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When it comes to implementing new financial concepts, retail investing tends to lag behind institutional investing. So it's not surprising that the more retail-dominated life-cycle funds (also called *target-date funds*) have been slow to adopt many of the investment techniques embraced by institutional investors. This matters because in recent years life-cycle strategies have been one of the fastest-growing areas among defined contribution plans, which themselves have gained market share from the more institutionally managed defined benefit plans. According to a recent Morningstar survey, life-cycle assets grew almost tenfold over the last 10 years—from \$71 billion in 2005 to \$671 billion in 2014.[1] Assuming this trend continues, we argue that it is becoming increasingly important for life-cycle strategies to modernize.

Life-cycle strategies are in many ways good vehicles for retirement saving. They are relatively cost efficient, allow easy access to a broad range of investments, and avoid concentrated investments in employer stock. However, these funds contain a number of shortcomings—including, chiefly, that they are poorly diversified in various ways. In this article, we will discuss five dimensions of underdiversification, propose improvements to address each, and present simulated evidence on how better diversification could have improved retirement savers' historical performance.

The first two shortcomings—the underutilization of global investment opportunities in stock and bond markets (home bias), and a lack of inflation-protection assets—are well known. It's perhaps too harsh to say life-cycle funds have ignored these problems. They have gradually improved on them by increasing allocations to non-U.S. assets and to inflation-hedging assets. However, at current levels, there still remains some home bias and limited inflation protection; more can be done.

The third shortcoming is perhaps most important—excess concentration in one risk source, the stock market. To be fair, this problem is not unique to life-cycle funds. Many traditional portfolios, such as a 60/40 stock/bond allocation, suffer from this same problem. Even though from a "dollars invested in each asset class" perspective, a 60/40 portfolio looks well diversified, when viewed from a risk perspective, the 60/40 portfolio is highly concentrated in equities because stocks are so much more volatile than bonds. We show that this concentration makes traditional life-cycle investors particularly sensitive to what happens to equity market valuations over their lifetime. Providing better risk balance across asset classes is one way to improve those portfolios by reducing the overreliance on equity market performance.

The fourth shortcoming is risk concentration in particular episodes of equity market turbulence. Markets are not equally volatile at all points in time, but the asset allocation of target-date portfolios does not recognize this. Couple this shortcoming with the fact that these funds are overly sensitive to stock markets (and U.S. stock markets, at that), and the performance of life-cycle strategies will be especially sensitive to episodes of heightened U.S. stock market volatility. By dynamically targeting portfolio volatility, one can effectively offset fluctuations in market volatility by adjusting position sizes. As a result, it's possible to make portfolios less sensitive to the most volatile periods.[2]

Finally, the fifth shortcoming is that life-cycle strategies rely primarily on long-only asset class risk premia as return providers, underutilizing the diversification and return potential of long/short strategies called *alternative risk premia* or *liquid alternatives*. We will discuss one example, *trend-following strategies*, which invest dynamically in many liquid asset markets based on recent return trends. These strategies have historically had a tendency to perform well in equity market downturns (see Hurst, Ooi and Pedersen [2014]). Thus, they nicely complement equity holdings. There are other liquid—and illiquid—alternatives that life-cycle investors could consider; but because we have more than a century of data on trend-following strategies, they are especially suitable for our empirical analysis.

With these five improvements in mind, we propose what we believe is a better life-cycle strategy. Traditional life-cycle strategies gradually adjust the equity/bond mix as investors age, in order to reflect the notion that investors can tolerate more risk when they are young and less as they age.[3] However, basic financial theory tells us that if we are able to borrow and lend, then our asset allocation decision should be independent of the amount of risk that we wish to take. Portfolio construction should occur in two steps: An investor first finds his or her optimal portfolio and then scales that portfolio to achieve the desired risk target, which will decline with age.

In our analysis, we will demonstrate that by using this approach and incorporating each of the five improvements listed above, this alternate life-cycle strategy could have produced significantly better retirement outcomes than the traditional approach; this holds for every cohort saving for 40 years and retiring between 1942 and 2014. We further decompose the added value of this approach versus the traditional approach and find that each of the above five improvements adds incremental value. Finally, we find that this approach produces outcomes that are less dependent on what happened to equity market valuations over the relevant savings period.

DATA DESCRIPTION

In our main analysis, we use monthly returns on 10 equity indexes, 8 global bond indexes, and 29 commodity futures contracts compiled by Hurst, Ooi, and Pedersen in 2014. The return series start as early as 1900, with narrower universes that gradually expand as data became available for more investments. All series end in the fourth quarter of 2014. For a list of all return series used, see Appendix A; for trading cost estimates used, see Appendix B.

Asset Classes

We use the above-named instruments to construct three asset class portfolios: global stocks, global bonds, and commodities. In some examples, we follow the practice of many life-cycle strategies and treat U.S. and international (non-U.S.) equities and bonds separately. With this split, we have five asset classes: U.S. equities, U.S. bonds, international equities, international bonds, and commodities. Within the international stock and international bond portfolios, we use GDP-weighted portfolios of all available investments at each point in time. Similarly, when we use global stock and global bond portfolios (including both U.S. and international), we use GDP weights. Thus, for global portfolios, we add the United States as one country, but with roughly 50% weight.[4] Within commodities, we use equally weighted portfolios. The choice of using GDP-weighted and equally weighted portfolios is driven by the lack of good market capitalization data over this very long time period. However, it's worth noting that during periods when we have both GDP-weighted and cap-weighted stock and bond composites, the performance is very similar.

Trend Following

We also explore the impact of adding a 10% allocation to a long/short trend-following strategy, as in Hurst, Ooi, and Pedersen [2014]. In this trend-following strategy (also called *managed futures*), the equity indexes, bond indexes, and commodity futures cited above—as well as some currency forwards—are each month positioned long or short, depending on whether the previous months' and year's return was positive or negative.

Traditional Life-Cycle Strategy

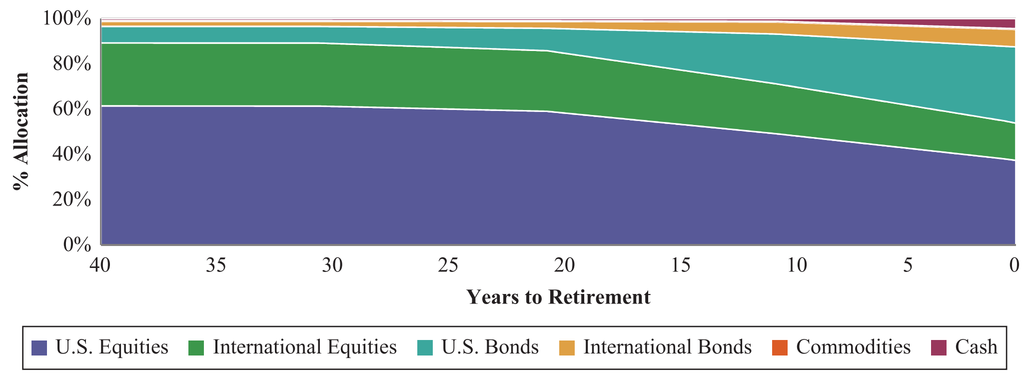
To develop a proxy for the traditional life-cycle approach, we take data from the websites of three of the largest providers of these strategies. Each firm provides different recommended asset allocations, depending on how far an investor is from retirement. Given these allocations, we construct our traditional life-cycle strategy by averaging the recommended asset allocations from each provider. Exhibit 1 shows these averages versus the number of years to retirement. Although we show averages, it's worth noting that there are no substantial differences in the recommendations across the three firms.

As we see in Exhibit 1, all providers show significant home bias toward U.S. stocks and bonds. Also, all three have a relatively small allocation to commodities (in fact, only Fidelity recommends any direct investment in this asset class). Finally, the dynamic risk-taking path is broadly similar across the three providers: All recommend close to 90% equities when investors are young and move to about 50% equities near retirement.

What does this mean in terms of portfolio risk? Exhibit 2 plots an estimate of the volatility of the average recommended portfolios versus years to retirement. Estimates are based on the average recommended asset allocation at each point in time and a full sample covariance matrix calculated using monthly data from

EXHIBIT 1

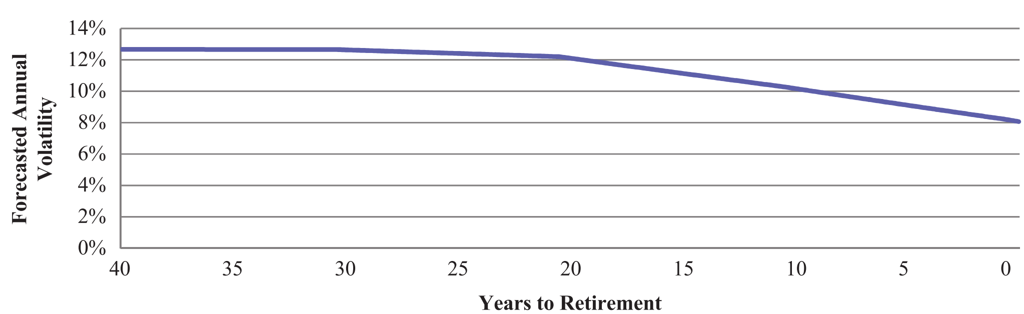
Traditional Recommended Asset Allocation vs. Years to Retirement, Averaged From Three Top Providers' Fund Weights, 2014



Source: AQR and the three large providers of target-date funds: Fidelity, Vanguard, and T. Rowe Price as of 2014.

EXHIBIT 2

Estimated Annual Volatility of Recommended Asset Allocation vs. Years to Retirement



Source: AQR and the three large providers of target-date funds: Fidelity, Vanguard, and T. Rowe Price as of 2014.

1900 through 2014.[5] As we would expect, these strategies target higher risk early on, and they decrease target risk as the investor approaches retirement. The estimated portfolio volatility is highest at the beginning, at 12.7%, and decreases to 8.1% near the end (a volatility level that seems surprisingly high for near-retirees).

LIFE-CYCLE STRATEGIES ARE UNDERDIVERSIFIED

In this section, we present more in-depth discussion of the five distinct ways in which traditional life-cycle funds are potentially underdiversified and propose possible remedies for each (see Exhibit 3). We also document how these remedies could have enhanced risk-adjusted returns historically.

Home Bias

Life-cycle funds are home biased. Averaging data across the three major providers, we see that target-date funds allocate 69% of their equity component to U.S. stocks, well above their 49% weight in a global market-cap portfolio (in 2014). Home bias is even more pronounced in bonds: Domestic allocation of bonds averages 79%—more than double the 34% share that U.S. bonds have in global bond markets.

This type of home bias is not uncommon. Despite a large body of evidence showing the benefits of international diversification, many investors remain heavily biased toward domestic investments. A common criticism of international equity investing is that global markets seem to crash at the same time—seeming to fail investors exactly when they need it most. In response to this critique, Asness, Israelov and Liew [2011] show that international diversification works—eventually (that is, at long horizons). Using monthly data since 1950, they show that the worst losses of global equity portfolios over 5- to 10-year horizons were significantly smaller than the average worst losses of domestic portfolios. Thus, even though international diversification may at times disappoint over short horizons, over long horizons it seems to work as one would expect. Given that investors who are saving for retirement are almost by definition long-horizon investors, we believe that international diversification makes sense for life-cycle portfolios.

EXHIBIT 3

Five Shortcomings of Traditional Life-Cycle Strategies with Proposed Improvements

Shortcoming / Source of Underdiversification	Proposed Solution / Source of Better Diversification
Home-Biased Investing	Invest Globally
Few Inflation-Protection Assets	Add, e.g., Commodities
Equity Risk Concentration	Risk-Balanced Allocations
Sensitivity to Volatile Periods	Dynamic Volatility Targeting
No Diversifying Alternative Premia	Add, e.g., Trend Following

Source: AQR.

Lack of Inflation Protection

Traditional life-cycle strategies consist largely (98%) of stocks and bonds. Among others, Ilmanen, Maloney, and Ross [2014] find that neither asset class is a good inflation hedge. They present evidence that global stocks and bonds have historically had much lower risk-adjusted returns when inflation was "up" (i.e., above average or merely above the consensus forecasts) rather than "down." Because commodities display the opposite pattern, they are naturally a useful complement to stock-and-bond portfolios. A meaningful allocation to commodities—or other inflation-protection assets on which we have shorter historical data—can help life-cycle investors when inflation picks up. (We do not focus on the deflation risk, though we note that equity-oriented portfolios suffered and bonds performed well amid the deflations of the U.S. Great Depression in 1930s and of Japan after the Nikkei bubble, suggesting that the equity-inflation relationship is not linear.)

We should recognize, however, that inflation protection may be available from the saver's nonfinancial holdings (human capital, eventual Social Security checks, home equity).[6] The design of life-cycle investing should take into account those nonfinancial assets, even if they are not part of life-cycle funds.

Equity Risk Concentration

Life-cycle funds may appear nominally diversified, but even near retirement, when they have their lowest allocation to equities, their performance is still driven primarily by one risk source: global equity market direction. The variance decomposition in Exhibit 4 shows that even at retirement, when life-cycle funds have their

EXHIBIT 4

Capital and Risk Allocation of a Typical Life-Cycle Fund Near Retirement

	Equities	Bonds	Commodities
Capital Weights	56.3%	43.3%	0.4%
Risk Contributions	91.7%	8.1%	0.2%

Source: AQR and the three large providers of target-date funds: Fidelity, Vanguard, and T. Rowe Price as of 2014.

lowest allocation to equities (roughly half), equities still account for about 90% of total portfolio risk.[7]

Perhaps younger investors can tolerate their even greater equity risk concentration—they have time on their side—but for near-retirees, large equity losses are a more severe problem, as the 2008–2009 experience attests. It is not surprising that during that time, older savers were most likely to capitulate and sell their target-date funds near the market trough. Better risk balance across asset classes or other diversifying return sources can help mitigate this problem.

Qian [2005], Hurst, Johnson, and Ooi [2010], and Roncalli [2014], among others, present a risk-parity approach to asset allocation that allocates equal risk weight to each of the underlying asset classes. Portfolios constructed in this manner have outperformed traditional allocations over most long samples studied (see Asness, Frazzini, and Pedersen [2012]). This result stands in contrast to the capital asset pricing model (CAPM), which would predict that the market-cap-weight portfolio should deliver the highest risk-adjusted return.

Asness, Frazzini, and Pedersen [2012] argue that the reason risk-parity portfolios beat traditional market-cap-weight portfolios is that investors are leverage averse, echoing the pioneering work of Black [1972]. Thus, if investors want more return, they shift their portfolios to higher-risk investments. This behavior pushes up the expected return on low-risk versus higher-risk investments. Asness, Frazzini, and Pedersen [2012] argue that risk parity outperforms the market in the long run because risk parity overweights these low-risk investments that have better risk-adjusted returns.[8]

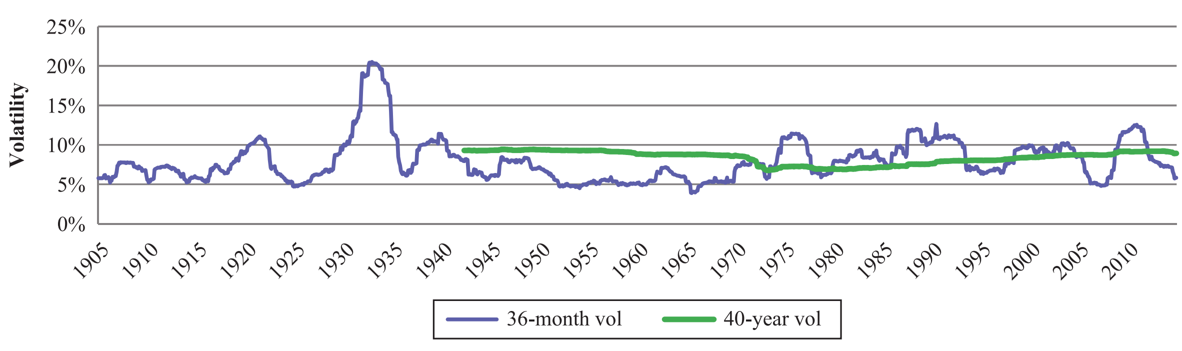
Excess Sensitivity to Volatile Periods

Life-cycle savers are often poorly time-diversified in the following sense: Despite a long savings period, they are especially sensitive to capital market performance during the last decade before retirement—when their savings pot is largest. Here we emphasize another underappreciated feature. Like all strategies that target nominal allocations, traditional life-cycle strategies are overly sensitive to episodes of high market volatility, such as in 2008, 1973–4 and 1929–32. Exhibit 5 provides one illustration by plotting 3-year and 40-year rolling volatilities of a 60/40 global stock/bond portfolio. Clearly, the Great Depression in the early 1930s was the period of greatest volatility: The 3-year rolling volatility was nearly triple the median volatility over the whole period.

Dynamic volatility targeting can be used to lessen excess sensitivity to volatile periods. For example, a strategy that targets constant portfolio volatility over time reduces nominal position sizes when near-term

EXHIBIT 5

Rolling Historical Volatility of Hypothetical 60/40 Global Stock/Bond Portfolio Since 1905



Source: AQR. For the 60/40 portfolio, we use GDP-weighted combinations of stock and bond country indexes listed in Appendix A.

volatility forecasts (based on recent volatility) increase. As a result, portfolios should be more evenly affected by capital market fluctuations over time and not dominated by episodes of high market volatility.

Lack of Diversifying Alternative Strategies

Despite a large and growing body of literature suggesting that risk premia exist in places other than traditional long-only asset classes, traditional life-cycle strategies make no or minimal allocations to alternative risk premia.[9] Here we focus on just one: trend-following strategies that have the longest period of data. Hurst, Ooi, and Pedersen [2014] examine trend-following strategies using over 100 years of data and find strong evidence of a positive reward to these strategies. Although trend following is not the only documented alternative risk premium, it's one that appears to be a particularly interesting complement to traditional portfolios, because it has some tendency to perform well during severe market downturns. Despite favorable empirical evidence over decades and centuries, some investors question the sustainability of trend-following strategies because their explanations are mainly behavioral rather than risk-based.[10]

Empirical Benefits of the Five Improvements

We end this section by quantifying the incremental benefits from the five improvements highlighted in Exhibit 3 and discussed in this section. This analysis uses more than a century of data (1903 to 2014), but does not yet apply the full machinery of life-cycle investing with evolving risk targets. (We will later show performance in more relevant terms for pension savers: average real retirement wealth across various cohorts. Those results reflect for each cohort a typical saving path over 40 working years and a portfolio with the equity share declining with age.) Here, we start from a simplified traditional life-cycle portfolio and then show how the long-run Sharpe ratio improves stepwise for five other increasingly diversified portfolios (but we do not yet include a 40-year saving path or evolving portfolio risk).

In Exhibit 6, the first column shows the Sharpe ratio of the most simplified average traditional life-cycle portfolio that is fully home-biased—thus 80% in U.S. stocks and 20% in U.S. bonds. Next, we go global: The second column corresponds to a global version of this portfolio that replaces U.S. stocks and bonds with their GDP-weighted global counterparts. The third column shows the impact of replacing 10% of this stock-and-bond portfolio with (equally weighted) commodity futures—an effort to provide better inflation protection.[11] The fourth column corresponds to a risk-parity portfolio that targets equal risk contributions between global stocks, global bonds, and commodities. However, in this case, we target constant long-term volatility from each underlying asset class. That is, we do not dynamically manage the positions as short-term measures of volatility change. As such, this fourth case tells us how much benefit we get from allocating risk more equally across asset classes (on average over time) without trying to manage the amount of risk we take through time. The fifth portfolio manages the risk we take through time by dynamically adjusting the portfolio's nominal positions to target constant short-term risk exposure to each asset class.[12] The sixth and last column adds a 10% allocation to a trend-following strategy (with the same volatility target as the risk-parity portfolio).

Exhibit 6 shows that each improvement boosts the long-run Sharpe ratio incrementally. Stated in returns for a portfolio with 10% volatility, the combined improvements would have historically doubled the annual return

EXHIBIT 6

Stepwise Sharpe Ratio Improvement Across Hypothetical Portfolios: ***Sharpe Ratios Net of Estimated T-Costs, January 1903–December 2014***

Traditional U.S.	Traditional Global	Trad. Global + 10% Commodities	Risk Parity Without Vol. Targeting	Risk Parity With Vol. Targeting	Risk Parity + 10% Trend
0.38	0.51	0.54	0.61	0.64	0.76

Source: AQR. The raw data series used are described in Appendix A and the trading cost estimates in Appendix B.

over cash from 3.8% for the traditional U.S. portfolio to 7.6% for the risk parity +10% trend portfolio.

These results are presented net of expected trading costs using historical cost estimates described in Appendix B. Gross Sharpe ratios would be about 0.01 higher for traditional portfolios and up to 0.03 higher for risk-parity portfolios if trading costs related to risk targeting were not deducted. We do assume significantly higher trading costs (t-costs) in the olden days than in 2000s, but, admittedly, we do not have good data on historical costs, especially going back to the early part of our sample. Moreover, severe investability challenges include capital controls across countries, leverage constraints, and lack of liquid futures markets or index funds. Most academic and practitioner studies ignore t-costs for similar reasons. As a result, we recognize that distant historical analyses cannot really tell us whether certain strategies would have worked in practice. However, we can say that if the gross-of-t-cost market return patterns observed historically hold up going forward, then they are particularly attractive today, given lower trading costs and better investability.[13]

Most of the improvements we propose to life-cycle strategies are not "macro consistent"—that is, all investors cannot simultaneously improve their portfolios without moving market prices. The one exception is home bias: All investors across the world could, in principle, improve their portfolio diversification and reduce portfolio risk by trading with each other and replacing their home-biased portfolios with global market-cap portfolios. However, in other cases (as in all active strategies), investors with opposite opinions or preferences would be needed to take the other side, so these ideas could not be adopted by everyone.

COMPARING TWO LIFE-CYCLE STRATEGIES

The defining characteristic of life-cycle strategies is the glide path of evolving risk-taking over time. Life-cycle strategies assume investors' risk tolerance varies with age. Young investors generally have higher risk tolerance but as they grow older, risk tolerances go down.[14] So far, we have only presented empirical results for average life-cycle portfolios. In this section, we will incorporate the glide path of declining risk tolerance as well as the growing retirement savings pot. The goal is to compare the traditional life-cycle approach to a risk-parity approach that incorporates the above improvements.

Traditional life-cycle strategies reflect evolving risk preferences by investing more heavily in equities early on and gradually shifting toward more bonds as investors approach retirement. As we saw in Exhibit 1 above, our proxy for the traditional approach holds almost exclusively (always above 95%) stocks and bonds and gradually reduces the stock weight from about 90% to about 50% during a 40-year savings period. By concentrating in stocks in the early years to achieve more return, this strategy represents a classic case of leverage aversion in action.

A risk-balanced or risk-parity approach to life-cycle investing captures the declining risk tolerance with age by adjusting the portfolio volatility over time instead of applying the glide path of declining equity/bond mix. The risk-parity approach holds the same risk-diversified mix of investments through life, but with a gradually declining volatility target over time. Thus, our risk-parity strategy maintains equal risk allocation to each of the underlying asset classes and reduces the target total portfolio volatility with age from 12.7% to 8.1%. In this way, it matches the estimated volatility path of the traditional approach depicted in Exhibit 2 above.[15]

Although the traditional approach is a straightforward application of the weights in Exhibit 1, the risk-parity life-cycle strategy warrants some further explanation. Below is a step-by-step discussion of our portfolio construction methodology:

- We start by constructing three model subportfolios, one for each asset class: (GDP-weighted) global equities, (GDP-weighted) global bonds and (equally weighted) commodities. Note that we do not incorporate trend following at this stage. We'll save that for later in this article.
- Because we want each asset class to have equal risk representation in the overall portfolio, we next estimate the riskiness of each asset class portfolio using rolling volatilities with an exponentially weighted data window (1-year center of mass). Using these estimates (based on information that was in principle available to investors at the time of investing), we scale each asset class portfolio to target an arbitrary, constant 10% ex ante volatility at each point in time. (We will later scale the size of the entire portfolio to match a risk target.)
- Then we add the excess-of-cash returns, net of estimated trading costs, of these three portfolios together

to create a combined portfolio with equal volatility contributions from the three components.[16]

- The next step is to estimate the volatility of this combined risk-parity portfolio using rolling correlations between the asset classes (the same exponentially weighted window). When correlations are higher, our risk estimate for the portfolio will be higher. Correlations are floored at zero.[17]
- We then use this estimate to scale our risk-parity portfolio to match the volatility path of the traditional life-cycle strategy shown in Exhibit 2. Instead of shifting the asset allocation as the investor ages, our risk-parity life-cycle strategy changes the risk target by changing the amount of leverage we apply to our risk-parity portfolio. Thus, from a risk level perspective, our risk-parity life-cycle strategy exactly matches the traditional life-cycle strategy, but it gets there in a very different way.
- As a final step, we impose a constraint that each asset class's leverage is capped at 200%.[18] In practice, this means that we will not reach exactly equal risks across asset classes because, at times, volatilities are low enough that our capped positions will not reach their full risk target.

Empirical Results

We next turn to our horse race. We have monthly returns from January 1900 through December 2014, and we need the first three years to get initial risk estimates. Therefore, in our study, the first cohort invests from January 1903 to December 1942 (we'll call this the *1942 cohort*). The next cohort invests from January 1904 to December 1943 (the *1943 cohort*), etc. In this manner, we are able to track the performance of 73 investor cohorts with 40-year life-cycle strategies. Although we have over 100 years of data, we should note that we have less than three independent/non-overlapping observations (though more data than most studies!).

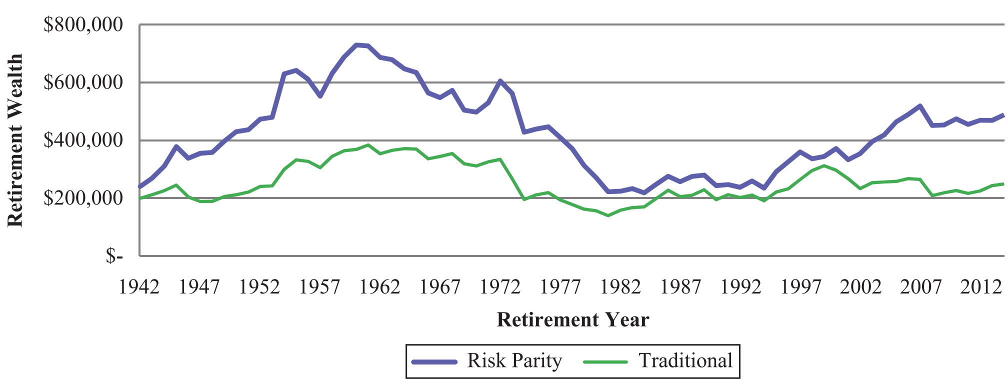
We now compare the performance of the two life-cycle investing approaches using a savings strategy that begins by investing $1,000 in real terms (net of consumer price inflation [CPI]). At the end of December every year through the 40-year accumulation period, the savings strategy contributes a gradually increasing amount (2.8% real p.a.), peaking at a final contribution of $3,000 in real terms.[19] Our key performance measure below in Exhibit 9 will focus on how much money (in real terms) each cohort ends with at retirement.

Exhibit 7 presents the results, and they look compelling. In this historical simulation, the risk-parity strategy achieved higher ending values at retirement for every cohort. The advantage is greatest for cohorts retiring between 1960 and 1980. The average (real) retirement wealth across 73 cohorts is $253,000 for the traditional life-cycle strategy and $427,000 for the risk-parity strategy (69% higher). These numbers correspond to internal rates of return of 6.0% and 8.3%, respectively.[20,21]

Overall, cohorts retiring in the early 1940s and early 1980s were unlucky and achieved relatively low

EXHIBIT 7

Real Ending Wealth for Hypothetical Traditional vs. Hypothetical Risk-Parity Life-Cycle Strategies by Retirement Year



Source: AQR. The raw data series used are described in Appendix A and the trading cost estimates in Appendix B.

retirement wealth (in both approaches) because capital markets were not benign in the final decade before their retirement (when the saving pot was largest). Conversely, cohorts retiring between 1960 and 1972 or between 1997 and 2007 can be considered lucky, given capital market tailwinds at the end of their savings window.

Interpreting the Results

Why does the risk-parity approach perform so well? To answer this, let's study the returns of the key building blocks during each 40-year saving cohort. Exhibit 8 plots the 40-year rolling real returns of the three constituent asset class portfolios. Because our leverage cap over this 114-year period is binding only for the bond portfolio, we present two lines for bonds: In addition to the unlevered bond portfolio, the "levered max 200%" line shows the returns of a global bond portfolio that is scaled to match the volatility of global equities, but with its leverage capped at 200%.[22]

Over this period, each asset class has its day in the sun. Equities are especially important for cohorts retiring in the 1950s and '60s. Commodities are the saving grace for investors retiring in the mid-1970s and thereafter. Bonds have lower returns because of the leverage constraint, but are essential for cohorts retiring in the 2000s. We clearly cannot explain risk parity's consistent outperformance in Exhibit 7 by levered bonds or commodities outpacing equities with regularity. If anything, Exhibit 8 shows that equities are often the highest-returning asset class. The real reason for risk parity's consistent outperformance over the traditional life-cycle approach is its better diversification. When asset classes are lowly correlated, risk parity can take larger exposures in all asset classes to target the same portfolio volatility as the more risk-concentrated traditional approach.

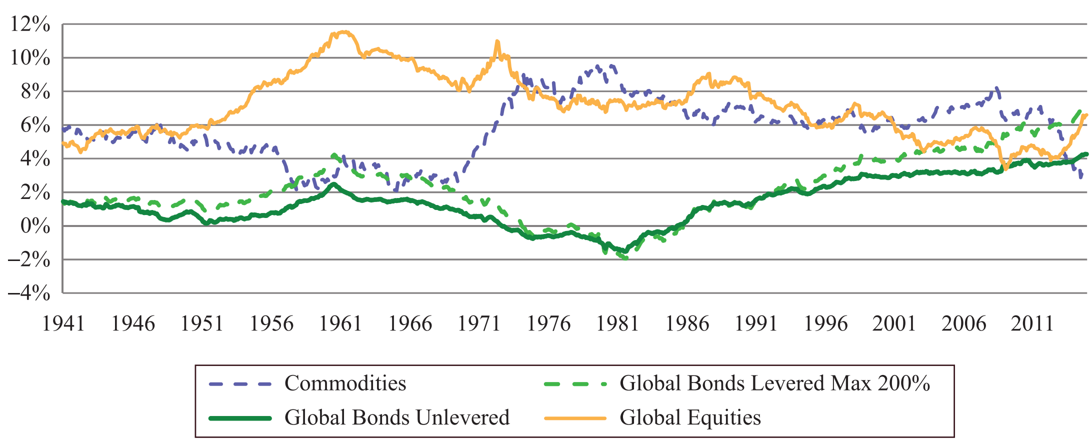
Detour: Traditional Life-Cycle Strategies' Overreliance on Equity Valuations

Because of the high concentration of equities in traditional life-cycle strategies, changing equity market valuation during a 40-year savings period can be an important factor in the success of these strategies. An investor can get lucky by saving and buying assets at bear-market prices and selling, or annuitizing, at the end of a bull market (or he can get unlucky by doing the opposite). For example, the lucky cohort retiring in 1999 experienced the largest increase in the U.S. equity market's Shiller P/E ratio during the 40-year saving window, which helped in achieving a relatively high retirement wealth. Not surprisingly, the traditional life-cycle strategy turns out to be especially sensitive to changing valuations—the correlation across cohorts between the changing valuation ratio and the terminal wealth was 0.59.

What does this look like for the risk-parity strategy? The terminal wealth for the risk-parity strategy also varies positively with changing equity market valuations—but

EXHIBIT 8

40-Year Rolling Real Returns of Hypothetical Unlevered Global Equities, Bonds, Commodities, and Global Bonds Levered to Match Equity Market Volatility but Subject to 200% Leverage Cap



Source: AQR. The raw data series used are described in Appendix A and the trading cost estimates in Appendix B.

more mildly, with a 0.34 correlation. The contrast is even starker if we relate retirement wealth across cohorts with the change in the Shiller P/E ratio during the final decade before retirement (this decade matters most as the savings pot is largest): The correlation is 0.43 for the traditional life-cycle strategy and 0.00 for the risk-parity strategy.

Clearly, if investors want to minimize their exposure to the lucky or unlucky draw of what happens to equity market valuations over their 40-year savings period, the risk-parity strategy appears to be the better option.

Incremental Impact of Each Method of Better Diversification

Although it's great to see that, as a group, the above approaches to adding diversification improve investors' final savings versus a traditional life-cycle strategy, it is also important to see how each method contributes on its own. To do this, we repeat the horse race above, but decompose the above risk-parity life-cycle strategy into five life-cycle strategies that each incorporate one improvement at a time in a stepwise manner.

1. *Traditional U.S.*: This strategy represents our straw-man starting point. Note that this strategy is actually a bit worse than the traditional approach used above because it's limited to only U.S. stocks and bonds (nothing global and no commodities). However, it follows the same stocks-versus-bonds glide path as the traditional life-cycle strategy (whose performance we show for comparison in the last row of Exhibit 9).
2. *Traditional global*: This strategy takes (1) above and goes global. That is, it holds the same stocks-versus-bonds weights as strategy (1), but instead holds GDP-weighted global stock-and-bond portfolios.
3. *Traditional global + 10% commodities*: This strategy starts with (2) above and adds a 10% allocation to commodities.
4. *Risk parity without dynamic volatility management*: This strategy moves into balancing risk across asset classes. We consider this strategy close but not fully risk balanced, because it uses a full sample (114-year) covariance matrix to construct the portfolio and manage the portfolio's volatility glide path. That is, we manage the portfolio without the benefit of dynamically targeting volatility. Compared to strategy (3), this portfolio will hold larger allocations to bonds and commodities and will use leverage to attain comparable risk targets to a traditional approach.
5. *Risk parity with dynamic volatility management*: This strategy uses full risk parity and dynamically manages volatility using the methodology described above.
6. *Risk parity with dynamic volatility management + 10% allocation to managed futures*: This strategy takes (5) above and adds a 10% risk allocation to managed futures (a diversified composite of trend-following positions), dynamically adjusting weights so that the total portfolio still targets the same volatility path as risk parity. So far, our empirical horse race has been limited to long-only asset classes. However, as we have noted, long/short strategies (including what we call *alternative risk premia*) are potentially highly attractive diversifiers for many portfolios, including those of life-cycle investors. Here we focus on one such strategy, trend-following.

EXHIBIT 9
Horse Race Step-by-Step

	Annual Real Return	Annual Vol	Sharpe Ratio	Avg. of best 10%	Mean	Avg. of worst 10%	Max Drawdown	Worst Year
Traditional U.S.	6.4%	15.2%	0.40	$318,199	$231,025	$141,963	–50%	–42%
Traditional Global	6.5%	10.6%	0.59	$391,911	$261,361	$180,731	–38%	–30%
Trad.Global + 10% Commodities	6.5%	10.0%	0.63	$365,794	$259,681	$191,530	–34%	–27%
Risk Parity w'out Vol. Targeting	7.9%	12.3%	0.63	$621,466	$386,278	$203,063	–35%	–27%
Risk Parity with Vol. Targeting	8.3%	11.4%	0.71	$679,636	$426,510	$232,218	–28%	–24%
Risk Parity + 10% Trend	10.0%	11.6%	0.84	$1,058,551	$658,445	$369,402	–26%	–23%
Traditional Official	6.5%	12.1%	0.52	$366,735	$252,560	$165,435	–43%	–35%

Source: AQR. The raw data series used are described in Appendix A and the t-cost estimates in Appendix B.

Exhibit 9 presents the results of our exercise.[23] In line with Exhibit 6, it shows that every performance and downside risk metric is incrementally improved at almost every step. For example, on perhaps the most important criterion, the mean retirement wealth across cohorts, every addition is helpful, except for the second (commodity addition). In that case, the mean drops marginally from $261,000 to $260,000, but all downside risk statistics look better. Adding a small allocation to managed futures ("trend") provides a meaningful improvement in every statistic.

A More Realistic Proposal: Modest Portfolio Reallocation toward Risk-Diversifiers

For most investors, it would be unrealistic to make a total change away from the traditional life-cycle investment strategy, given the leverage and unconventionality in our alternative strategies (a topic we return to in the next section). Thus, our final empirical exercise analyzes the impact of more modest, and thus more realistic, portfolio reallocations in what we argue is the right direction. Exhibit 10 uses the traditional global portfolio as the baseline because removing home bias does not require leverage and is today a rather conventional choice. It compares a 100% allocation to the traditional global strategy to alternate strategies that allocate (1) 80% to the traditional global strategy and 20% to risk parity (with volatility targeting) and (2) 80% to the traditional global strategy and 10% each to risk parity and trend following. All portfolio statistics improve with each step (albeit less than above because the reallocations are modest). Exhibit 11 shows that every cohort would have achieved a higher retirement wealth.

PRACTICALITY AND RISKS OF USING LEVERAGE

Although we believe that the approach we propose is superior to the current industry practice, we have no illusions about the difficulty of changing standard practice. Better diversification attracts people—but the

EXHIBIT 10

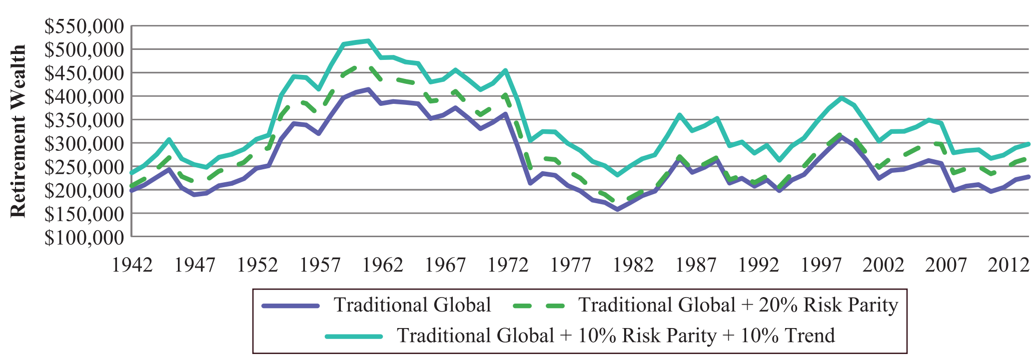
Horse Race With Small Modifications to the Traditional Global Life-Cycle Strategy

	Annual Real Return	Annual Vol	Sharpe Ratio	Avg. of best 10%	Mean	Avg. of worst 10%	Max Drawdown	Worst Year
Traditional Global	6.5%	10.6%	0.59	$391,911	$261,361	$180,731	–38%	–30%
Trad. Global + 20% Risk Parity with Vol. Targeting	6.8%	10.3%	0.65	$439,216	$289,320	$194,526	–35%	–29%
Trad. Global + 10% Risk Parity + 10% Trend	7.5%	9.6%	0.77	$489,377	$341,560	$247,816	–30%	–25%

Source: AQR. The raw data series used are described in Appendix A and the trading cost estimates in Appendix B.

EXHIBIT 11

Real Ending Wealth by Retirement Year for Three Hypothetical Life-Cycle Strategies: *100% Traditional (Global); 80% Traditional / 20% Risk Parity; and 80% Traditional / 10 % Risk Parity / 10% Trend*



Source: AQR. The raw data series used are described in Appendix A and the trading cost estimates in Appendix B.

use of leverage in the proposed approach scares them, as does the general unconventionality of any new approach. Leverage does involve risks, but careful evaluation suggests that these risks are manageable and generally well-rewarded in modestly leveraged, liquid strategies (and remember, leveraging the best portfolio is absolutely consistent with the most basic financial theory, unlike the traditional approach).

Investors who want higher returns in today's low-yield world can choose between two types of risk: the concentration risk of an equity-dominated portfolio or the leverage risk of a well-diversified portfolio.[24] However, they cannot choose higher returns without taking one of these risks. We believe leverage is the more manageable risk—and the one that in theory and practice is actually rewarded over the long term. The fact that concentration is conventional does not make it any less perilous. Unlike studies that propose leveraging equity portfolios for young savers (e.g., Ayres and Nalebuff [2013]), ours does not propose increasing portfolio risk above that of traditional life-cycle funds: We propose taking the same amount of portfolio risk in a more diversified way.

To be clear, we do not recommend that individual retirement savers or even target-date funds apply direct leverage to their portfolios. Instead, they may consider making some allocations into funds that apply some leverage (perhaps by using futures). Not all leverage is equal; certain types of leverage are more dangerous than others. Examples include levering up illiquid positions or intentionally taking more risk when risky assets appear unattractive in order to hit a return objective. Other types of leverage are more benign—for example, moderately levering up liquid assets or holding exchange-traded futures, while leaving plenty of free cash in the portfolio—and are much less likely to cause investors to delever in bad times. Not surprisingly, we endorse the latter, more benign type of leverage. We believe it is a superior form of risk-taking compared to concentrating in only equities, but by no means do we regard it as riskless.

CONCLUSION

If investors are able to use leverage, modern portfolio theory tells us that investors can construct their optimal portfolio by making two separate decisions: 1) What is the most efficient or highest Sharpe ratio portfolio? 2) How much risk should they take? With leverage, the second decision should be simply a matter of deleveraging (by investing some of the portfolio in cash) or leveraging the optimal portfolio to reflect the investor's risk preference (volatility target).

Traditional life-cycle strategies seem to ignore this advice and invest in highly concentrated portfolios. We believe that they suffer from many forms of underdiversification—home bias, insufficient inflation protection, equity risk concentration, excess sensitivity to volatile periods, and lack of diversifying long/short strategies. One explanation for why people still follow this arguably suboptimal traditional approach seems to be extreme leverage aversion: Because of their aversion, many investors let equity market directional risk dominate their portfolios and thus forfeit opportunities for better risk diversification.

Our risk-parity-based life-cycle strategy tries to address these issues. Our approach simply leverages the same well-diversified portfolio to achieve a given risk target, which in a life-cycle strategy declines over the participant's life. In this way, the risk-parity life-cycle strategy may at all times achieve superior diversification when compared with the traditional approach.

Having said all this, we note that empirical studies of lifetime investing strategies are difficult. After all, we are looking at 40-year strategies with a little over 100 years of data—so, in reality, we have less than three truly independent data points. However, the available data clearly agree with the theoretically motivated premise that holding better-diversified portfolios through the pension accumulation period should result in better long-run investment outcomes. A careful comparison of traditional and risk-parity life-cycle investing shows that the latter approach would have resulted in greater ending wealth for all cohorts in this study. Even mild portfolio shifts to this direction would have helped retirement savers.

APPENDIX A

DATA DETAILS

Our study uses monthly returns on 10 equity indexes, eight global bond indexes, and 29 commodity futures contracts compiled by Hurst, Ooi, and Pedersen [2014]; see Exhibit A1.

The return histories start as early as January 1900, but the asset universe is narrow in the early decades. In order

EXHIBIT A1

Monthly Return History Since…
(until December 2014 unless otherwise noted)

Australian 10-Year Bond	01/1986
Canadian 10-Year Bond	01/1950
Australian 10-Year Bond	01/1986
Canadian 10-Year Bond	01/1950
French 10-Year Bond	01/1900
German 10-Year Bond	01/1968
Italian 10-Year Bond	01/1900
Japanese 10-Year Bond	01/1980
U.K. 10-Year Gilt	01/1900
U.S. 10-Year Treasury Note	01/1900
Aluminum	02/1979
Brent Crude	05/1989
Cocoa	02/1965
Coffee	04/1974
Copper	02/1977
Corn	01/1900
Cotton	01/1925
Gas Oil	11/1984
Gold	01/1970
Heating Oil	01/1979
Hogs	03/1966
Lard	01/1900 (to 11/1951)
Live Cattle	02/1965
Natural Gas	05/1990
Nickel	02/1993
Oats	01/1900
Platinum	02/1984
Pork	01/1900 (to 03/1922)
Rye	01/1900 (to 11/1951)
Shortribs	01/1900 (to 07/1929)
Silver	02/1965
Soybeans	01/1949
Soymeal	01/1953
Soyoil	01/1953
Sugar	02/1965
Gasoline	01/1985
Wheat	01/1900
WTI Crude	04/1983
Zinc	02/1991
Australian SPI 200	01/1900
Canadian S&P/TSE 60	01/1934
French CAC40	01/1900
Germany DAX	01/1958
Italian FTSE MIB	01/1925
Japanese Topix	01/1925
Netherlands AEX	01/1951
Spanish IBEX 35	04/1940
U.K. FTSE100	01/1923
U.S. S&P 500	01/1900

to compile a continuous series going as far back as possible, Hurst, Ooi, and Pedersen [2014] combined data from various sources (Global Financial Data, Ibbotson, Datastream, Morgan Markets, CBOT, CSI, and Bloomberg).

Finally, all non-U.S. stock and bond returns are hedged for exposure to foreign currency,[25] and all the return series are adjusted by U.S. CPI. Thus, we are evaluating real returns to U.S.-based investors.

APPENDIX B

COST ESTIMATES

We present results net of estimated trading costs used in Hurst, Ooi, and Pedersen [2014].[26] We assume five to seven times higher costs before 1990s than in recent years, reflecting the notion that costs have fallen sharply over time. Specifically, we assume equity trading costs per notional trade of 34 bps until 1992, then falling to 6 bps for the past decade. For bonds, the assumed trading costs fall from 11 bps to 2 bps; and for commodities, from 58 bps to 10 bps. The use of steeply higher costs in the past is a deliberately conservative choice. In line with most of the literature, we do not include holding costs (of, say, rolling between futures contracts or into newly issued bonds) or financing costs of levered positions beyond the cash rate.

The actual costs to investors reflect both the costs above and the dollar turnover over time. We estimate that traditional life-cycle investors who retired recently experienced about 6 bps annual performance drag due to trading costs (we ignore the fact that in practice, target-date funds often invest through active managers with higher fees and costs). Cohorts that retired between the 1940s and 1970s experienced about 10bp annual drag due to trading costs.

For risk-parity investors who retired recently, the annual drag from trading costs was 30 bps. Cohorts that retired between the 1940s and 1970s suffered an annual drag of 40–50 bps. These costs are higher because higher turnover, but they are still small enough not to overturn the large performance edge risk parity has historically offered.

We make no claim on the past investability of these investment strategies. Capital controls in many countries, lack of index funds or active futures markets, and impediments to leverage would have made global diversification challenging in the traditional approach, and more so in the risk-balanced approach. We add some realism by constraining bond portfolio leverage to 200%, and by using sharply higher trading costs in the past. This study is intended to show how various investment strategies designed today would fare in at least one realistic potential path of history—the path followed by markets over the past 100 years.

ENDNOTES

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[1]Bary [2014] quoted these numbers. More recently, BrightScope [2015] estimated that the size of target-date funds (narrowly defined: Investment Act of 1940 funds) has grown to over $700 billion, whereas a broader definition of target-date assets (which includes collective investment trusts and pooled separate accounts) is closer to $1.1 trillion and is predicted to exceed $2 trillion by 2020.

[2]We will highlight below another aspect of underdiversification in time dimension. Retirement savers are especially sensitive to market movements in the years when their saving pots are largest. Thus, changing equity market valuations during the decade before the retirement will largely determine whether the saver belongs to the lucky or unlucky cohort.

[3]We take here as a given the industry consensus shape of the glide path in which equity weights decline with age. This shape has been challenged in several studies; for a review of many alternatives, see the appendix of Chilewich, et al. [2013].

[4]In 2014, the GDP-weight of the United States was 47% (equities; among 10 countries) or 50% (bonds; among 8 countries). In 1955, the corresponding weights were 54% and 63%. Before the 1950s, the U.S. weight in the global portfolio was even larger because we lack market return data from many countries.

[5]The pattern is broadly similar if we calculate our covariance matrix using monthly data over the last 50 years.

[6]Poterba [2014] shows that for most households at retirement age, Social Security and housing wealth constitute a larger part of total wealth than their retirement savings and other financial assets.

[7]Capital allocations are based on the 2014 average of Fidelity, Vanguard, and T. Rowe funds' recommended non-cash allocations for investors near retirement. Risk contributions reflect these capital weights and covariances between asset class returns using data between 1900 and 2014. These risk contributions show the fraction of portfolio variance contributed by each investment based on typical variance-decomposition calculations.

[8]Although one can certainly debate exact parity (equal risk) allocations, it is harder to argue against the broader notion that better risk balance should result in better portfolio diversification and smaller tail risks.

[9]Alternative risk premia are the returns of systematic long/short strategies in liquid asset classes, such as value, carry, and momentum style premia.

[10]Another idea, not explored here, is to improve the long-only equity and bond portfolios in life-cycle funds by tilting the cap-weight portfolios with over- and underweights toward well-rewarded styles such as value, momentum, and quality.

[11]Data availability over a century forces us to use only commodities as inflation-protection assets. Today investors have a broader range of assets available and should use a more diversified portfolio for inflation protection. For the 1975–2014 period, we also have return data on real estate (averaging direct real estate NCREIF index and listed REITs FTSE-NAREIT index) and on U.S. inflation-linked bonds (10-year TIPS since 1997, earlier using AQR's proprietary proxy for it). Our diversified inflation-protection portfolio makes equal allocations to real estate, linker bonds, and commodity futures. Over this 40-year period, a traditional global stock/bond portfolio had a Sharpe ratio of 0.50; allocating 10% to commodities would have raised it to 0.51, whereas allocating the same amount to the diversified inflation-protection portfolio would have boosted it to 0.52. The last three Sharpe ratios (with risk-parity portfolios) were 0.7–0.8 when using commodity futures; each of them rose by 0.03–0.05 when a diversified inflation-protection portfolio was used instead.

[12]Risk parity with volatility targeting (the fifth bar) targets 10% portfolio volatility at every point in time based on a recent covariance matrix (using exponentially weighted moving averages with 1-year center of mass). Risk parity without volatility targeting (the fourth bar) averages the weights through time and holds the same notional asset class allocations at every date instead of adjusting them based on recent volatilities or correlations. In both cases, correlation estimates are floored at 0 and the leverage of each asset class is capped at 200% (this leverage constraint is only binding for bonds).

[13]There is no guarantee that these model portfolios will come to market or be profitable.

[14]Academics have long debated the relationship between risk tolerance and age. Classical research by Paul Samuelson and Robert Merton suggested that investors should hold a constant share of their wealth in risky assets irrespective of their investment horizon or age. However, such behavior is optimal under quite restrictive assumptions. The main arguments in favor of greater risk tolerance for young investors are that i) human capital is a larger share of the total wealth of the young, suggesting that they can take more risk with their financial wealth (as long as their human capital is more bond-like than equity-like and thus not highly correlated with financial wealth); ii) young people have more flexibility to work harder to supplement financial returns if risky-asset returns disappoint; and iii) young investors have more time

to benefit from any mean reversion in risky-asset returns. Of course, there are counterarguments, but greater risk tolerance for the young has clearly won out in the marketplace and is what we focus on here.

[15]Note that the life-cycle funds' portfolio volatility of 8.1% at retirement is surprisingly high and reflects the still-high equity weight (and more fundamentally, the common tendency of investors to look at their portfolios' dollar allocations and ignore their more extreme risk allocations).

[16]A technical reader may note that our methodology targets equal volatilities rather than equal risk contributions across asset classes. We also tested the latter approach based on trailing correlations. The results (not shown) do not change much but, if anything, are even better for risk-parity life-cycle funds.

[17]Correlations are floored because we recognize the estimation error in correlations and the possibility of regime changes. Investors may not want to count on the persistence of this exceptional diversification, especially when it makes them vulnerable to sudden correlation spikes.

[18]This constraint is binding only for bonds and reduces their relative risk contribution. Especially in early 1900s, the relative volatility between global stocks and bonds was extremely high. Thus, to achieve full risk balance across asset classes, bonds would have had to be levered 6–12 times in the early 1900s and 2–5 times between the 1920s and 1960s. As noted, we do not claim that our strategies were investable in the distant past, but tenfold leverage seems both infeasible and intolerable for leverage-averse investors. Thus, we decided to maintain some realism by adding the 200% leverage cap.

[19]We assume contributions increase because of the general growth in real wages in the population and the cross-sectional increase in wages when aging (together up to 2% p.a.), as well as assumed tendency for workers to save a larger fraction of their income—rising from 8% to 12% as they age. As a robustness check, relative performance between risk parity and traditional is virtually unchanged if we simply save a fixed $1,000 real each year.

[20]The performance gap would be even wider if we had not subtracted trading costs or constrained bond leverage. The internal rate of return numbers can be computed using, for example, the Solver function in Excel.

[21]Sadly, such returns (for both cases) may be harder to achieve in the 21st century, given today's low starting yields for both stocks and bonds—but that is a topic for another day.

[22]Note that Exhibit 7 is not directly comparable to Exhibit 8, given different underlying saving schemes. Exhibit 7 shows cumulative wealth after 40 years of returns, including growing yearly contributions, whereas Exhibit 8 shows average return of a single initial outlay over a 40-year window. Thus, for Exhibit 7 the asset class performance near the end of the 40-year window matters more for ending wealth than the performance in early years when the saving pot was small.

[23]In the first three columns of Exhibit 9, we calculate arithmetic mean return, volatility, and arithmetic Sharpe ratio of every strategy variant for each of the 73 cohorts. Per statistic, what is reported is then the mean of the 73 data points. Those statistics do not reflect the growing size of the saving pot over the 40-year savings period. The last five columns do reflect the growing pot, including the last two columns which show % losses in wealth, not % returns. Note that compared to Sharpe ratios in Exhibit 6, the Sharpe ratios in Exhibit 9 are averaged across 73 cohorts, and reflect the full life-cycle machinery of a gradually increasing savings pot and dynamically reducing portfolio risk through time.

[24]Nominal weights of 60/40 between stocks and bonds imply at least 90/10 risk weights, given stocks' higher volatility. Moreover, traditional alternative investments, such as private equity and hedge funds, contain so much equity beta that they have been poor diversifiers. In contrast, the risk-parity approach weighs market risk premia so that each asset class matters equally to the portfolio risk. An unlevered risk-parity portfolio would be better diversified and thus offer higher risk-adjusted returns than an equity-dominated portfolio. However, because an unlevered portfolio would involve low volatility and return level, it is often levered to a similar level of volatility as a 60/40 stock/bond portfolio.

[25]The exact calculation for our currency-hedged total returns is to take the total return in the country's local currency in excess of the local short-term interest rate and then add the U.S. T-bill total return. This ignores a *cross-product term*, which is the return on the foreign currency times the local market returns, as well as any private-Treasury spreads in money markets, and therefore is only a close approximation to the true currency-hedged return.

[26]Trading costs are economically comparable in both papers; however, Hurst, Ooi and Pedersen [2014] costs for bonds are quoted in terms of bonds with a duration averaging 4; ours are for a duration averaging 7. As a result, the quoted costs per notional trade differ by a factor of 7/4 between the two papers.

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