



Alternative Thinking

Strategic Portfolio Construction: How to Put It All Together

This issue discusses strategic portfolio construction, focusing on top-down decisions where mean-variance optimization, always to be used carefully, is of even more limited use. How to combine traditional asset classes with illiquids or with alternative risk premia? How much illiquidity, leverage and shorting to allow? The answers — and thus major portfolio choices — are largely driven by investor beliefs and constraints.

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Executive Summary

- Portfolio construction is challenging even when combining comparable investments such as stocks and bonds. When mixing traditional and alternative assets, the challenges are multiplied.
- Mean-variance optimization is a useful tool for portfolio construction but it has many pitfalls. Its use is particularly limited when we make top-down portfolio decisions on characteristics beyond the mean-variance framework. For example, how much leverage and shorting to allow, or how much to allocate to illiquid investments, when an optimizer often keeps saying “more”? How should we weigh across diversifiers of traditional portfolios: alternative risk premia versus illiquid assets or versus manager-specific alpha?
- Such top-down decisions are largely driven by investor-specific beliefs and constraints. We describe portfolios both based on formal optimizations and on such subjective judgments.

Introduction

Portfolio construction is such a broad topic that it requires a book to cover the subject thoroughly.¹ Here we only cover some big themes on strategic asset allocation decisions (ignoring liabilities). Mean-variance optimization (MVO) is the classic portfolio construction workhorse, and its key insights are a natural starting point. One way to summarize unconstrained MVO is that as a baseline it allocates equal volatility to different investments but then it tilts toward investments with higher Sharpe ratios and/or lower correlations.

MVO works best when the allocation problem involves a small number of comparable investable assets, and the assumptions underlying MVO are broadly satisfied. Even then, estimation errors in

inputs — especially expected returns — are a serious problem, and unconstrained MVO can suggest extreme positions. The Appendix discusses MVO’s uses and pitfalls in more detail.

MVO is less useful when we combine traditional and alternative investments. We show the MVO results for allocating between traditional asset classes, illiquid “endowment alternatives” (defined below), and liquid alternative risk premia, given plausible inputs. Not surprisingly, unconstrained optimizers love illiquidity, leverage and shorting in a portfolio. Investors often respond to unreasonable results by adding constraints: ruling out leverage and shorting or setting even narrower ranges for acceptable weights. Indeed, when top-down decisions involve portfolio characteristics beyond the MVO framework, real-world choices are largely driven by investor-specific beliefs and constraints.² We finish with some practical portfolio proposals based on such inherently subjective considerations and only loosely guided by optimization results.

MVO and Top-Down Portfolio Decisions

Some of the most important top-down decisions in portfolio construction depend on investor tolerance for portfolio volatility, leverage, illiquidity and short-selling (as well as views on how well these features are rewarded by markets). Only the first, the mean/volatility tradeoff, is naturally captured in the MVO framework. Examples below show that, under common investor assumptions, MVO tends to load up on leverage, illiquidity and shorting unless these are explicitly constrained or penalized. While it is possible to impose such constraints, MVO does not help in deciding how tight the constraints should be.

We present MVO results for several universes, starting from the simple stocks (EQ) vs. bonds (FI) case, then broaden to include illiquid “endowment alternatives” (EALTS, such as private equity, hedge funds, real estate) and later also liquid alternative

¹ Some excellent books already exist; see *Successful Investing Is a Process* by Lussier (2013) and *Portfolio Construction and Risk Budgeting* by Scherer (2015). These topics are also covered in *Active Portfolio Management* by Grinold and Kahn (1999), *Asset Management* by Ang (2014) and *Introduction to Risk Parity and Budgeting* by Roncalli (2014).

² In geek-speak they end up at corner solutions where your basic beliefs and constraints are fully driving the outcome and the optimizer is not adding any value.



risk premia (ARP, such as a diversified portfolio of long/short style premia or hedge fund risk premia).³ We contrast the results of long-only, leverage-constrained optimizations (capital weights must be non-negative and sum to 100%) and unconstrained versions, as well as the impact of two different volatility targets.

The results, of course, reflect the inputs we give. Our expected return, volatility and correlation estimates are shown in **Exhibit 1**. We use forward-looking estimates, partly guided by long-run historical experience, but erring to the conservative side versus history with one exception: We use a deliberately optimistic assumption for the Sharpe ratio of EALTS so as to make our case for ARP rest mainly on diversification (as opposed to Sharpe ratio).⁴ We welcome readers to challenge these assumptions.

Choice 1: Risk Tolerance

In the MVO context, investors quantify their subjective risk tolerance by selecting an acceptable target level of portfolio volatility. Another way to quantify risk tolerance is setting the target equity weight in the EQ/FI allocation. In that spirit, we first check the optimal weights at low (6%) and high (10%) volatility levels when allocating between just these two asset classes (with no shorting or leverage allowed, but allowing investments in cash). **Exhibit 2** shows that the optimal EQ/FI capital weights are 34/62% and 66/34%, respectively, at these volatility

levels.⁵ Throughout this article we show both capital and risk allocations in two bars for each portfolio.⁶

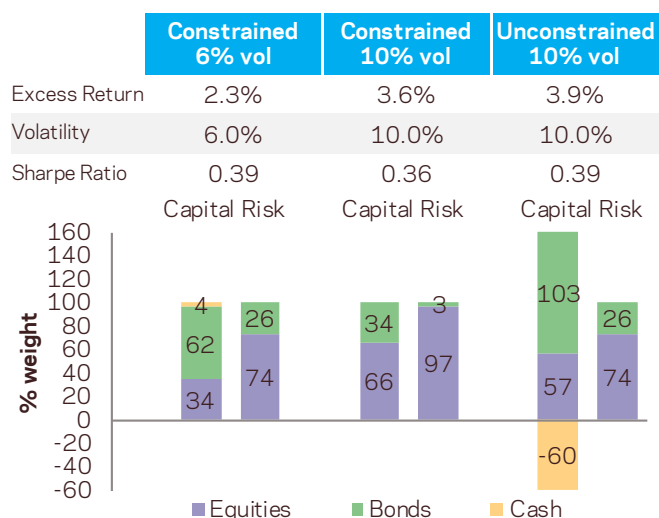
Exhibit 1 | Our return and risk assumptions

	Excess Return	Volatility	Net SR
Equities (EQ)	5.0%	15%	0.33
Fixed Income (FI)	1.0%	5%	0.20
Endowment Alts (EALTS)	7.0%	10%	0.70
Alt Risk Premia (ARP)	7.0%	10%	0.70

Correlations

	EQ	FI	EALTS	ARP
Equities	1.0			
Fixed Income	0.0	1.0		
Endowment Alts	0.7	0.2	1.0	
Alt Risk Premia	0.1	0.1	0.1	1.0

Exhibit 2 | Optimizing the equity-bond allocation



Source: AQR. Expected returns (excess over cash) for equities and fixed income are broadly in line with those in 2015Q1 *Alternative Thinking*. There is no guarantee, express or implied, that long-term return and/or volatility targets will be achieved. Realized returns and/or volatility may come in higher or lower than expected.

The low-risk portfolio has a higher Sharpe ratio while the high-risk portfolio has a higher expected return (we always show expected returns as excess

³ For more on style premia, see Asness, Imlanen, Israel and Moskowitz (2015): "Investing with Style", *Journal of Investment Management* 13(1). For more on hedge fund risk premia, see Berger, Crowell, Israel and Kabiller (2012): "Is Alpha Just Beta Waiting To Be Discovered?"

⁴ The Sharpe ratio we assume for EALTS is lower than raw data would suggest, as we use volatilities and correlations for de-smoothed returns of illiquid assets, for better comparability. The seven percent excess return may appear modest given the combination of equity premia, illiquidity premia and manager alpha. We note that our assumptions are for net-of-fees performance of all managers, not just the top-quartile. Moreover, EALTS contains subsets with higher expected returns than 7% (private equity), intermediate expected returns (hedge funds, private credit), and lower expected returns (real estate, infrastructure). For a discussion on historical returns (as well as biases in them), see Imlanen (2011) and Ang (2014, op.cit.).

The Sharpe ratio we assume for ARP is much lower than historically observed in, say, Asness et al. (2015) (op.cit.). Low correlations across premia is central. Even if single market-neutral premia strategies have low Sharpe ratios (say, 0.2), diversification across them can magnify the ARP portfolio Sharpe ratio.

⁵ Though with no leverage allowed and only two asset classes the MVO is only solving to match the desired volatility in this case.

⁶ Risk allocations measure each investment's contribution to the portfolio's variance. They account for both variances and correlations. (Variance is the square of volatility, or standard deviation, so variance contributions tend to be more heavily weighted towards riskier assets than volatility-adjusted weights.)

over cash). However, if we allow leverage (third column), the optimal portfolio is the tangency portfolio (see Box) levered up to the required volatility level — this offers somewhat higher expected returns than the leverage-constrained version at the higher volatility target.

Choice 2: Leverage vs. Concentration

Exhibits 2 and 3 are also helpful for discussing the second major top-down choice — between leverage and concentration. Many investors are leverage averse or constrained. To achieve higher risk and return targets they do not lever up the tangency portfolio but, instead, climb up the efficient frontier toward more-concentrated positions in the riskier asset class.⁷ That is, in our figure they travel up the purple not the green line (see Exhibit 3 in the Box). These investors forfeit the higher risk-adjusted returns available on the capital market line in favor of raw returns. Investors with lower risk and return targets may be able to benefit from optimal diversification without needing to face this tradeoff between leverage and concentration. Since investors are presumably willing to own cash (with leverage being the willingness to “short” cash) they can move along the green not purple lines as long as they are staying to the left of the tangency point.

For most investors, leverage constraints lead to portfolios that are not well diversified by risk. This has a negative impact on expected returns (in the case of Exhibit 2, the difference between the leverage-constrained and -unconstrained portfolios is 0.3%), and this gap increases at higher levels of volatility. Assuming cost-effective access to leverage, a portfolio on the capital market line will be expected to outperform a portfolio on the efficient frontier. Empirical evidence over long periods has tended to support this theory.⁸

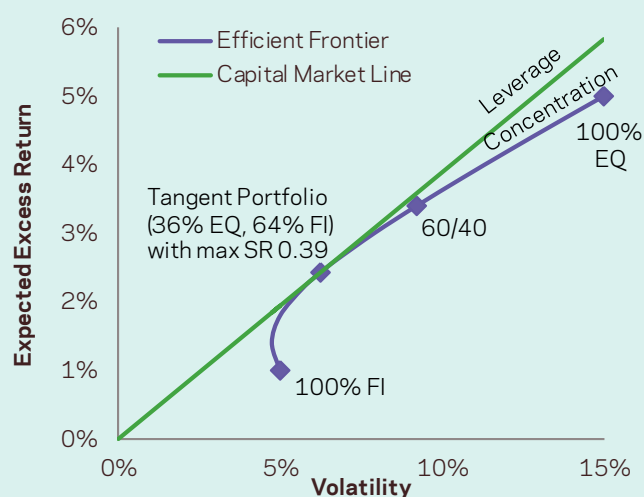
⁷ In a two-asset case, the efficient frontier contains just varying mixes of EQ and FI. In multi-asset cases, this frontier contains the highest-return portfolios at each vol level. Adding more risky assets to the investment universe tends to move the frontier northwest (higher Sharpe ratios).

⁸ See, for example, Asness, Frazzini and Pedersen (2012): “Leverage Aversion and Risk Parity,” *Financial Analysts Journal* 68(1).

On Leverage and Modern Portfolio Theory

If cash is not available for borrowing and investing, all that investors can do is choose from the efficient frontier of risky assets a portfolio that best suits their risk tolerance. The availability of cash allows investors to do better (see Exhibit 3). The pioneers of modern portfolio theory showed more than 50 years ago that the selection of an optimal portfolio involves two separate decisions: first, what is the most efficient or maximum Sharpe ratio portfolio among risky assets, and second, how much risk to take? Investors who agree on the opportunity set should all agree on the first portfolio (in theory; of course that’s an unrealistic assumption). It is called the tangency portfolio, as it is at the tangent of the line between cash and the efficient frontier (called the capital market line). Thus it has the highest Sharpe ratio among all portfolios on the efficient frontier. With cash available for investing or borrowing, the second decision is a matter of mixing cash and the tangency portfolio: deleveraging or leveraging the optimal portfolio along the capital market line to reach a volatility level that reflects investor risk tolerance. Given the inputs above, the tangency portfolio has 36/64% EQ/FI weights (cf. last column in Exhibit 2 where this portfolio is levered with 60% of cash).

Exhibit 3 | Classic portfolio choice with two risky assets



Source: AQR. For illustrative purposes only.

It may seem puzzling that so many institutions choose risk concentration but there are clear reasons for it. In fact, the average investor must do it because a global wealth portfolio of all assets displays concentrated equity risk. Despite this, financial markets have not rewarded equities with a uniquely high Sharpe ratio, likely due to prevalent leverage aversion.⁹ Less-constrained investors may be able to diversify better and enhance returns at the expense of the constrained majority.

Choice 3: Liquid vs. Illiquid

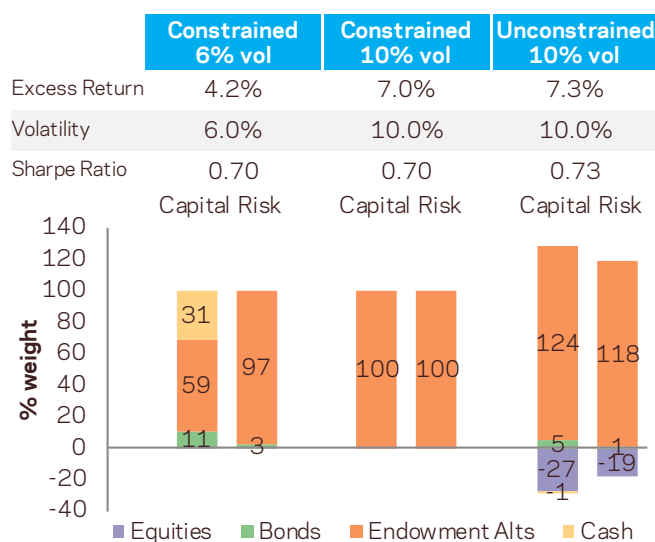
Another important top-down choice relates to illiquid investments. It would be possible to harvest illiquidity premia within equity and bond markets by overweighting less-liquid securities, but here we represent illiquidity with “endowment alternatives” (EALTS) due to their popularity among endowments. The best-known proponent, the Yale Endowment, allocates more than 75% of its portfolio to alternative assets such as private equity, hedge funds and real estate. Harvesting illiquidity premia is only a part of the motivation; another is that well-selected (“first-quartile”) managers can better add alpha in less-competitive private markets.¹⁰

We assume that a well-diversified combination of EALTS can offer 7% excess returns (even after fees) at 10% volatility, for an admirable Sharpe ratio of 0.7. However, in line with historical experience of less-impressive diversification benefits, we assume an equity market correlation of 0.7 (see Exhibit 1).

If one simply looks at historical data on EALTS they would show too low estimate of volatility and correlations. This is because these returns are

artificially smoothed as the assets are not regularly (by choice or impossibility) marked-to-market. Our volatility and correlation assumptions are meant to be our guess of the real values if they were not artificially smoothed.¹¹ Optimizers would be even more impressed with the smooth raw series and would make even larger allocations in EALTS than below if we used such inputs.

Exhibit 4 | *Optimizing the allocation between equities, bonds and “endowment alternatives”*



We think our Sharpe ratio assumption for EALTS is generous, although some readers may be even more optimistic. Let’s see how the optimizer feels adding only EALTS as an option besides EQ and FI. **Exhibit 4** shows that it really likes EALTS. For the 6% volatility target, the optimizer mixes 59% of EALTS with cash and bonds, with no room for public equities. For the 10% volatility target, the optimizer selects only EALTS. These cases restrict shorting or direct leverage (though embedded leverage typical of these assets is allowed). For an

⁹ Leverage aversion is so common because many investors recall infamous episodes caused by excessive leverage. While leverage can help investors diversify better, it undoubtedly carries a risk that must be carefully managed. One key precept is to avoid mixing leverage and illiquidity, a combination responsible for several past portfolio blowups.

¹⁰ Not all investors can select those first-quartile managers, and illiquidity premia may not be as high as often expected — if investors overpay for the artificial smoothness in illiquid-asset returns. We expect to tackle some of these topics in future research.

We do think that the illiquidity premium is one important source of long-run returns, among many. It especially suits investors with a long horizon and limited liquidity needs, and who use little leverage, shorting, dynamic rebalancing or risk targeting (these do not mesh well with illiquids).

¹¹ See Meng and Zhang (2015) “Illiquidity Premium, Transaction Costs and Risks of Illiquid Assets”, Ilmanen (2011) Expected Returns chapters 11 and 18, and Ang (2014, op.cit.) on illiquidity premia and de-smoothing of returns. Meng and Zhang document long-run equity market correlations of 0.74 for de-smoothed private equity and 0.35 for de-smoothed real estate. The correlation between the returns of the HFRI hedge fund index and the S&P500 index over the past 20 years is 0.76.

unconstrained 10% volatility target, the optimizer does not demand much leverage but does increase the EALTS position to 124% by shorting public equities. While this is not realistic for most, or recommended (shorting equities to lever up illiquid assets is about as scary as it sounds), it is also not surprising; this is indeed typical of MVO. When an optimizer sees two assets with dissimilar Sharpe ratios and a high correlation, it wants to create a long/short position to exploit this opportunity — investing more in EALTS but hedging part of equity-directional risk by shorting the inferior asset.

Choice 4: Long-Only vs. Long/Short

If MVO prefers an asset with a higher Sharpe ratio than traditional assets despite a high correlation with them, how would it treat an investment which combines a high Sharpe ratio with low correlations? Yes, we'd expect love at first sight.

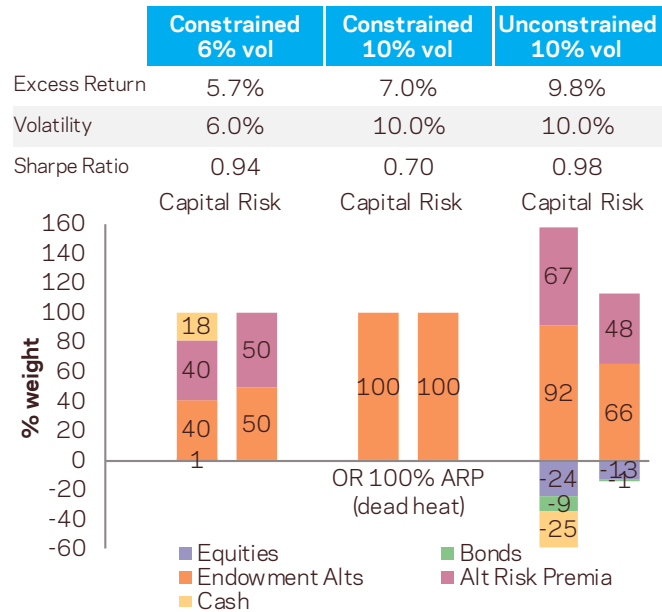
Here we add alternative risk premia (ARP) to the investment universe alongside the existing triplet. We assume similar expected return and volatility and the same 0.7 Sharpe ratio as for EALTS, but thanks to shorting it is possible to create market-neutral positions that have only 0.1 correlation with major asset class returns.¹² (ARP can actually target zero correlation but we assume a mild positive number here.) To us, a highly diversified composite of long/short premia has the most credible shot at achieving a sustainably high Sharpe ratio. One challenge is that when ARP's high Sharpe ratio reflects diversification and thus volatility reduction, it will require meaningful leverage to achieve the 10% volatility target (we assume 8x: 4x longs and 4x shorts — though we assume some of these strategies involve bonds which explains much of the required leverage; without bonds assumed Sharpes would be somewhat lower and leverage much lower).

The main message from **Exhibit 5** is that in reasonable situations, MVO prefers the better-diversifier ARP over EALTS, though both have a

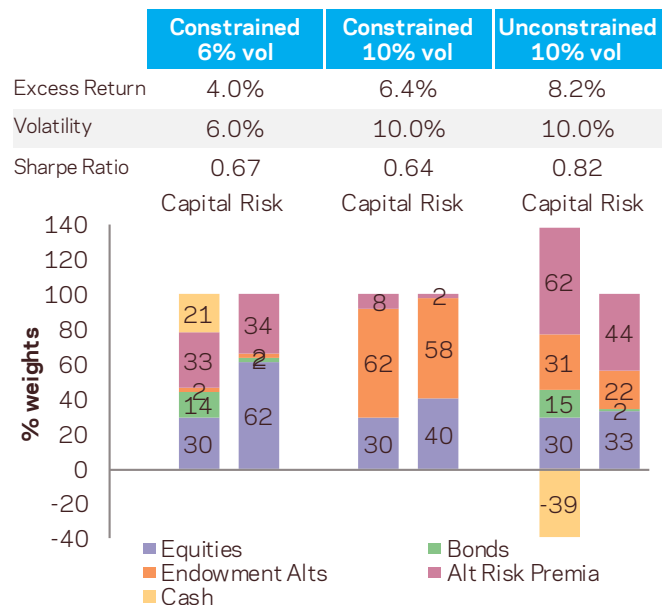
useful role. But it is worth going through some counterintuitive results where ARP are not favored, as these illustrate complications with MVO.

Exhibit 5 | Optimizing the allocation between equities, bonds, endowment alts and alt risk premia

Panel A: Unconstrained equity allocation



Panel B: Minimum 30% equity allocation



Source: AQR. There is no guarantee, express or implied, that long-term return and/or volatility targets will be achieved. Realized returns and/or volatility may come in higher or lower than expected.

¹² Here we refer to shorting within ARP. The optimizations in Exhibits 2-5 will allow or not allow shorting of the asset classes as described.

Again, equities are trumped and get assigned zero or negative weights, and bonds do not fare much



better. For the 6% volatility target, the optimal portfolio takes roughly equal risk in EALTS and ARP. For the constrained (no leverage or shorting) 10% volatility target, the optimizer chooses either full investing in EALTS or in ARP. Any diversification between them would preclude reaching the risk target although it would significantly boost the portfolio Sharpe ratio.

For an *unconstrained* 10% volatility target, the optimizer takes large partly levered positions in both EALTS (92%) and ARP (67%), funded by shorting EQ, FI and cash, and the portfolio Sharpe ratio rises from 0.7 to 0.98. (This involves levering up the tangency portfolio, which in this case has 8% volatility.) The optimizer takes a bigger position in EALTS than in ARP because much of the former's risk can be hedged by taking a short position in equities. Being a good diversifier becomes a disadvantage for ARP in this quirk of MVO.

For a more-realistic example (Panel B), we impose a constraint that public equities are at least 30% of the portfolio (see, we're doing exactly what most other investors do, starting with assumptions and MVO and then saying "well we can't do that!" and imposing constraints). The constrained low-risk portfolio now ignores EALTS in favor of the better-diversifying ARP, but the constrained higher-risk portfolio (second column) must allocate mostly to EALTS in order to reach its 10% volatility target. If the leverage constraint is removed (but the 30% floor for equities retained, third column), the optimizer makes its largest allocation to the diversifying ARP (62%) and uses leverage to reach the risk target.

In practice, the choice between EALTS and ARP is largely beliefs-based. One could argue for either of them having the higher ex-ante Sharpe ratio. As noted, we believe ARP have the edge, but many investors' — especially endowments' — portfolio choices suggest an opposite belief. Here we assume equal Sharpe ratios and focus on the impact of different equity market correlations.¹³ Using these

admittedly coarse assumptions ARP has the edge except when shorting public equities is allowed and EALTS can then deliver its "alpha" without market exposure. Given the same Sharpe ratio, we might expect that better diversifiers to traditional portfolios should be favored, but leverage-constrained MVO may not always do this (if the worse diversifiers have higher unlevered volatility).¹⁴

Choice 5: Systematic vs. Idiosyncratic Returns

There are other potential top-down decisions besides tolerance to volatility, illiquidity, leverage and shorting. We finish this section with a discussion on what used to be called the "active versus passive"¹⁵ or "alpha versus beta"¹⁶ allocation debate. A more modern debate is that on allocating between systematic and idiosyncratic return sources when seeking uncorrelated value-added to traditional asset classes. For example, an institution launching an absolute return program in liquid assets must decide how much to allocate to systematic ARP where the return sources are known and studied versus manager-specific "alpha."

The allocation decision between ARP and discretionary hedge funds is largely *beliefs-based*. Investors who believe that markets are inefficient

achieve it through superior internal diversification. ARP portfolios harvest well-rewarded style premia with modest Sharpe ratios (say, 0.2 each) and magnify the portfolio Sharpe ratio to an extent possible only when diversifying across market-neutral strategies (and, finally, they use leverage to reach higher portfolio volatility targets).

¹⁴ Here we focus on EALTS vs ARP when assessing the "long-only vs. long/short choice". Another relevant choice is between smart beta and ARP. Investors can harvest style premia through both long-only smart beta portfolios and long/short ARP portfolios. The latter have higher expected Sharpe ratios and are better diversifiers, but they may also involve too much leverage and shorting for large portfolio allocations. Which approach is better? The answer varies across investors and for some it may be 'both.'

¹⁵ We would argue that any deviations from the market cap portfolio (the only portfolio that all investors can hold simultaneously) are active positions. Thus it is wrong to equate "systematic" with "passive." Systematic investors can achieve "implementation alpha" by better portfolio designs and execution.

¹⁶ Investors increasingly appreciate that the alpha-beta dichotomy is too simple and stale for two reasons. First, ARP are a third broad return source between market risk premia ("beta") and alpha, so they do not fit well within the old alpha-beta boundaries. ARP are systematic and may be widely known, thus resemble market-risk premia, but they are uncorrelated with static market risk premia, thus resemble alpha. Second, hedge funds and other active managers offer a *blend* of several return sources: market risk premia, ARP, manager-specific alpha, and even some illiquidity premia.

¹³ As a crude recap, we argue that EALTS earn most of their above-market Sharpe ratio through illiquidity premia, whereas ARP portfolios

and that they can identify in advance superior managers who can exploit these inefficiencies are more likely to make large hedge fund allocations. Investors who believe more in cost-effective harvesting of diverse systematic return sources tend to favor ARP.¹⁷ These two avenues need not be mutually exclusive, and the optimal allocation may well be a mixture. While we can debate the relative Sharpe ratios, empirically it does seem clear that hedge fund portfolios have much higher equity market correlations than ARP portfolios do (suggesting the allocation decision is similar to that between EALTS and ARP above).

Problems With MVO

We have already seen hints of problems with MVO. MVO is most useful when its underlying assumptions are broadly satisfied (e.g., if investors care only about portfolio means and variances) and when we have reasonable inputs for the optimizer. The Appendix discusses the two related problems for the MVO: model errors and estimation errors.

When it comes to top-down portfolio allocation decisions, constraints are particularly important. We saw above that even with reasonable inputs, unconstrained MVO can give results that are unacceptable to most real-world investors: zero or negative allocations to traditional asset classes and heavy exposure to illiquidity, leverage and shorting. No wonder then that constraints against these features dominate actual portfolios.

We made a closely related observation in *Alternative Thinking* 2013Q1 when we showed that an unconstrained MVO, with reasonable inputs, would allocate 23% of its risk to long-only market risk premia and 77% to long/short ARP. In reality, most

investors allocate most risk to the former. We argued that four “Cs” — conviction (beliefs), constraints (against leverage and shorting), conventionality and capacity — drive real-world portfolio choices.

How can investors then select reasonable constraints? Sometimes laws, regulations or liabilities force the constraints on them. More often, constraints are self-imposed, perhaps guided by peer behavior, which creates a self-perpetuating convention or gradual herding among institutions.¹⁸

We think a better approach is for investors to consider carefully their key beliefs and preferences, and then make judgmental choices that combine the broad spirit of optimization — diversify widely but tilt toward higher risk-adjusted returns and diversifiers — with investor-specific priors and constraints. Optimization results above give helpful direction to the allocations, but rarely the final result.

Putting It Together — Practical Examples

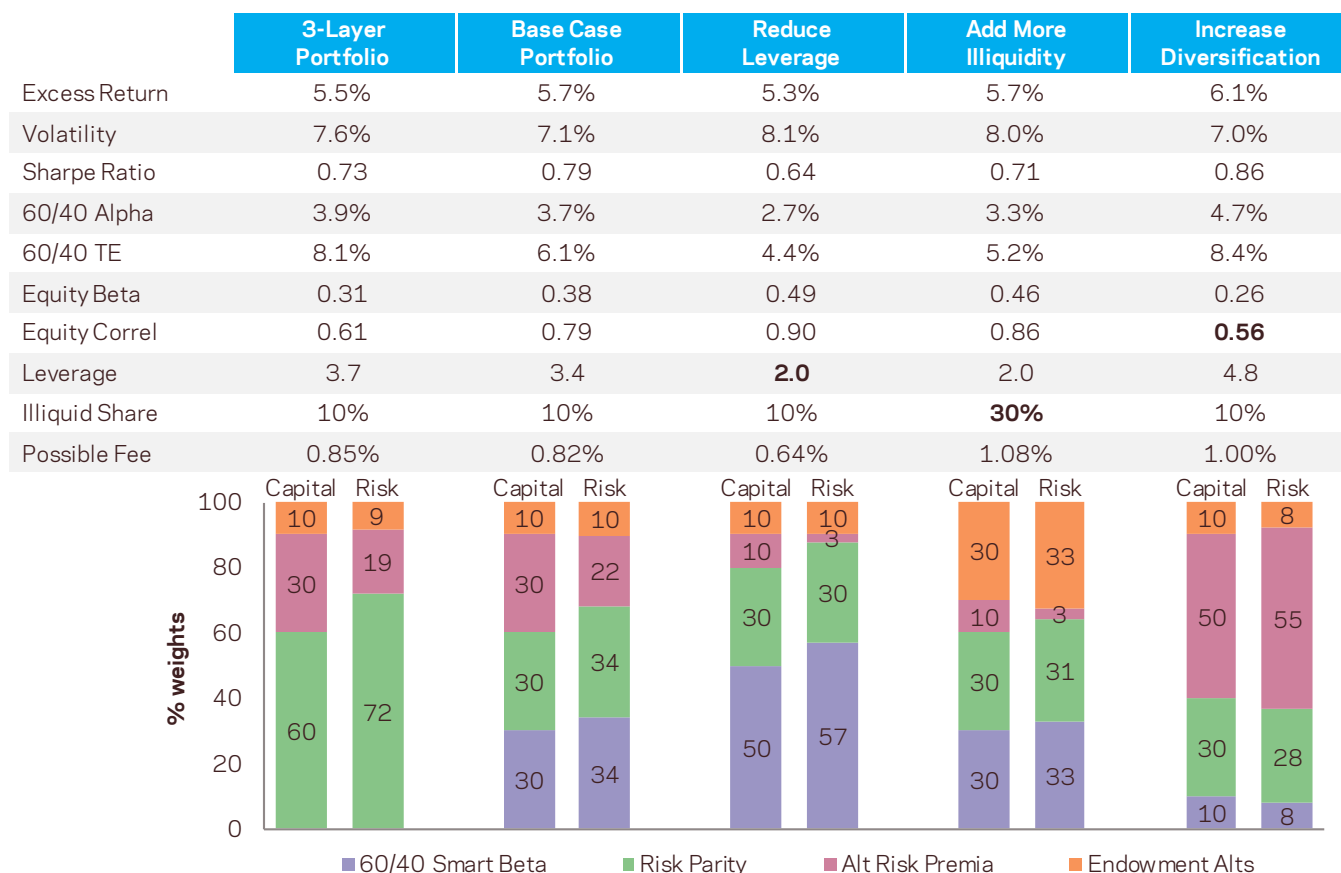
In this last section, we show some concrete examples of how we let our beliefs — and preferences and constraints — guide our top-down decisions on portfolio allocations, and how other institutions might make different choices. The portfolios below are not optimized but heuristic, due to the subjective nature of beliefs and constraints.

Exhibit 6 shows several portfolio allocations and the portfolio statistics based on our inputs listed below. (As before, these are forward-looking assumptions, guided by historical experience.) All these portfolios are consistent with our core beliefs in that they involve more aggressive diversification than traditional portfolios (and use leverage and shorting to get there). But they all differ in some respect from one another; investors can judge from portfolio characteristics as well as from performance and risk statistics which one fits them best.

¹⁷ Belief in market efficiency is not a key distinction: both hedge funds and ARP harvest various risk-based return premia and market inefficiencies. It is also ironic that the putatively idiosyncratic hedge fund “alphas” are in practice more correlated with market risk premia than are systematic ARP (since the latter are designed to be market-neutral). For single hedge funds, idiosyncratic alpha may be as large as beta, but it diversifies away in a broad multi-manager portfolio, leaving a much bigger role for systematic exposures. This feature explains how single hedge funds may have equity correlation of, say, 0.3-0.4, on average, while a broad hedge fund portfolio may have 0.7 equity correlation.

¹⁸ What is deemed conventional among institutional investors does evolve over time, albeit slowly. Examples include rising equity allocations since the 1960s and the growing role of alternatives in 2000s.



Exhibit 6 | Five candidate portfolio allocations, with performance and risk statistics**Inputs**

	Excess Return	Volatility	Net SR	Leverage*	Possible Fee
60/40 Smart Beta	5.0%	10%	0.50	1	0.3%
Risk Parity	4.5%	10%	0.45	2	0.4%
ARP	7.0%	10%	0.70	8	1.2%
EALTS	7.0%	10%	0.70	1	2.5%

Correlations

	60/40 SB	Risk Parity	ARP	EALTS
60/40 Smart Beta	1.0			
Risk Parity	0.6	1.0		
ARP	0.1	0.1	1.0	
EALTS	0.7	0.6	0.1	1.0

Source: AQR. Simulated portfolio data is based on correlation, risk, return, leverage, and fee assumptions as detailed above. For illustrative purposes only. 60/40 Smart Beta is a style-tilted long-only portfolio of equities (60%) and bonds (40%). Risk Parity is a portfolio of risk-balanced market risk premia. ARP is a portfolio of risk-balanced long/short style premia. EALTS is a portfolio of illiquid "endowment alternatives." * On leverage levels, see footnote 20. There is no guarantee, express or implied, that long-term return and/or volatility targets will be achieved. Realized returns and/or volatility may come in higher or lower than expected.

The first portfolio has three layers: 60% risk-balanced market risk premia (risk-balanced is often called risk parity and is itself a near-optimal portfolio of liquid market risk premia under some simplifying assumptions), 30% liquid ARP, and 10% illiquid EALTS. This could be our portfolio advice to institutions seeking a valuable addition to their more traditional portfolio, or for a small minority of

less constrained investors, it could be a recommendation for the total portfolio.¹⁹

¹⁹ The first portfolio is already a step removed from ideals and toward reality. A fully unconstrained portfolio recommendation might include equal risk allocations to 8 to 10 lowly correlated return sources (for example, three market risk premia, five style premia, plus allocations to illiquidity premia and idiosyncratic manager alpha). Such a portfolio offers even more extreme diversification but would be too unconventional and involve even more leverage and shorting than the portfolios shown here.

The first portfolio has attractive expected return characteristics (5.5% over cash, Sharpe ratio 0.73), but is still too unconventional for most institutions' total portfolio (under our assumptions its tracking error vs. 60/40 is 8% and employs look-through, i.e., counting the leverage in the risk parity and ARP strategies themselves, leverage of 3.7)²⁰.

The second portfolio is our base case. We split the market risk premia component, investing half of the 60% allocation in a long-only 60/40 stock/bond portfolio with style tilts (denoted by "smart beta," even if we are not fans of this term)²¹ and retaining half in risk parity. This change makes the portfolio more realistic for a total portfolio solution, as the tracking error to 60/40 is now 6% and leverage somewhat reduced. Expected return is actually mildly higher (5.7%) and volatility lower, resulting in a higher Sharpe ratio (0.79). However, this portfolio is a worse diversifier to traditional 60/40 portfolios (offering slightly lower alpha and a much higher equity correlation).²²

Note that this base-case portfolio harvests style premia through both the long-only 60/40 portfolio and the long/short ARP portfolio. Investors who want large style exposures may find that their

constraints against leverage and shorting limit their allocations to ARP portfolios, and they may want to add style exposures further through smart beta.

We now analyze three tweaks to the base case portfolio and assess the impact of each on expected performance and risk characteristics.²³

Reduce leverage: Compared to the base case, we shift 20% from levered long/short style premia (ARP) to unlevered long-only style-tilted portfolios (60/40 Smart Beta). This switch actually gives us the most conventional portfolio (lowest tracking error vs. 60/40 of 4.4%, highest equity correlation of 0.90), and the look-through leverage falls to 2. This option likely has the biggest capacity as well. Of course, there is a cost: the Sharpe ratio drops from 0.79 to 0.64, while expected return drops from 5.7% to 5.3%.

Add more illiquidity: Compared to the base case, we again shift 20% from ARP, this time to illiquid EALTS. Because we do not penalize EALTS for embedded leverage, this variant too shows portfolio leverage at 2. Compared to the previous switch, this one shows a smaller decline in the Sharpe ratio (to 0.71) and a smaller increase in equity correlation (to 0.86). Fees are meaningfully higher.

Increase diversification: Compared to the base case, we do the opposite shift to that in "reduce leverage," now *increasing* ARP by 20% at the expense of the long-only style-tilted allocation. Not surprisingly, better diversification results in the highest Sharpe ratio (0.86), lowest volatility and lowest equity correlation (0.56) but also the highest leverage and tracking error versus 60/40. This portfolio would be the most consistent with our core beliefs, but also the least conventional. The previous three portfolio options may be more palatable for most investors.

²⁰ Leverage is a topic that would require a longer treatment. Many investments ranging from public and private equities to hedge funds contain embedded leverage, but we follow here the common practice of quoting their leverage as 1. In contrast, we quote the leverage of risk parity and ARP (2 and 8, respectively) at higher standards, given their better transparency. The look-through leverage is the gross sum of all the long and short positions divided by assets under management (or the fund NAV). Using the same treatment as we use for hedge funds, we could instead quote these as investments with leverage 1 since for end-investors these are unlevered investments into levered vehicles (which, moreover, contain plentiful free cash).

²¹ Smart beta portfolios are long-only portfolios tilted toward one well-rewarded style such as value (and less often toward multiple styles). We believe that a multi-style approach is superior and that an efficient multi-style-tilted 60/40 stock/bond portfolio can achieve about 5% excess return over cash and 0.5 net Sharpe ratio (if we assume 3.5% and 0.35 for the "passive" cap-weighted 60/40 portfolio). These assumptions are conservative compared to the historical evidence on the excess returns of style-tilted equity and bond portfolios; see, for example, Frazzini, Israel, Moskowitz and Novy-Marx (2013) "A New Core Equity Paradigm" and Israel and Richardson (2015) "Investing with Style in Corporate Bonds."

²² Being more conventional (having a lower tracking error versus traditional portfolios) and being a worse diversifier (having a higher equity market correlation) are really two sides of the same coin. There is an inherent tradeoff between these two goals. Whether an investor cares more about conventionality or about absolute diversification benefits tells whether it is more peer oriented or total return oriented. Over the long run, we believe the latter approach will give better portfolio performance.

²³ We actually tried one more tweak, lowering costs, but will just summarize the key results here. Compared to the base case, using cap-weighted 60/40 instead of smart beta 60/40 would lower typical portfolio fees by 0.08% but would also cut net expected return by 0.5% and Sharpe ratio by 0.06. Given our input assumptions, style tilts are worth doing.



Concluding Thoughts

Many investors ask for our thoughts on “putting it all together.” This article does not give definitive answers, or even recommend a particular framework for making such decisions. Instead, we start from our quantitative home territory (inputs such as expected returns, volatilities and correlations) and then explore how investor-specific beliefs and constraints can inform and interact with formal optimization methods. We will continue to study this topic as part of a two-way discussion with investors, in an effort to help them build portfolios that best meet their own particular investment objectives, constraints and beliefs.

Appendix: MVO — Its Uses, Pitfalls and Remedies

MVO works well with many portfolio problems

Although MVO is an effective tool for portfolio construction, many investors eschew using it, partly because of problems described below. Others would stick with MVO and try to deal with these problems by some modifications. We take a middle path in the main text when describing MVO results but then arguing that certain top-down decisions are so complex that they may be better done heuristically, only guided by the insights from optimizations.

Yet, to be clear, we find optimizers very useful in many portfolio problems, and they are central to both our research and portfolio construction process. We cannot go deep into these topics here, but we offer one illustration of a portfolio problem where MVO gives helpful insights and reasonable portfolio weights. Here we allocate risk across four style premia: value, momentum, carry, defensive. The optimizer maximizes the portfolio Sharpe ratio while requiring that weights amount to 100%.

- We assume that each of the four strategies targets 15% annual volatility. In the baseline case, we assume all the strategies have equal long-term expected Sharpe ratios of 0.4 (thus 6% expected return over cash), and that all the strategy pairs are uncorrelated. Not surprisingly, an optimizer would give equal risk allocations to all the strategies. With four uncorrelated return sources, expected portfolio volatility would be 7.5% and the portfolio Sharpe ratio would double to 0.8.
- If we change the inputs so that one style pair, let's say value and momentum, have an appealing negative correlation of -0.5, the optimizer assigns twice as large risk weight (33% vs. 17%) to these better complements than to the other two style premia. The improved diversification reduces portfolio volatility further and boosts the Sharpe ratio to 0.98.
- We can also ask what Sharpe ratio we'd have to assume for value and momentum if all styles were uncorrelated, to give the same risk weights as the negative correlation did (33% vs. 17%). The answer is 0.57, so the -0.5 value-momentum correlation is as valuable as a 0.17 rise in the

Sharpe ratio of these two premia (both assumptions give an equal boost to the optimal portfolio's Sharpe ratio).

This is just one example where an optimizer provides insights about the worth of good diversifiers, apart from its core task: mapping inputs into actionable outputs (portfolio weights). The optimizer gave reasonable solutions partly because the inputs were reasonable and the portfolio constituents were comparable in the MVO framework, so we didn't need to impose constraints.

Problems with MVO ... and some remedies

MVO uses information on means, volatilities and correlations. The general optimality condition for the unconstrained case is that in the maximum Sharpe ratio portfolio the ratio of marginal contribution of return to marginal contribution of risk is the same for all assets. This condition ensures that we cannot improve the portfolio by marginal reallocations between constituent assets.

MVO is most useful when (i) its underlying assumptions are broadly satisfied (investors care only about portfolio means and variances or these two moments capture well the investment opportunity set — this requires near-normal return distributions and liquid investments) and (ii) we have reasonable inputs for the optimizer. We address next the two corresponding classes of problems for the MVO: (i) model errors and (ii) estimation errors.

Model errors: Clearly, investors care about portfolio characteristics beyond mean and variance. We have highlighted above liquidity, leverage and shorting aversions; other preferences may include higher moments (e.g., skewness) or ethical considerations.

The biggest error may be that we are dealing with the wrong question. Perhaps a broader question is needed than the typical optimization across financial asset holdings. A corporate pension plan may broaden its perspective from asset optimization, include pension liabilities and optimize the asset-liability surplus, or the corporate



sponsor may go further and try to optimize even broader enterprise risk. Likewise, the optimization problem of a retirement saver may be broadened to include human capital and the housing wealth.²⁴

Estimation errors: Even for the simple MVO question we need to use reasonable inputs for expected returns, volatilities and correlations. Estimation error is the difference between the estimated value and the true value of a parameter. Expected returns are especially susceptible to estimation errors (returns are harder to predict than volatilities and correlations) *and* the optimizer output (portfolio weights) is especially sensitive to expected return inputs. Recall in Exhibit 5 the optimizer's reaction to seeing two strongly positively correlated assets with different Sharpe ratios: it sought to create a large long/short position between EALTS and EQ to exploit this apparent opportunity. Unconstrained optimizers (not just MVO) can often create extreme positions, concentrated risk and high turnover due to estimation errors.

Thus, care is needed when choosing the inputs.²⁵ Using historical volatilities and correlations may be reasonable in many cases, but plain use of historical average returns without any adjustment is a recipe for trouble. Shrinking estimates toward some reasonable priors (such as zero or group average) often helps. It may make sense to give bigger weight to the inputs you trust than those you don't.

In practice, estimates of means are the least reliable, or the most susceptible to estimation errors. Thus, several popular portfolio construction approaches avoid using the expected return information. Implicitly, these often assume that risk-adjusted

returns are similar across assets (or that the estimates are highly uncertain).²⁶ Full MVO would be superior if expected return or risk-adjusted return differences between assets were so large that we could reliably estimate them.

Instead of giving up on expected return information, one could try to make the optimization better behaved. As noted, shrinking inputs may help. Another possibility is to use robust optimization, a formal technique that directly considers the uncertainty in data (estimation errors).

In many practical portfolio construction problems, AQR uses a modified optimization approach to make the outputs reasonable, while anchoring them to our unconstrained views and risk models. We first generate a theoretical portfolio based on our predictive models that we would hold in the absence of constraints or transaction costs; we use this portfolio's weights and an input covariance matrix to reverse-engineer our unconstrained implied expected returns. In the second step we put these implied expected returns together with our covariance matrix as well as constraints and transaction cost estimates into a robust optimizer that comes up with the final optimal weights.

Even if we can tame estimation errors, model errors remain a problem for top-down portfolio decisions which are not neatly captured by just mean and variance. Extending MVO to capture some missing element (say, illiquidity, leverage aversion or higher moments) results in a more complex model where more parameters need to be estimated, increasing the potential for estimation errors. And in practice, we cannot deal with all these dimensions in one model, so constraints must be imposed.

²⁴ Potential model errors don't stop there. Investment returns are hardly normally distributed (though downside risk measures may be nearly proportional to volatility). Multiple risk sources, time-varying expected returns, multiperiod hedging needs, capacity concerns and market frictions are other challenges hard to fit to the MVO framework.

²⁵ The choice of investable assets also matters. Investments with problematic characteristics (illiquid assets; options and other assets with highly asymmetric distributions; investments with limited histories or prone to structural changes) should be avoided in typical MVOs. Ideally, there are a reasonable number of distinct investable blocks with high intra-group correlations and low inter-group correlations. A large number of assets creates problems unless factor structure is used to drastically reduce the number of estimated parameters in a covariance matrix.

²⁶ The simplest portfolio construction approach is equal weighting (1/N). The next simplest, equal volatility weighting, uses only volatility as an input, while equal risk contribution (full risk parity) uses both volatility and correlation inputs. The latter two weighting schemes coincide with the maximum Sharpe ratio portfolio (full MVO result) if all assets have equal Sharpe ratios and zero or equal correlations. Two other weighting schemes, the minimum variance and maximum diversification portfolios, also use volatility and correlation inputs. Their optimality condition is different, however, resulting in larger weights for low-risk assets.

Disclosures

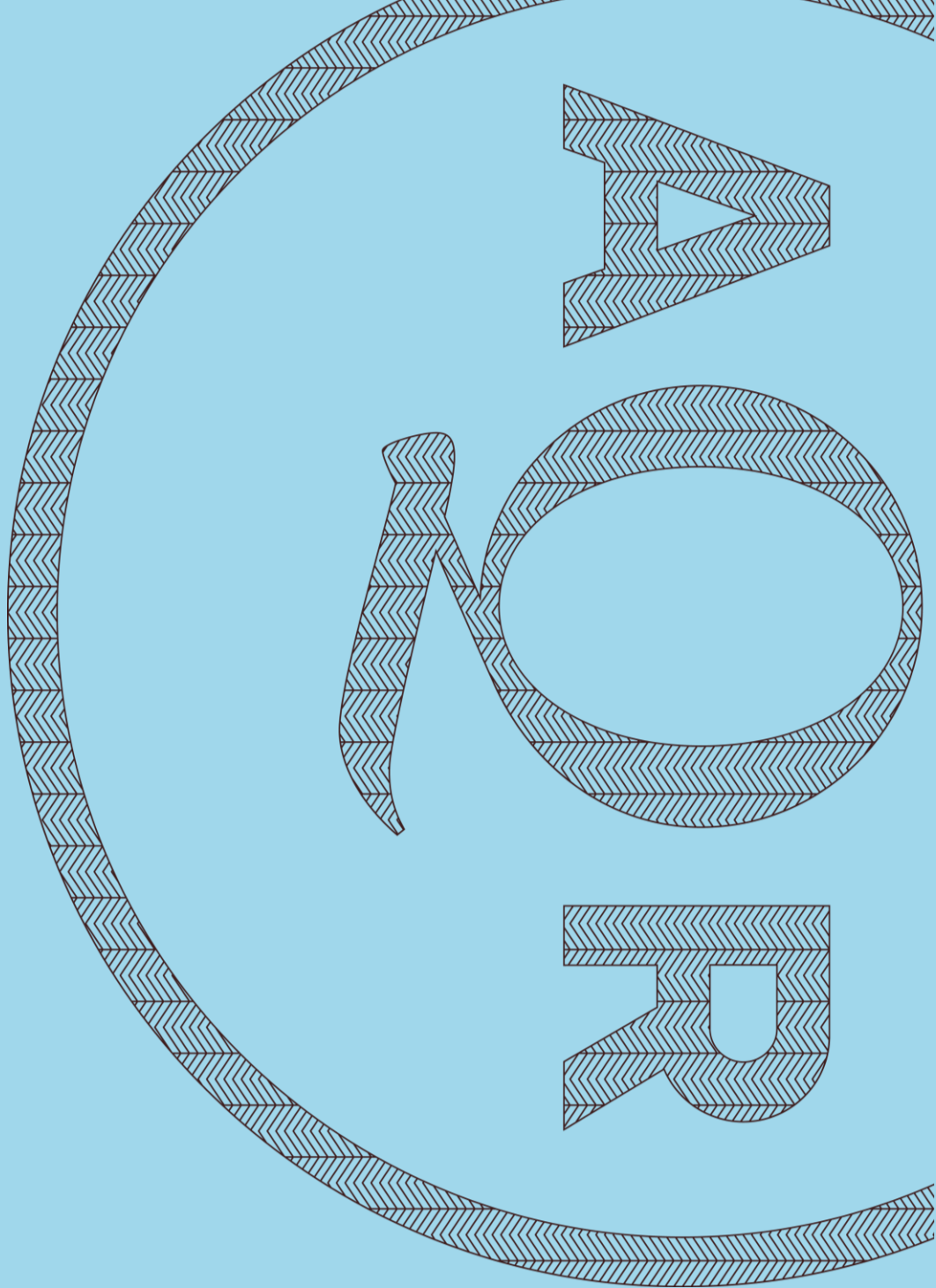
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