyzing a Composite of Small-Cap Value Managers (January 1980–December 2014)

Panel A: Regression Results

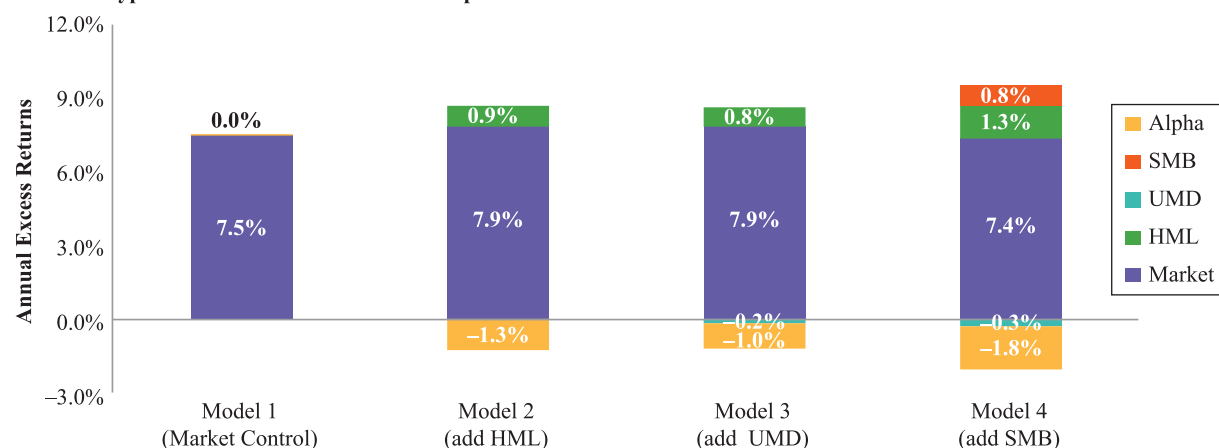
	Model 1 (Market Control)	Model 2 (Add HML)	Model 3 (Add UMD)	Model 4 (Add SMB)
Alpha (ann.)	0.0%	-1.3%	-1.0%	-1.8%
<i>t</i> -statistic	0.0	-1.0	-0.8	-2.1
Market Beta	0.96	1.01	1.01	0.95
<i>t</i> -statistic	40.5	42.2	41.3	57.1
HML Beta		0.23	0.23	0.36
<i>t</i> -statistic		6.6	6.2	14.3
UMD Beta			-0.02	-0.04
<i>t</i> -statistic			-1.0	-2.3
SMB Beta				0.54
<i>t</i> -statistic				22.4
R ²	0.80	0.82	0.82	0.92

(continued)

EXHIBIT D1 (continued)

Analyzing a Composite of Small-Cap Value Managers (January 1980–December 2014)

Panel B: Hypothetical Portfolio Return Decomposition



Notes: AQR analysis based on the Morningstar universe of small-cap value mutual funds. The composite returns are net of management and performance fees. The academic explanatory variables are the contemporaneous monthly Fama–French factors for the market (MKT-RF), value (HML), momentum (UMD), and size (SMB). The portfolio returned 7.5% in excess of cash, on average, over the period, the market returned 7.8% excess of cash, HML returned 3.6%, UMD returned 7.3%, and SMB returned 1.6%.

ENDNOTES

We would like to thank Cliff Asness, Marco Hanig, Lukasz Pomorski, Lasse Pedersen, Rodney Sullivan, Scott Richardson, Antti Ilmanen, Tobias Moskowitz, Daniel Villalon, Sarah Jiang, and Nick McQuinn for helpful comments and suggestions.

¹Applying styles across multiple asset classes provides greater diversification. In addition, the effectiveness of styles across asset classes helps dissuade criticisms of data mining (Ilmanen [2011]; Asness, Moskowitz, and Pedersen [2013]; Asness et al. [2015b]). Past performance is not indicative of future results.

²Although a holdings-based approach may provide more precise estimates at each point in time (and may also indicate how managers vary their exposures over time), it may not be a feasible approach for many investors. It requires significant resources and infrastructure and often leads to challenges with data—an investor may not always have access to the holdings, or holdings may be available at a lag or may only be partially available (i.e., 13F filings do not reflect short positions). In contrast, a regression-based approach is more accessible and still provides useful information in analyzing exposures. Ultimately, where possible, investors should look at both approaches for a more robust sense of portfolio factor exposures.

³The long-only style tilt portfolio will still have significant market exposure. This type of style portfolio is often referred to as a *smart beta* portfolio.

⁴CAPM says the expected return on any security is proportional to the risk of that security as measured by its market beta.

⁵More generally, the economic definition of alpha relates to returns that cannot be explained by exposure to common risk factors (Berger et al. [2012]).

⁶Specifically, these factors are constructed as follows: SMB and HML are formed by first splitting the universe of stocks into two size categories (S and B) using NYSE market-cap medians and then splitting stocks into three groups based on book-to-market equity (highest 30% [H], middle 40% [M], and lowest 30% [L], using NYSE breakpoints). The intersection of stocks across the six categories are value-weighted and used to form the portfolios SH [small, high book-to-market equity (BE/ME)], SM [small, middle BE/ME], SL [small, low BE/ME], BH [big, high BE/ME], BM [big, middle BE/ME], and BL [big, low BE/ME]), where SMB is the average of the three small stock portfolios (1/3 SH + 1/3 SM + 1/3 SL) minus the average of the three big stock portfolios (1/3 BH + 1/3 BM + 1/3 BL), and HML is the average of the two high book-to-market portfolios (1/2 SH + 1/2 BH) minus the average of the two low book-to-market portfolios (1/2 SL + 1/2 BL). UMD is constructed similarly to HML, in which two size groups and three momentum groups (highest 30% [U], middle 40% [M], and lowest 30% [D]) are used to form six portfolios, and UMD is the average of the small and big winners minus the average of the small and big losers (Fama and French [1996]).

⁷The trade-off is that some, perhaps a lot, of this variation may in fact be random noise. Past performance is not indicative of future results.

⁸Because unconstrained long-short factors can capture the underlying styles more efficiently, long-only portfolios are essentially penalized when long-short factors are used in the regression; this is because the regression expects the long-only portfolio to harvest returns to the same extent that long-short factors can. In these cases, investors should be cautious with their interpretation of alpha.

⁹Fama-French HML uses lagged prices. See the section on other factor design choices.

¹⁰See Frazzini et al. [2013] for more detail on how to construct a multistyle portfolio. Note that we have followed a similar multistyle portfolio construction approach here. To build our portfolio, we rank stocks based on simple measures for value (book-to-price using current prices) and momentum (price return over the last 12 months) within the U.S. small-cap universe (Russell 2000 Index). We compute a composite rank by applying a 50% weight to value and 50% to momentum. We then select the top 25% of stocks with the highest combined ranking and weight the stocks in the resulting portfolio via a 50/50 combination of each stock's market capitalization and standardized combined rank. Portfolio returns are gross of transaction costs, unoptimized, and undiscounted. The portfolios are rebalanced monthly.

¹¹One of the most common mistakes in running factor analysis is forgetting to take out cash from the returns of the left- and right-hand-side variables. For a long-only factor or portfolio, such as the market, one must explicitly do this. A long-short factor is a self-financed portfolio whose returns are already in excess of cash.

¹²We have also included an error term (ϵ), which is the difference between actual realized returns and expected returns. More specifically, the error term captures the unexplained component of the relationship between the dependent variable (e.g., the portfolio excess returns) and explanatory variables (e.g., the market risk premium).

¹³All risk premiums in this article are returns in excess of cash.

¹⁴Market beta $\times$ Market risk premium = $0.96 \times 7.8\%$.

¹⁵Even though value has a negative univariate correlation with the portfolio (as seen in Exhibit 2), we can see that after controlling for market exposure (in Exhibit 3), the portfolio loads positively on value. We will discuss the importance of multivariate over univariate regressions for a multifactor portfolio later in the article.

¹⁶See the section on other factor design choices in which we discuss how HML can be viewed as an incidental bet on UMD; this affects regression results by lowering the loading on UMD (as HML is eating up some of the UMD loading that would otherwise exist). We correct for this in Appendix B

and show a higher loading on UMD. See also Frazzini et al. [2013] and Asness et al. [2014] for more information on the most efficient way to gain exposure to momentum.

¹⁷Asness, Krail, and Liew [2001] showed that the value-added (alpha) from hedge funds can be largely explained by market (beta) exposure.

¹⁸An important caveat: Even with a large number of observations (i.e., more than five years), alpha can be difficult to assess with conviction.

¹⁹Note that alpha goes away when one includes a purer measure of value based on current price; this is shown in Appendix B and described in the section on other factor design choices.

²⁰See Berger et al. [2012], who discussed the concept of "alpha to you."

²¹Specifically, these implementation issues drop out of the covariance. Implementation issues, such as fees and transaction costs, are relatively stable components (constants), which mathematically do not matter much for higher moments such as covariance.

²²See Endnote 6 for more information on how the academic factors are constructed.

²³Despite its large number of stocks, the small-cap group contains roughly 10% of the market cap of all stocks (Fama and French [1993]).

²⁴Note that this graph is meant to be descriptive of the types of issues that may arise when analyzing non-risk-targeted portfolios. We cannot say for certain how much of the relation shown here is noise, or if it is predictable.

²⁵This is a reasonable assumption. See Asness and Frazzini [2013].

²⁶This equation applies for betas using a univariate regression—that is, with a single right-hand-side variable. Multivariate regression betas may differ from univariate betas because they control for the other right-hand-side variables, which means that they take correlations into account.

²⁷Some investors may be familiar with the work that Sharpe [1988, 1992] did on style analysis. This approach constrains the regression so that the coefficients are positive and sum to one. In this case, the coefficients (or betas) can be used as weights in building the *replicating* portfolio. In other words, a portfolio with factor weights equal to the weighted average of the coefficients should behave similarly in terms of its returns.

²⁸The lower relative loading is partly driven by differing volatilities, but it is also a result of the fact that HML can be viewed as an incidental bet on both value and momentum. We correct for this by using a purer measure of value; this is shown in Appendix B and described in the section on other factor design choices.

²⁹Although for a multifactor portfolio, investors should focus on partial correlations, which provide insight into the

relationship between two variables while controlling for a third. Alternatively, for a long-only portfolio, investors can look at correlations using active returns—that is, net out the market or benchmark exposure.

³⁰This approach is similar to decomposing portfolio risk by using portfolio weights, correlation, and volatility estimates. We have included an example of how to do this for a simple two-factor portfolio in Appendix C.

³¹Note that volatility is the square root of variance.

³²In this case, the idiosyncratic, asset-specific risk would account for 30% of the overall variance of the portfolio. This example focuses on a single-factor model in which we can ignore factor correlations. If we were to apply the same approach for a multifactor model, factor correlations would matter, and we would need to incorporate the covariance matrix. This approach requires matrix algebra and is computationally intensive, so we have omitted the calculation.

³³Note that it is more accurate to look at the adjusted R^2 when comparing models with a different number of explanatory variables. By construction, the R^2 will never be lower and could possibly be higher when additional explanatory variables are included in the regression, and the adjusted R^2 corrects for that. With a large number of observations, the two measures will be similar; this is the case with our regression because we use monthly data over 35 years (meaning a large sample size with 420 observations).

³⁴It is important to note that the t -statistic increases with more observations; that is, as the sample size grows very large, we are more certain about our beta estimates.

³⁵A t -statistic of two generally represents a reasonable standard of significance (which implies statistical significance at a 95% confidence interval under the assumption of a normal distribution) if there is no look-ahead bias. Generally, the higher the t -statistic, the more confident we can be about our beta estimates.

³⁶Note that we have used assumptions that are broadly representative of the historical volatilities and correlations for HML and UMD. But the example applies for any set of assumptions. It is for illustrative purposes only.

³⁷This composite was obtained from Morningstar as of June 2015.

³⁸Note that it is not surprising to see a low negative momentum loading because we are only looking at a value portfolio rather than a 50/50 value/momentum portfolio (as we did earlier in the article).

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The information presented is supplemental to a Global Investment Performance Standards (GIPS)-compliant presentation; however, because AQR does not currently manage accounts in the strategy presented, a GIPS-compliant presentation is not available. A complete list and description of all firm composites is available upon request.

The white papers discussed herein can be provided upon request.

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