Portfolio Rebalancing

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Common Misconceptions

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Decisions relating to portfolio rebalancing, while often considered secondary to deciding on the allocations themselves, can be considered an active investment strategy and have important implications for expected (and realized) portfolio returns and risk.

In this article we address common misconceptions about the role and implications of rebalancing, particularly in the context of actively-managed portfolios. These include the so-called "rebalancing premium" and the impact of rebalancing on the expected performance of risk-targeted and levered portfolios. A companion article (Ilmanen and Maloney (2015)) examines the rebalancing of strategic asset portfolios.

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Executive Summary

Rebalancing might seem at first glance to be just a necessary chore of portfolio management, but it is nonetheless a topic surrounded by controversy. Inconsistent use of terminology and varied interpretations of the underlying math have spread more confusion than understanding, and anecdotes are too often presented as definitive data.

In this article we refute four prevalent misconceptions related to portfolio rebalancing and seek to clarify the practical implications of each of these topics. Our conclusions may be summarized as follows:

1. Rebalancing is not generally, as some have suggested, a source of smart beta outperformance. It does alter the distribution of possible return outcomes for a portfolio, but this is more correctly and usefully interpreted as a risk-reduction effect from maintaining better diversification.
2. While rebalancing to constant capital allocations maintains long-term risk characteristics better than 'buy and hold', dynamic risk targeting does an even better job.
3. There is no reliable evidence that risk-targeted portfolios suffer a drag on returns from selling 'after the horse has bolted.' Evidence of the risk benefits, on the other hand, is pervasive.
4. Levered portfolios require an additional rebalancing process, but this does not cause any special "drag" effects beyond the normal compounding effects of investments at a similar risk level, and some additional transaction costs. Moderately levered portfolios of liquid instruments can be suitable for long-term investors, subject to their risk preferences.

An appendix covers several additional topics, including a simple illustration of several rebalancing processes applied to a dynamic portfolio.[1]

Misconception 1: "Rebalancing is a Source of Smart Beta Outperformance"

Some smart beta managers have suggested that periodically rebalancing a portfolio to its target weights is itself a source of returns,[2] and even the main source of expected outperformance. Rebalancing is said to "harvest a premium" that does not require mean reversion in prices.

Fact: Rebalancing does not earn positive returns – and is not a positive expected return strategy – unless prices are mean-reverting at a frequency the rebalancing process can capture. It does, however, help to maintain diversification and so alter the distribution of possible outcomes for the portfolio. The claim that rebalancing can "harvest a premium" even in the absence of mean reversion (and regardless of the target weights) is based on a stretched interpretation of the math, as we illustrate below.

Details

It is a curious fact that a periodically rebalanced portfolio earns a positive return ("turns water into wine") if the component assets all end up at their starting prices.[3] The resolution of this seeming paradox is that such an outcome already implies some mean reversion (the assets have reverted to their starting prices). It also requires the assets to have positive arithmetic mean (AM) returns.[4]

1 This article focuses on rebalancing in response to inputs other than changing investment signals or expected returns. The question of how and when to apply new investment signals – weighing the costs of holding a sub-optimal portfolio against the costs of trading to new targets – is highly dependent on the strategy and its investment universe.

2 Based on an extensive literature on "diversification return" perhaps starting with Booth and Fama (1992) and including Bernstein and Wilkinson (1997), Fernholtz (2002) and Willenbrock (2010). See Ilmanen (2011) for a concise overview. For the smart beta context, see for example Azuelos and Yasenchak (2014). For an analytical framework that attributes relative performance of rebalanced portfolios to two opposing components, see Hallerbach (2014).

3 Erb and Harvey (2006) use commodity futures as an example.

4 AM is always higher than GM (where volatility is non-zero), and in this case it is the assets' GM returns that are zero. An asset whose price rises from

In the above case, the outperformance generated by rebalancing is equal to the "diversification return," usually defined as the difference between the geometric mean return (GM) of a portfolio and the weighted average GM of its components. But this equivalence does not usually hold. Indeed, the practical relevance of "diversification return" is limited by the fact that in general the weighted average GM is not the return of any investable portfolio. The buy-and-hold portfolio, which is investable, may have a higher or lower GM than the portfolio rebalanced to constant weights.[5]

While diversification return should not be considered a return premium, we believe it does highlight why AM and GM rates of return are both incomplete measures of expectation which are better understood in the context of the *distribution of terminal wealth outcomes*.[6]

Rebalancing reshapes the distribution of terminal wealth outcomes, by neutralizing compounding effects within the portfolio. It prevents winning investments from earning higher weights and losers from decaying to lower weights. In other words, a rebalanced portfolio forgoes the very best buy-and-hold outcomes (the right tail of the distribution), where winning investments keep on winning and compounding their gains, and losers fizzle out to small, inconsequential weights. But it compensates by outperforming in many other outcomes where performance across investments is less divergent.[7] Importantly, rebalancing also tends to maintain a lower risk-level than buy-and-hold by preventing the concentration of risk among winning investments.

For these reasons, rebalancing tends to narrow the distribution of terminal wealth outcomes for the portfolio and make it less positively-skewed. If all the assets have equal expected returns, rebalancing increases the median wealth outcome while leaving the mean unchanged: **Exhibit 1** illustrates this using simulated data. (In the case of, say, a portfolio of stock and bond allocations with dissimilar expected returns, rebalancing actually reduces the mean expected terminal wealth by preventing dominance of the higher-return asset, but the narrowing and reshaping effect is even more pronounced.)

Exhibit 1 – Simulated Impact of Rebalancing on Terminal Wealth (TW) Outcomes (10-Year Horizon)

Measure	Buy & Hold	Rebalance	Impact
Mean TW	$2.71	$2.71	Same mean
Median TW	$2.51	$2.54	Higher median
90th percentile	$4.11	$4.05	Lower best cases
10th percentile	$1.55	$1.59	Higher worst cases
90th-10th Range	$2.56	$2.46	Narrower range
Volatility	12.1%	11.5%	Lower volatility
Max Drawdown	-18.3%	-16.9%	Smaller drawdowns

Source AQR. For illustrative purposes only, not indicative of actual investments. 50,000 simulations based on the assumption of a $1 portfolio of three uncorrelated assets with constant expected volatility of 20%, expected arithmetic Sharpe ratio 0.5, normally-distributed serially-independent returns, and daily rebalancing. Gross of transaction costs.

This change in the shape of the distribution of terminal wealth means that a rebalanced portfolio is more likely to realize positive returns – and more likely to realize returns equal to or exceeding the mean – over the investment horizon. These are risk-related qualities that many investors would prefer in their portfolios, so it is no surprise that most do choose to periodically rebalance.

$100 to $110 (+10%) and then falls back to $100 (-9%) has a positive arithmetic mean return.

5 Even a rebalanced portfolio of assets which follow a true random walk is associated with a positive diversification return, and yet in this hypothetical situation it is not possible to apply investment skill or "harvest a premium" as the results are purely random.

6 For a discussion of return estimation, see for example Jacquier, Kane and Marcus (2003), Hughson, Stutzer and Yung (2006), and references therein. Some recent articles (for example Chambers and Zdanowicz (2014)) have questioned the usefulness of rates of return in general and especially GM returns, preferring to focus solely on mean expected terminal return or terminal wealth (TW). We have sympathy for this position but believe it is important to consider the distribution of possible outcomes, and not just the mean. We also emphasize that the use of terminal return assumes the investor can 'stay the course' regardless of the path of returns. Qian (2014) also considers the impact of rebalancing on terminal wealth distributions, and defines a "wealth Sharpe ratio" as mean excess TW / standard deviation of TW.

7 See Wise (1996) for an analysis of the probability of outperformance.

Some authors have interpreted these effects as a return premium for buying losers (effectively selling portfolio insurance) and forgoing the best outcomes by selling winners, i.e., applying a strategy which itself has a negatively-skewed return distribution.[8] But we believe it is more useful (and mathematically correct) to describe these effects as the risk-reduction benefits of maintaining better diversification. Contrarian rebalancing does not harvest returns in the sense of an increased mean expected wealth,[9] *unless* prices tend to mean-revert at a frequency the rebalancing process can capture.[10] Rebalancing also incurs costs, which has implications for smart beta portfolios as we discuss in the appendix.

Misconception 2: "Constant Capital Equals Constant Risk"

While investors may not explicitly assert the above, traditional rebalancing processes that aim to maintain benchmark capital weights (for example, 60% stocks and 40% bonds) do imply a belief that capital weights determine the relevant risk characteristics of a portfolio.

Fact: The riskiness of assets varies over time,[11] and the future riskiness of assets is easier to predict than their future returns. While rebalancing to constant capital allocations does indeed help to maintain long-term risk characteristics (see Ilmanen and Maloney (2015)), dynamic risk targeting does an even better job. Seemingly passive buy-and-hold portfolios tend to have the most variable and least predictable risk outcomes.

Details

Risk rebalancing is the practice of rebalancing to target risk allocations, rather than capital allocations, based on dynamic risk estimates for each investment. So long as the risk estimates don't change, this is exactly the same contrarian process as capital rebalancing – selling winners and buying losers – with the same tendency to outperform if investments experience mean-reverting performance, and the same impact on the distribution of terminal wealth outcomes (raising the median and reducing the positive skew, as detailed above).

When risk estimates do change, some additional rebalancing will be required to maintain risk allocations. Below we provide empirical evidence that many investments' risk characteristics are time-varying with some predictability. A risk-rebalanced portfolio may therefore maintain its diversification and risk characteristics better than a capital-rebalanced portfolio.

Exhibit 2 shows realized volatility contributions through time for three stock/bond portfolios over a 41-year period. Portfolio (a) allocates 20% to equities and 80% to bonds at the start of the period, and does not rebalance. This portfolio's volatility gradually becomes dominated by equities. Portfolio (b) starts with the same allocation and rebalances to maintain it. This portfolio achieves a more consistent risk allocation, though changing asset volatilities cause significant variations. Portfolio (c) allocates by risk, not capital, and rebalances to maintain equal risk allocations through time. It achieves the most consistent risk contributions, by

8 As described by Perold and Sharpe (1988), Harvey et al (2014) and Hillion (2016). Harvey et al suggest a rebalanced portfolio is susceptible to larger drawdowns than a buy-and-hold portfolio (in contrast to the results in Exhibit 1 above). This is true for two portfolios entering a period of sustained investment losses with the same weights, as the authors illustrate. However, this analysis misses the tendency of rebalanced portfolios to be already better diversified at the onset of such periods. When considered in the context of longer investment horizons, as above, rebalanced portfolios tend to experience smaller drawdowns because of their lower average risk level.

9 As emphasized by Chambers and Zdanowicz (2014). Mean expected wealth may be the most relevant measure of expectation when considering components of a wider portfolio. For example, if the investments' next-day returns were negatively correlated to their past-month returns, a portfolio rebalanced to fixed weights daily or monthly would capture a larger positive expected return than one rebalanced annually.

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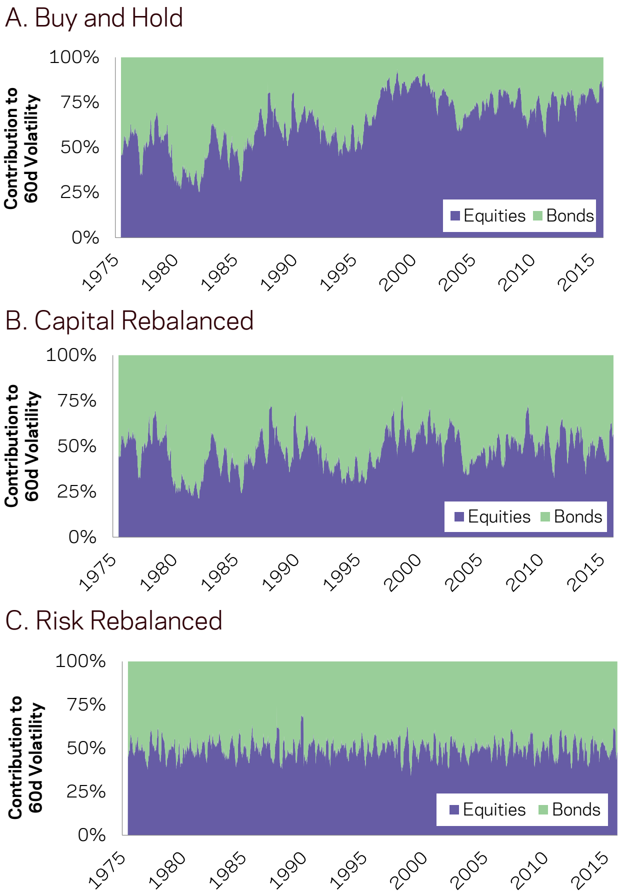
11 A result formalized and brought to wide attention by Engle (1982) but dating back many decades.

Exhibit 2 – Realized Volatility Contributions for Three Rebalancing Approaches, 1975-2015



Source: AQR and Bloomberg. Starting capital allocations for (a) and (b) are 20% equities, 80% bonds. Based on daily returns of U.S. equities (S&P500 Index) and U.S. 10-year Treasuries, and daily rebalancing. Risk estimates for risk-rebalanced portfolio are based on rolling 60-day volatility.

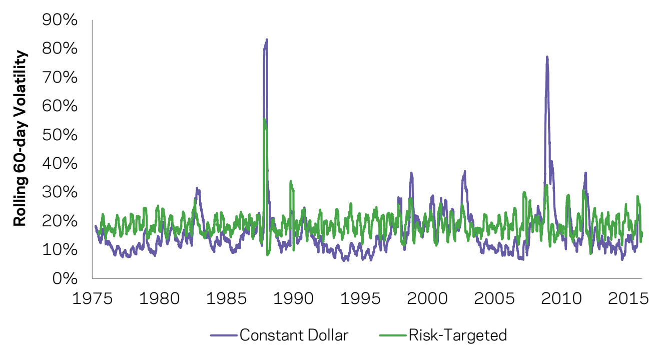
adjusting target weights based on recent relative asset volatilities.

What if *all* of the portfolio's components become more risky? *Risk-targeted* (RT) portfolios go one step further than risk-rebalancing by scaling each investment individually and allowing variations in the total capital invested, rather than merely adjusting relative allocations.

The main objective of risk-targeting is to provide more stable realized volatility. A portfolio should be designed with the worst case in mind, and therefore narrowing the range of outcomes gives investors more scope to seek higher returns on average. It also increases diversification over time, reducing the likelihood of short periods dominating long-term performance.

Exhibit 3 compares the rolling 60-day volatility of a constant-dollar investment in U.S. equities to that of a simple risk-targeting strategy based on past 60-day volatility. The risk target is set to match the full-sample constant-dollar volatility of 18% for visual comparison. The volatility of volatility is reduced by 54%.

Exhibit 3 – Realized Volatility of U.S. Equities, 1975-2015

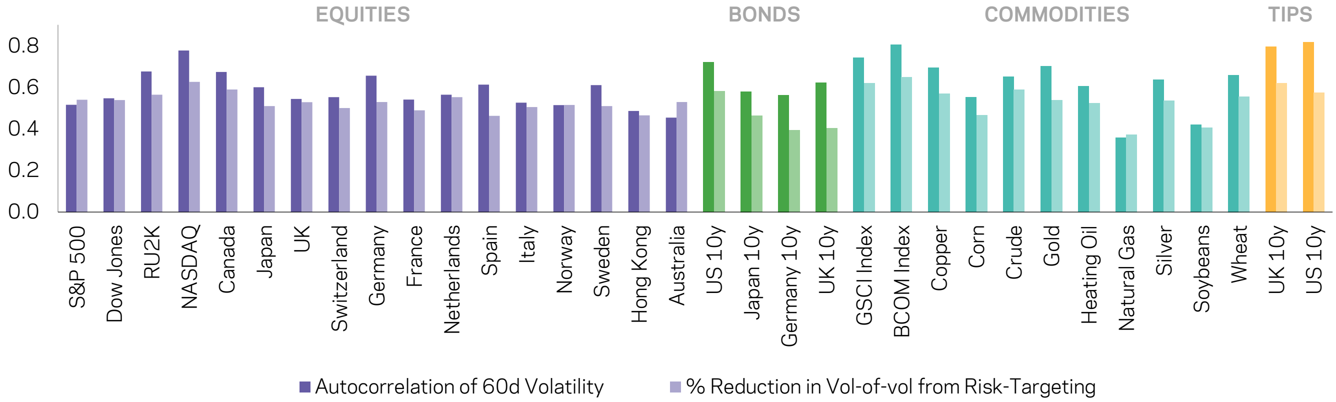


Source: AQR and Bloomberg. Based on daily returns of the S&P500 Index.

The benefits of RT are not only found in equities. **Exhibit 4** shows the corresponding reduction in volatility of volatility for 34 assets, using the same simple strategy. Reductions vary from 37% to 65%, with the amount of reduction achieved being closely related to the persistence or autocorrelation of volatility for that asset. RT works when volatility is persistent, and volatility is persistent everywhere we look.

Although even naïve risk-targeting can produce more stable realized volatility, all risk targeting processes are not equal. Each depends on a risk model, consisting of estimates of the volatility of each asset and of correlations between them. Estimates are typically based primarily on historical returns and must balance responsiveness with the need to minimize estimation noise and unnecessary turnover.

Exhibit 4 – Volatility Persistence and the Impact of Risk-Targeting, 1975-2015



Sources: AQR and Bloomberg. Hypothetical strategies based on daily returns from 1975 where available; some series start later. Based on daily rebalancing, gross of transaction costs and fees. Autocorrelation is based on non-overlapping 60-day periods. Hypothetical data have inherent limitations, some of which are disclosed in the back.

The portfolio volatility target is usually a constant number corresponding to the investor's risk appetite.[12] The strategy may target this constant risk level at every rebalance, or, more likely, it may permit short-term variation in the target depending on investment signals, constraints and other risk controls, but be calibrated with the aim of achieving the target over the long-term.

Misconception 3: "Risk-Targeted Portfolios Sell When Assets are Cheap and Suffer a Drag on Returns"

Some contrarian-minded investors assume that risk-targeted strategies are doomed to sell investments too late, 'after the horse has bolted.' Indeed it has been suggested that for patient long-term investors, spikes in volatility are more likely to signal a good time to buy than to sell. Some research has claimed to support the idea that this causes a drag on risk-targeted returns.

Fact: We find no reliable evidence that risk targeting impairs risk-adjusted returns – and for a risk-targeted investment, risk-adjusted returns are what matter. The empirical evidence for any risk/return interaction is mixed and depends on the investment, time period and method for estimating risk. Some of the literature has misinterpreted this evidence. By contrast, the risk *benefits* of risk-targeting – more stable volatility outcomes – are supported by clear evidence across many investments.

Details

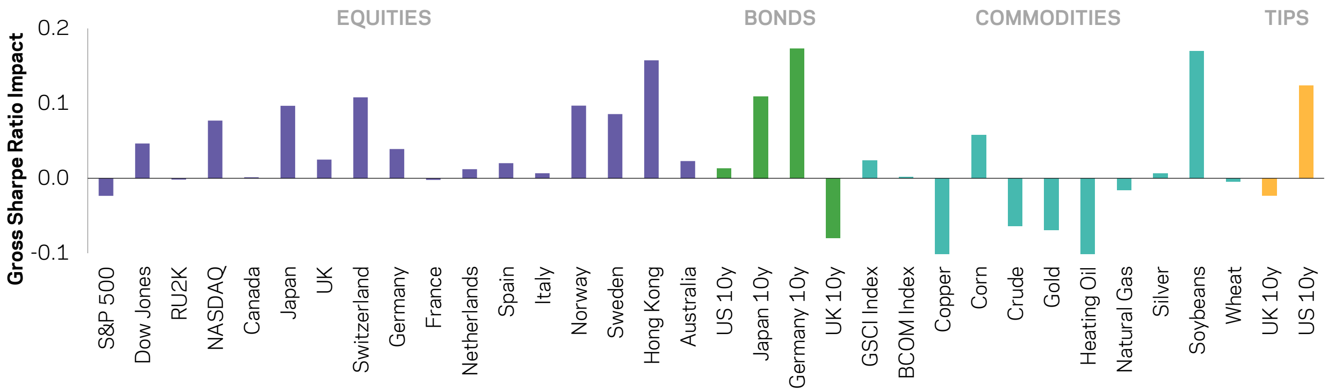
Risk-targeted (RT) portfolios sell investments when they become more volatile. Therefore if different levels of recent volatility predict higher or lower subsequent returns on average, the RT process may have an impact on expected returns. The myth stated above implies two separate assumptions about this relationship, both commonly held but both requiring scrutiny.

The first is that RT portfolios *sell investments after they fall in price*. This is true only if increasing volatility coincides with falling prices. While evidence suggests that equities do indeed exhibit this well-known tendency, other asset classes such as bonds and commodities do not (see Exhibit A2 in the appendix).

The second questionable assumption is that *falling prices imply higher future returns* (or Sharpe ratios). The well-documented success of trend-following

12 Andersen, Bianchi and Goldberg (2014) also consider time-varying risk targets, e.g., based on the realized volatility of a benchmark portfolio. The practical usefulness of this approach is limited, given that the main reason for risk targeting is that investor risk tolerance does not increase with market volatility.

Exhibit 5 – Gross Impact on Sharpe Ratio from Simple Risk-Targeting, 1975-2015



Sources: AQR and Bloomberg. Hypothetical strategies based on daily returns from 1975 where available; some series start later. Based on daily rebalancing, gross of transaction costs and fees. Sharpe ratios calculated using 3-month T-Bills as risk-free rate. Hypothetical data have inherent limitations, some of which are disclosed in the back.

strategies, which are based on the opposite belief, already hints that asset price behavior is not so simple. Both reversal and momentum effects are observed in many asset classes, tending to operate at different time horizons, which may explain why empirical evidence of the return impact of RT (based on different estimation horizons) has tended to be mixed.[13] Moreover, value effects may be better harnessed by cross-sectional strategies than timing strategies.[14]

Exhibit 5 shows the gross Sharpe ratio impact for the same simple risk-targeting strategy used in Exhibit 4. The impact on Sharpe ratios is much less consistent than the impact on the stability of realized volatility. For this particular strategy and sample period, the Sharpe ratio impact tends to be positive in equities (the S&P 500 being a notable exception), and mixed in other asset classes. In all cases the impact is modest (-0.1 to +0.2).

Not only is the empirical evidence mixed, but ex ante expectations are unclear. Should we expect an investment to earn a constant Sharpe ratio through time, with periods of elevated risk compensated by higher returns? Or are variations in expected returns likely to be unrelated to risk, implying a lower prospective Sharpe ratio during volatile periods?[15]

Whatever our expectations, it is important to recognize that while time-varying volatility has a predictable component, it also has an unpredictable component. This is why variations in realized volatility can be significantly reduced but not eliminated. We would expect this unpredictable element to dilute any ex ante Sharpe ratio impact implied by the above assumptions. For a risk-targeted portfolio of diversifying assets or strategies, it may be reasonable to assume a very modest long-term Sharpe ratio advantage due to improved diversification through time and across the portfolio: RT is after all primarily a risk management strategy, not a return-seeking strategy.

Finally, it is important to note that risk-targeting requires trading and incurs costs. It may

13 Andersen, Bianchi and Goldberg (2014) suggest that a risk-targeted stock-bond risk parity portfolio would have significantly underperformed a fixed-leverage portfolio over a long period, due to a detrimental interaction between the RT portfolio's leverage (related to recent volatility) and its subsequent returns. However, this was a misinterpretation of their own results (see Asness, Hood and Huss (2015) for details). Other studies, based on other periods and other assumptions, have reported the opposite effect, especially in equities (see for example Perchet et al (2014) and Moreira and Muir (2016)).
14 See Asness, Ilmanen and Maloney (2016).
15 Both of these scenarios imply a positive Sharpe ratio impact from risk targeting, through mild time diversification in the first case and favorable timing in the second. However, any such benefits would be diluted by (1) the fact that we can forecast risk only imperfectly and (2) transaction costs. A third scenario, where elevated risk is associated with a higher Sharpe ratio, implies a negative impact but is the least well-supported by empirical evidence.

therefore be most appropriate for the most liquid instruments and for managers with cost-effective execution infrastructure including cost-optimized rebalancing and patient trading algorithms.

Misconception 4: "Levered Portfolios Underperform because of Variance Drag Associated with Rebalancing, and are Unsuitable for Long-Term Investors"

Some commentators have suggested that leverage – which can be used by managers to offer the same strategy at different risk levels – incurs a "variance drag" or a "negative rebalancing return" that makes such levered portfolios unsuitable for long-term investors.

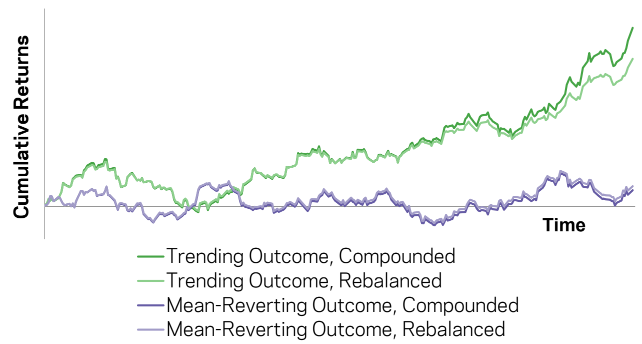
Fact: Levered portfolios can outperform or underperform the scaled performance of their underlying unlevered reference portfolio, due to compounding effects that depend on the investment outcome. Variance drag, as most commonly discussed, is a misleading concept. Many moderately levered portfolios are suitable for long-term investors, subject to their risk preferences.

Details

To understand the implications of rebalancing levered portfolios, we must first be clear on the effects of compounding. Most portfolios, whether fully-invested or risk-targeted, are allowed to compound gains and losses. Mathematically, this equates to an absence of contrarian rebalancing at the portfolio level – profits are not withdrawn but reinvested, and losses are not replaced.

Exhibit 6 compares the simulated performance of a typical compounding portfolio with that of a portfolio that rebalances to a constant NAV (i.e., does not reinvest gains and losses, but exchanges them with an external pool of capital). The compounding portfolio outperforms during periods of persistent positive or negative performance (the green lines show one such outcome), but lags during choppy, mean-reverting performance (purple lines). Compounding also, as we mentioned previously, creates a positively-skewed distribution of outcomes for terminal wealth.

Exhibit 6 – Effects of Compounding on Trending and Mean-Reverting Investment Outcomes



Source: AQR. For illustrative purposes only, not indicative of actual investments. One-year simulations of daily returns with zero expected autocorrelation and 15% expected volatility. The two simulations were selected to have trending and mean-reverting returns respectively. For each simulation, the compounded version applies normal compounding while the rebalanced version rebalances to its starting value daily, i.e., does not reinvest gains and losses.

There are two main approaches to using leverage. One is to explicitly target a leverage ratio (many levered ETFs do this). The other is used by risk-targeted strategies, where some variable amount of leverage is required to meet the risk target. The two approaches have different objectives and very different risk characteristics, but they both require an additional rebalancing process that is specific to levered strategies. If the net value of the portfolio changes significantly, some rebalancing will be required to regain the leverage or risk target. Positive returns will cause the portfolio to fall below its target risk level, and the manager must buy more investments to regain it.[16]

16 Consider a hypothetical $100 invested in a strategy with a volatility target of 10%, which it initially achieves by holding levered investments totaling $200 in assets with 5% forecast volatility. The strategy makes a 10% gain, while the market risk forecasts remain unchanged. The portfolio volatility

Similarly, after negative returns the manager must sell investments.

Unlike contrarian rebalancing *between* investments to maintain portfolio weights (which levered portfolios must also do, unless they are cap-weighted), this additional process has a momentum bias. It has been characterized in some research[17] as "incurring a negative diversification return." However, it is more meaningful to say that it *reproduces the compounding effects experienced by an unlevered investment at a similar risk level.*[18] As described above, these compounding effects can have a positive or negative impact depending on the price path: a 3x levered ETF earns (gross) less than three times the index return during a choppy year, but it outperforms three times the index return during a (positive or negative) trending year.

So gross of costs and ignoring potential differences in interventions by risk managers or counterparties,[19] levered portfolios do not suffer any special "drag" effects beyond the normal compounding effects of investments at a similar risk level. Furthermore, these compounding effects do not reduce the mean expected terminal wealth, unless investment performance is assumed to be mean-reverting. They do, however, require additional turnover and incur transaction costs, which has led some to question their suitability for long-term investors. A levered ETF offering 2x or 3x exposure to an equity index will have a very high risk level (30-50% annual volatility), and will indeed require very active rebalancing. But for a risk-targeted strategy with a volatility of 10-20% this effect (and associated transaction costs) is milder, since it scales with the square of the volatility. Such strategies — especially those using liquid instruments and employing cost-efficient trading processes — can indeed be suitable for long-term investors, subject to their risk preferences.

Concluding Thoughts

Well-managed dynamic rebalancing processes may lead to more predictable risk characteristics, while seemingly passive buy-and-hold portfolios may have the most variable and least predictable risk outcomes. The most dynamic portfolios require a combination of several separate rebalancing processes, each of which has its own rationale and its own effects on risk and return expectations (see appendix for an illustration).

In general, rebalancing does not "harvest a return premium," but it does help to maintain diversification and thereby change the distribution of possible wealth outcomes for a portfolio. The bottom-line return impact over any given investment period will depend on the price paths of investments during that period.

Rebalancing topics will no doubt continue to attract research and commentary, and there are often several possible interpretations of the same underlying math or empirical evidence. In this article we have challenged several misconceptions that we believe have arisen from unhelpful or erroneous interpretations, and aimed to substitute more convincing and intuitive explanations. Rebalancing is an essential part of all active investment management. Once the implications have been clearly understood and the most efficient processes implemented, managers and investors can then turn their attention back to the underlying strategy, which is the real source of expected returns.

is now approximately 5%*$210 = $10.50,which is only 9.5% of the new NAV of $110. The manager must invest an additional $10 to regain the 10% risk target. The same mechanism applies to portfolios explicitly targeting a leverage ratio.

17 See for example Qian (2012).

18 To see this, we can compare the example in footnote 15 to an unlevered $100 investment with a volatility of 10% (i.e., $10). After a 10% gain, the volatility of the unlevered investment is $110*10%=$11. This investment achieves by natural compounding the same new risk level that the levered strategy achieves by adding investments

19 Strategies levered to target higher risk levels may be subject to relatively tighter risk controls, as tolerance for large drawdowns does not necessarily scale with volatility; these controls may affect returns if they are triggered.

Appendices

Putting It All Together: An Illustration

In this article we discussed misconceptions surrounding several different types of rebalancing processes, each being an adjustment to portfolio weights that is made in response to market developments, separate from any views on expected returns. A levered, risk-targeted risk parity strategy is an example of a portfolio that employs all of these processes, though of course it trades only their net sum. In **Exhibit A1** we present a simplified illustration of how the processes may be combined. The notes in the last column describe what is happening at each stage.

Table (a) shows the starting portfolio, with equal risk allocations to two assets that might represent stocks and bonds (expected volatilities of 15% and 5%, expected correlation zero). To reach its risk target of 10%, the portfolio is approximately 1.9x levered.

Table (b) shows the impact of price changes and changes in risk estimates on the allocations and portfolio

Exhibit A1 – Worked Example of Rebalancing a Levered, Risk-Targeted Portfolio

a. Starting Portfolio	Asset A	Asset B	Fund	Notes
Price	$100.00	$100.00	$100.00	
Exposure	$47.14	$141.42	$188.56	Fund is 1.89x levered
Risk Estimate	15.0%	5.0%	10.0%	Risk target is 10%
Risk Allocation	50.0%	50.0%	100.0%	Equal risk allocation

b. Portfolio After Market Move	Asset A	Asset B	Fund	Notes
Price	$90.00	$105.00	$102.36	Asset A -10%, asset B +5%, portfolio +2.4%
Exposure	$42.43	$148.49	$190.92	Fund now 1.87x levered
Risk Estimate	16.0%	5.0%	9.8%	Asset A now riskier, portfolio below target
Risk Allocation	47.8%	52.2%	100.0%	Allocations have shifted

c. Required Rebalancing	Asset A	Asset B	Fund	Notes
1. Rebalance From Winners to Losers	$5.30	($5.30)	($0.00)	Contrarian shift from winner B to loser A
2. Rescale Portfolio Due to Leverage	$0.52	$1.57	$2.09	Scale portfolio due to change in NAV
3. Rebalance From Riskier to Less Risky	($2.30)	$2.30	($0.00)	Rebalance due to change in risk estimates
4. Rescale Portfolio Due to Risk Changes	($0.72)	($2.30)	($3.02)	Scale portfolio due change in risk estimates
TOTAL REBALANCE	$2.81	($3.74)	($0.93)	Net trades

d. Portfolio After Rebalance	Asset A	Asset B	Fund	Notes
Exposure	$45.24	$144.75	$189.99	Fund now 1.86x levered
Risk Estimate	16.0%	5.0%	10.0%	Portfolio back at 10% risk target
Risk Allocation	50.0%	50.0%	100.0%	Equal risk allocations re-established

Source AQR. For illustrative purposes only, not indicative of an actual portfolio or actual investments.

risk estimate. Asset A is down 10% and its risk estimate has increased, while asset B is up 5% and its risk estimate is unchanged. The allocations and portfolio risk level have moved away from their targets.

Table (c) shows the four different rebalancing processes that are required to regain the target allocations and risk level. These are the processes we have discussed in this article:

1. contrarian rebalancing from winners (asset B) to losers (asset A);
2. rescaling of the levered portfolio due to the change in NAV (all else equal, the increase in NAV had left the portfolio slightly underinvested);
3. risk-rebalancing due to changes in risk estimates (asset A is now riskier); and
4. rescaling of the portfolio due to changes in risk estimates (this and the previous process may be considered part of the same risk-targeting process).

Table (d) shows the portfolio after the net rebalancing trades: it has regained its target risk allocations and target portfolio risk level.

Additional Implications

We conclude by briefly covering two topics that follow from the discussions in the main article: (1) If smart beta outperformance is not directly attributable to rebalancing, is it simply a consequence of weighting by anything other than market capitalization? (2) Could risk targeting exacerbate market volatility by selling at times of market stress?

Smart Beta: Anything But Cap-Weighted?

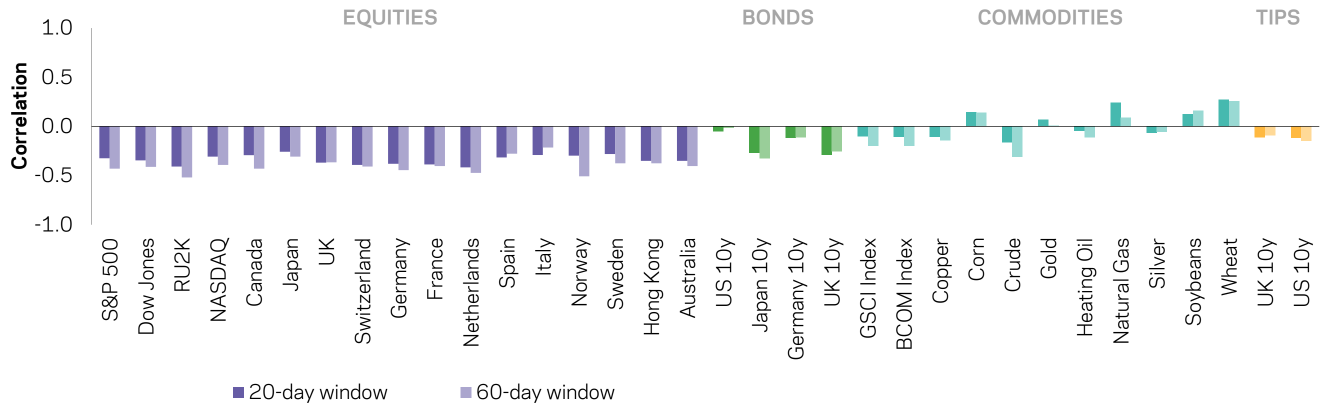
Many non-cap-weighted portfolios outperform on paper, gross of costs, but all (unlike the benchmark) require rebalancing and many are not implementable. Below we explain why we believe the best "smart beta" portfolios cost-efficiently implement the highest-expected-return tilts away from the market, including value, momentum and quality.

The classic (if imperfect) example of a buy-and-hold portfolio is a capitalization-weighted equity portfolio. This does not require rebalancing except for periodic adjustments to account for issuance, dividends, buy-backs and constituent changes. Cap-weighted portfolios have intuitive advantages relating to cost-efficiency, representativeness and macro-consistency. They also suffer from well-known disadvantages such as a tendency to overweight (and become dominated by) overvalued stocks, sectors or countries.

Arnott et al (2013) showed that many non-cap-weighted portfolios (even those weighted by the inverse of popular measures, or randomly generated) have earned higher returns and higher Sharpe ratios than a cap-weighted benchmark, gross of costs. The authors' interpretation was that high price predicts poor returns (as the well-known value and size factors seem to suggest) and therefore all portfolios that rebalance to prevent price determining weight will tend to outperform. Many "smart beta" managers, the authors imply, inadvertently harness and rely on this ubiquitous underlying price effect, while claiming a broader range of advantages for their products.

That analysis was gross of costs. In a world of transaction costs and capacity constraints, tilts away from

Exhibit A2 – Contemporaneous Correlation Between Volatility Change and Excess Return, 1975-2015



Sources: AQR and Bloomberg. Hypothetical strategies based on daily returns from 1975 where available; some series start later. Returns are excess of cash. Volatility changes are based on rolling volatilities with 20- or 60-day window to match change window. Hypothetical data have inherent limitations, some of which are disclosed in the back.

the market portfolio do not come for free. All non-cap-weighted portfolios require periodic rebalancing and so incur higher transaction costs. Some are illiquid or impractical for institutions because they give large weights to small-cap stocks. Therefore we believe that smart beta investors should seek out those tilts – or factors – with the highest expected net returns. Long/short evidence unambiguously supports the outperformance of factors such as value, momentum and quality (and not their inverses!), all of which are also supported by economic intuition.[20] Research on market frictions suggests that these factors are sufficiently robust and attractive to survive real-world costs.[21]

Could Risk-Targeting Exacerbate Market Volatility?

Could risk-targeting processes create feedback loops of selling during a crisis, similar to the effect of portfolio insurance in 1987? Risk-targeting trades have a momentum bias if increases in volatility tend to coincide with negative returns. **Exhibit A2** shows contemporaneous correlations between volatility changes and return for 34 assets and for two different horizons. Increasing volatility does indeed coincide with lower returns for equities, but for other asset classes the relationship is much weaker. For commodities, spikes in volatility are just as likely to be associated with price increases (risk-based rebalancing is actually contrarian during such episodes). Even in equities, risk-targeted capital currently represents such a small proportion of the total investor base that traditional stop-loss selling and non-quantitative increases in risk aversion are likely to be responsible for far more of any crisis selling.

The rebalancing of risk-targeted *long/short* strategies is more complex. For these strategies, directional market shocks do not necessarily generate net buy or sell adjustments. Strategy losses and increased volatility may cause reductions of both long and short positions and such deleveraging can result in further losses for similar strategies (as in the "quant crisis" of August 2007), but any impact from – or effect on – directional market moves is likely to be mitigated by the offsetting nature of long/short positions and trades.

20 See for example Asness, Moskowitz and Pedersen (2013) and Asness, Frazzini and Pedersen (2013). The findings of Arnott et al do not contradict this evidence. They do not apply factor tilts and reversed tilts, but rather test entirely price-agnostic weighting schemes and the inverses of those schemes, without regard to implementability. Alternative portfolio construction choices were explored by Amenc, Goltz and Lodh (2016), who found that factor-tilted portfolios did indeed outperform their inverses.

21 Frazzini, Israel and Moskowitz (2012).

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