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There is some conventional wisdom about today's term structure and how it applies to the mortgage-backed securities market:

> The current term structure is, by historical standards, quite steep. Fixed-income options models assume that on average forward rates are realized. That is, the models currently imply a bearish term structure flattening. Furthermore, mortgage prepayments tend to decline as interest rates rise; the models therefore assume future prepayments will be declining. Thus the models are unfairly prejudiced against bullish securities such as principal-only strips and inverse floaters. Conversely, the models are biased in favor of bearish securities such as interest-only strips and super-floaters.

This article demonstrates that this description, while partly true, is based on a misunderstanding of fixed-income option pricing models.[1]

Do fixed-income options models (FIOMs) actually assume that real-world forward rates are on average realized? With the yield curve so steeply sloped, this assumption is tantamount to assuming interest rates on average will rise.

In fact, FIOMs make few assumptions about what happens to real-world interest rates. They do assert that *in a risk-neutral world* forward rates on average would have to be realized.[2] The models then price

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options by pretending that investors are risk-neutral. Because the models price options by the absence of arbitrage, the "risk-neutral world" option price corresponds to the real-world option price.

There cannot be arbitrage opportunities in either world; thus the models convey real-world option prices. The models do not assume that real-world investors are risk-neutral, nor do they assume that real-world forward rates are expected to be realized.

An important insight is that although FIOMs are calibrated so that forward rates are on average realized, they also discount cash flows at the riskless rate. These actions are appropriate in a risk-neutral world, but not necessarily in the real world.[3]

For example, if expected returns are higher on bullish securities (e.g., long zeros have higher expected returns than short zeros), then FIOMs can discount the cash flows of bullish securities at a rate that is lower than their expected returns. This raises the prices of the securities. Yet the models also penalize these securities because they generate future discount rates that tend to rise with the forward curve, thereby lowering security value. If handled correctly, these effects are exactly offsetting, and the securities are correctly priced.

Option pricing models work on the principle of "no arbitrage." Risk-neutral pricing is used not because we believe investors are either risk-neutral or we expect forward rates to be realized. It is used because it conveniently replicates arbitrage-free pricing. The models make many assumptions, some of them questionable, but they do not assume that real-world forward rates are expected to be realized.

We explore these concerns by developing a basic fixed-income option model in a two-period world, demonstrating the equivalence of arbitrage-free and risk-neutral pricing for derivative securities.[4] We discuss relationships of real-world expected returns to the shape of the term structure and extend our arguments to more realistic FIOMs. Our description of option-adjusted spread (OAS) technology reveals some criticisms of the models that we do find valid.

THE BASIC MODEL

Our simple model presents ideas that relate directly to more realistic FIOMs. We assume that the current term structure (spot curve) is as shown in Exhibit 1.

Thus, the one-year forward rate from one to two years is:

EXHIBIT 1
The Assumed Spot Curve

Maturity	Treasury Zero Rate
One Year	7.5%
Two Year	10.0%

$$12.56\% = (1.1)^2/1.075 - 1 \tag{1}$$

A two-year zero purchased today will be a one-year zero in one year. If forward rates are exactly realized, then the one-year rate will be 12.56%, and the bond will be worth 100/1.1256 = 88.84. Today, this two-year zero is worth $100/1.1^2$ = 82.65. Thus, the return on the bond would be 88.84/82.65 – 1 = 7.5%. Note that if forward rates are exactly realized the one-year return on a two-year zero is equal to the current one-year rate.

One version of the expectations hypothesis states that the short-term expected return of all Treasury zeros is the same. FIOMs are usually calibrated so that short rates on average fit the forward curve.[5] Therefore, it is easy to assume that FIOMs must also be incorporating the expectations hypothesis. A simple FIOM will demonstrate that the expectations hypothesis is not assumed for the real world.

We assume that the one-year Treasury rate can either move up to 15% or down to 3.75% at the end of the year (that is, short rates can either double or halve). Short rates are, therefore, governed by a binomial process (note that we have made no assumptions about the probabilities of an up or down move):

15.00%
7.5% <
3.75%

The current price of a two-year zero-coupon Treasury bond is 82.64. At the end of one year this bond will be a one-year zero. Thus its price follows a binomial process:

86.96
82.64 <
96.39

A one-year zero will have a sure payoff of 100.0 at year-end:

$$93.02 \quad < \quad \begin{matrix} 100.0 \\ \\ 100.0 \end{matrix}$$

Now consider a one-year European call option on the two-year Treasury zero with a strike price of 90.0. It will have payoffs of:

$$C \quad < \quad \begin{matrix} 0.00 \\ \\ 6.39 \end{matrix}$$

We wish to solve for C, the call's price at time zero. To do so, we can replicate the call's payoffs by a portfolio of one-year and two-year zeros. We must simultaneously solve the equations:

$$A \times 100.0 + B \times 86.96 = 0.00 \qquad (2)$$

$$A \times 100.0 + B \times 96.39 = 6.39 \qquad (3)$$

A is the amount of the one-year zero to buy, and B is the amount of the two-year zero. Solving the equations results in A = -0.5892 and B = 0.6776. Thus buying 67.76 face value of a two-year zero and shorting 58.92 face value of a one-year zero will exactly replicate the value of the call option in any possible future state of the world. Assuming that there is no arbitrage, the value of the call option must equal the value of this portfolio, or 1.19.

There is an alternative, yet equivalent, method of deriving the option price. Imagine we live in a world where investors are risk-neutral, yet rates still evolve according to our binomial process. In this world investors are concerned only with the expected return of a security. It is therefore an equilibrium condition that all securities have the same expected return.

In this world, we may infer the probability of an "up" move in rates from the condition that the expected return on the two-year zero must equal the known return on the one-year zero. That is, we must solve the equation:

$$p \times 86.96/82.65 + (1 - p) \times 96.39/82.65 - 1 = 7.5\% \qquad (4)$$

Solving this equation results in p = 0.80; in this hypothetical world, there is an 80% chance that rates will rise, and a 20% chance rates will fall. Note that 0.80 is not the true probability of an upward move in real-world rates. It is the probability that would have to exist *if the world were risk-neutral.*

Since all securities in a risk-neutral world have the same expected return (the riskless rate), they can all be priced by discounting their expected cash flows at this rate. We may derive the price of the call option, in the risk-neutral world, by taking this expectation:

$$C = [0.80 \times 0 + 0.2 \times 6.39]/1.075 = 1.19. \qquad (5)$$

This method produces the same value for the call as the "no-arbitrage" method. This must always be the case. No-arbitrage is an equilibrium condition of both the real world and the risk-neutral world. A security priced by arbitrage methods must have the same value in both worlds. Because of the equivalence of the methods, any security that can be priced by arbitrage can be priced by discounting its expected cash flows in the risk-neutral world at the riskless rate.

Consider now some other common securities. Suppose there is a 10% coupon two-year mortgage security that allows no prepayment, has no default risk, and has the cash flows:

$$M \quad < \quad \begin{matrix} 10 \quad < \begin{matrix} 110 \\ \\ 110 \end{matrix} \\ \\ 10 \quad < \begin{matrix} \\ 110 \end{matrix} \end{matrix}$$

At the end of one year, we know the new one-year zero rate, and therefore we know the value of the mortgage (including coupon). The value of the mortgage follows the one-period binomial process:

$$M \quad < \quad \begin{matrix} 105.65 \\ \\ 116.02 \end{matrix}$$

We have shown that the current value of the mortgage must be the expected next-period value (using risk-neutral expectations) discounted at the current riskless rate:

$$M = [0.8 \times 105.65 + 0.2 \times 116.02]/1.075$$
$$= 100.21 \quad (6)$$

If we value the mortgage by direct arbitrage methods we arrive at the same result.

Now consider a mortgage prepayable at par. Rational mortgagees will prepay in the "down" rate case when the mortgage is worth 106.02, and not in the "up" rate case when the mortgage is worth 95.65 (of course, mortgagees are not always rational in the real world). The value of the mortgage (including its coupon) follows a one-period binomial process:

$$M < \begin{array}{l} 105.65 \\ 110.00 \end{array}$$

The value of the mortgage is:

$$M = [0.8 \times 105.65 + 0.2 \times 110]/1.075$$
$$= 99.09 \quad (7)$$

Thus, the value of the prepayment option is 1.12 (the difference between the values of the two mortgages).

It is just as easy for us to price interest-only (IO) or principal-only (PO) strips derived from this mortgage. The cash flows from these securities can be described as follows:

$$PO < \begin{array}{l} 0 < \begin{array}{l} 100 \\ 100 \end{array} \\ 100 < \begin{array}{l} 0 \\ 0 \end{array} \end{array} \qquad IO < \begin{array}{l} 10 < \begin{array}{l} 10 \\ 10 \end{array} \\ 10 < \begin{array}{l} 0 \\ 0 \end{array} \end{array}$$

Since we know both possible future one-year zero rates, we also know the processes that each security's value follows:

$$PO < \begin{array}{l} 86.96 \\ 100.00 \end{array} \qquad IO < \begin{array}{l} 18.70 \\ 10.00 \end{array}$$

We can find the current value of each derivative security by using risk-neutral valuation:

$$PO = [0.8 \times 86.96 + 0.2 \times 100.0]/1.075$$
$$= 83.32 \quad (8)$$

$$IO = [0.8 \times 18.70 + 0.2 \times 10.0]/1.075$$
$$= 15.78 \quad (9)$$

Derivative securities also include floaters and inverse floaters. Suppose the mortgage is split into two-thirds floater and one-third inverse floater. If the floater always receives the one-year rate, the inverse floater must have a coupon of:

$$\text{Inverse Coupon} = 30 - 2 \times \text{One-Year Rate} \quad (10)$$

Thus, the inverse floater has an initial coupon of 15.00%, and possible future coupons of 0% and 22.5%. The cash flows to the floater (FLT) and the inverse floater (INV) are as follows (in terms of 100 par value):

$$FLT < \begin{array}{l} 7.50 < \begin{array}{l} 115.00 \\ 115.00 \end{array} \\ 107.50 < \begin{array}{l} 0.00 \\ 0.00 \end{array} \end{array} \qquad INV < \begin{array}{l} 15.00 < \begin{array}{l} 100.00 \\ 100.00 \end{array} \\ 115.00 < \begin{array}{l} 0.00 \\ 0.00 \end{array} \end{array}$$

Again, since we know both possible future one-year zero rates, we also know the process that each security's value follows:

$$FLT < \begin{array}{l} 107.50 \\ 107.50 \end{array} \qquad INV < \begin{array}{l} 101.96 \\ 115.00 \end{array}$$

Using the risk-neutral valuation technique, the floater and inverse floater have values of 100.00 and 97.27, respectively.

In today's market, it would not be surprising to hear that the model treats the PO and the inverse floater unfairly, and is too generous to the IO, by assuming there is an 80% chance that rates will rise. Equivalently, we might hear that it is unfair to assume that the expected one-year spot rate in one year is 0.80 × 15.00 + 0.20 × 3.75 = 12.75.[6]

EXPECTED RETURNS

In a risk-neutral world all securities have the same expected return. If the yield of a two-period zero is higher than that of a one-period zero, then to equate expected one-year returns we must assume that rates are expected to rise.

However, the real world is not necessarily risk-neutral. It is very possible that investors actually require different expected returns on different maturity Treasuries. The higher yield of the two-year zero versus the one-year zero can mean one of two things:

1. Investors are indeed risk-neutral and expect rates to rise by next year.
2. Investors are not risk-neutral and insist on higher expected returns on longer-maturity zeros.

In the model described earlier, at no time did we specify the true probabilities of an up or down movement in rates. We did state that in a risk-neutral world the probabilities would have to be 0.80 and 0.20, but this is not necessarily the case in the real world. For instance, investors might assign equal probabilities (fifty-fifty) to up and down moves. If this is the case, then the expected returns for the first year on one-year and two-year zeros are different. These are shown in Exhibit 2.

EXHIBIT 2
The Expected Return Curve Assuming Fifty-Fifty Probabilities

Maturity	Expected Return
One Year	7.50%
Two Year	10.92%

Thus, under the assumption of fifty-fifty probabilities, investors actually require a higher expected return on longer-maturity zeros. If this were true, economists would refer to interest rate risk as a "priced" variable. That is, securities whose returns covary with changes in interest rates must have higher or lower expected returns than otherwise similar securities whose returns are uncorrelated with interest rate changes. In our example, securities that do well when rates fall (bullish securities) are required to have a higher expected return than those that do well when rates rise (bearish securities).

If the true probabilities are fifty-fifty, then the expected returns of the prepayable mortgage, and each of the derivative securities, are as shown in Exhibit 3.

The two most bullish securities, the inverse and the PO, have the highest expected returns. This is consistent with the zero-coupon curve. There is only one source of risk in our model, changes in the one-year interest rate.

The Treasury curve and our assumption of fifty-fifty probabilities together imply that securities that do poorly when rates rise must have higher expected returns (the converse also holds). This must be consistent across all securities.

For example, consider the IO. Its expected return is -9.06%. That is, investors expect to lose money by investing in the IO. This can be quite rational. In our assumed real world, investors are averse to the risk of rising rates, and therefore require higher expected returns for holding bullish securities. A bearish security actually reduces portfolio risk by acting as a hedge for other bonds. The IO is so bearish, and thus such a good hedge, that investors are willing to expect a loss on their investment in return for the hedging benefits they receive.[7]

Since we are assuming a one-factor model, all changes in Treasury prices are driven by changes in the factor (the one-year spot rate). If the one factor is priced, then each security will have an expected return in proportion to its exposure to the factor. In our fifty-fifty example, securities that covary negatively with the factor (bullish securities) will require a higher expected return.

We can now see the interplay between assuming forward rates are realized and discounting at the riskless short rate. Both of these actions are valid in the risk-neutral world; yet consider their effects on the IO. The assumption that rates on average rise ($p = 0.80$, not $p = 0.50$) clearly benefits the IO. This is why many practitioners assume FIOMs favor bearish securities.

In the real world, however, the appropriate

EXHIBIT 3
Expected Returns on Derivatives Assuming Fifty-Fifty Probabilities

Security	Expected Return
Mortgage	8.82%
PO	12.19%
IO	-9.06%
Floater	7.50%
Inverse	11.52%

discount rate for the IO is -9.06%. Yet, cash flows in the risk-neutral world of the FIOM will be discounted at the riskless short rate (7.5% for the first year). This clearly hurts the IO's value. These effects exactly offset each other, and the correct real-world arbitrage-free IO price is derived.

When misconceptions occur, it is usually the discounting effect that is ignored. When the yield curve is steeply sloped, FIOMs price bearish securities assuming rates tend to rise with the forward curve. However, it is less often noted that these securities are priced by discounting their cash flows at each period's riskless rate.

If we assume that real-world rates are not expected to rise with the forward curve, then the only explanation for an upward-sloping yield curve is that there are higher expected returns on bullish securities. Securities such as POs and inverse floaters are very bullish, and thus have higher expected returns (and equally higher discount rates). Securities such as IOs have lower expected returns.

If the models deem IOs too cheap, it is not because they mistakenly assume rates will rise (and prepays will fall). It is because IOs are being priced at premiums not commensurate with their bearishness.[8]

The bottom line is that FIOMs are not assuming that real-world spot rates on average will rise with the forward curve. Assumptions about the evolution of real-world spot rates are embedded in the Treasury zero curve, which is presumably determined by market equilibrium. FIOMs use risk-neutral techniques because they reproduce the results of arbitrage-free pricing. Under the assumptions of the model, market bears and bulls might disagree about which Treasuries are attractive, but they will agree on the prices of derivative securities.

MORE REALISTIC MODELS

Formally, if a derivative security has payoff f_T at time T, the value of this security now (time t) is:

$$\text{value} = \hat{E}[e^{-\bar{r}(T-t)} f_T] \tag{11}$$

Here $\hat{E}$ represents expectations in a risk-neutral world and $\bar{r}(T-t)$ is the geometric average value of the instantaneous short rate between t and T. (See Hull [1989].)

If we restate our simple FIOM using continuously compounded rates, then this equation is exactly the risk-neutral pricing equation we have been using. We derived the risk-neutral distribution for future rates (so we knew how to compute $\hat{E}$) and the payoffs for each security given each future rate (hence we knew f_T). We then applied Equation (11) to find each security's value.

Actual FIOMs are of course much more complicated than the model introduced earlier. They extend for many more periods than the two-period case we examined. They are calibrated so that interest rate volatility matches a desired input (usually an estimated volatility or an implied volatility). FIOMs may model some desirable features of the real-world term structure, such as mean reversion or the existence of additional stochastic variation beyond one factor. They use sophisticated cash flow models to examine complex securities such as mortgage pass-throughs and their derivatives. Finally, they incorporate empirical prepayment models that take into account the fact that a mortgage is callable and is not always called rationally.

Even though actual models are much more complex than our simple model, most are still based on Equation (11). For our simple model it was easy to find the expected value explicitly. In actual applications, limited computational power often precludes explicitly taking expectations. In this case, Monte Carlo simulation is often employed (see Boyle [1977]).

Given the risk-neutral distribution of future rates, we generate random numbers and take a sample path of future rates. Following this path, we use our cash flow and prepayment models to generate the security's payoffs across future periods. We then discount these payoffs at the generated short rate path to get one sample of the current value. Taking enough sample paths, and averaging values, leads to an approximation of Equation (11).

Instead of explicitly taking expectations, we can apply the simulation method to our simple model. For instance, to value our PO we can generate 100 samples of next year's one-year zero rate. Each sample will have an 80% chance of the one-year rate rising to 15% and a 20% chance of it falling to 3.75%.

In each case, we would observe which rate occurred, take the appropriate end-of-year value for the PO, and discount back at today's one-year interest rate (7.5%). With enough simulations, our calculated value will converge to the explicitly solved PO value.

The same procedure works for more complex models.

Some researchers have advocated using a risk-neutral interest rate path to generate discount rates, but a non-risk-neutral interest rate path to forecast prepayments. Using this framework, we would generate two interest rate paths: one path assuming the Treasury curve is not expected to change, and another path assuming forward rates are on average realized.

The logic here is that discount rates must on average rise in order to price Treasuries correctly, but we should not assume prepayments on average fall. Thus we use one path to discount cash flows and another to calculate prepayments.

In fact, this logic is flawed. To see this, imagine we priced the PO in this manner. There are four cases for the end-of-year PO value. We summarize these in Exhibit 4, along with the probability of each case occurring. The value of the PO would be found by simulating these probabilities.

With enough paths, this value will converge to the following:

$$\begin{aligned} PO = & [0.40 \times 86.96 + 0.40 \times 100.0 + 0.10 \times \\ & 96.39 + 0.10 \times 100.0]/1.075 \\ = & 87.84 \end{aligned} \qquad (12)$$

This value is substantially different from the value of 83.32 we derived earlier. The earlier value is clearly correct, as pure arbitrage arguments can demonstrate. Therefore, assuming a different interest rate process for prepayments and discounting is both inconsistent and incorrect.

Our model does not assume that there is only a 0.20 chance of the mortgage prepaying in the real world. In the risk-neutral world, however, there is only a 0.20 probability of interest rates falling, so *in the risk-neutral world* there is only a 0.20 probability of prepayment. The mortgages would indeed be prepaid only when rates fall; to assume any other behavior leads to incorrect arbitrage pricing.

A valid criticism is that the models assume prepayment behavior is deterministic, given the interest rate path, and consistent across investor attitude toward risk. Thus, no two states of the world have the same interest rate path and different prepayment behavior.

This is indeed a shortcoming of the models, but not a shortcoming addressed by the correction we have just examined. (See Roberts et al. [1992].) Given we assume that prepayments are a deterministic function of rates, we cannot assume one rate for discounting and another for prepayment forecasting.

EXHIBIT 4

Valuing the PO by Fixing the Forward Rate "Bias"

Probability	Interest Rate for Discounting	Interest Rate for Prepayments	End-of-Year PO Value
0.40	15%	15%	86.96
0.40	15%	3.75%	100.00
0.10	3.75%	15%	96.39
0.10	3.75%	3.75%	100.00

If one assumes that, given interest rates, prepayment models are either biased or very noisy predictors, then one can seriously question the output of FIOMs for mortgages. But this criticism has nothing to do with forward rates and the slope of the yield curve.

OPTION-ADJUSTED SPREAD

We now relate our prior discussion to a type of FIOM commonly used in the mortgage-backed securities market, the OAS model (see Hayre and Lauterbach [1991] for a detailed description).

The prices generated by FIOMs do not always correspond to prices observed in the real world. When they do not match, we can calculate the constant spread that must be added to the short rate in all periods so that the model will generate the security's observed price. This spread is called the option-adjusted spread (OAS).

For example, recall that we calculated the value of the PO as 83.32. If we observe a market price for the PO of 80.08, we determine the option-adjusted spread by the equation:

$$80.08 = 0.8\left[\frac{100}{(1.075 + OAS)(1.15 + OAS)}\right] + 0.2\left[\frac{100}{(1.075 + OAS)}\right] \qquad (13)$$

Solving this equation produces an OAS of 250 basis points (bp) for the PO.

Taken literally, the models imply that any OAS other than zero indicates an arbitrage opportunity.

That is, a non-zero OAS implies we can lock in a sure profit by dynamically hedging with Treasuries.

There are several reasons to expect OASs to differ from zero without necessarily giving rise to an arbitrage. FIOMs commonly make the assumption of perfect capital markets. This entails frictionless buying, and frictionless short-selling, of each of the securities considered. To the degree that the real world differs from this paradigm, a sure arbitrage cannot be guaranteed, and non-zero OASs become possible. Other real-world frictions such as liquidity concerns and the bookkeeping cost of receiving frequent payments may also play a role.

We have assumed that the securities we analyze are default-free. Many derivative securities admit at least the possibility of default. Thus, they can sell at some discount to their zero OAS price (i.e., at a positive OAS).

FIOMs assume that their specification of the interest rate process matches the process in the risk-neutral world. If the actual process were different, OASs would not necessarily be zero.

Perhaps most importantly, FIOMs assume that the prepayment rate of mortgage securities depends only on the state variables used in the model, and that the parameters of the prepayment function are estimated correctly. In other words, the models assume that prepayments are a known deterministic function of the interest rate path. If this is not the case, as it almost assuredly is not, then prepayment error and uncertainty can lead to non-zero OASs.

Notice that we have not included the assumption that forward rates are expected to be realized as a problem of OAS models. OAS models are a special case of FIOMs. Thus, our discussion of forward rates and FIOMs applies equally well to OAS models. OAS models do not assume that real-world forward rates are expected to be realized.

CONCLUSIONS

While FIOMs ostensibly assume that forward rates are on average realized, they also discount cash flows at the path of riskless short-term rates. Both these actions are valid only in a world of risk-neutrality. Because derivative securities are priced by arbitrage arguments, their value must be the same in the risk-neutral and the real world. Thus the models deliver real-world prices, while they do not assume that either of the two assumptions is correct in the real world. Thus, a common criticism of the models is misguided.

An upward-sloping yield curve indicates that either investors expect rates to rise or they require a higher expected return on longer-term securities. Either way, it is appropriate that FIOMs assume forward rates are on average realized in the risk-neutral world.

FIOMs price all securities relative to Treasuries. When a FIOM produces a negative OAS for, say, a principal-only strip (PO), some practitioners apologize for the model. They might say that the model unfairly hurts POs by assuming rates will rise (and prepayments will fall). We have shown this to be false.

Assuming the model is calibrated correctly and is internally consistent, a negative OAS indicates that a comparable PO could be constructed by dynamically trading Treasury bonds at less cost. This follows from the equivalence of risk-neutral pricing and arbitrage-free pricing. Thus, irrespective of the forward curve, the PO is a "rich" bond.[9] The role of risk-neutral pricing comes from its equivalence to this replication strategy, not from any assumptions about future rates or preferences.

We do not claim that opinions about forward rate realization are irrelevant. How bullish or bearish you are depends directly on your opinion about future spot rates. If you expect future spot rates to be lower than forward rates, then you also expect the return on long bonds to exceed short bonds. If you feel this extra expected return is commensurate with the risk of longer bonds, you will buy these bonds. Thus, opinions on forward rate realization are relevant. They are not relevant, however, for pricing derivative securities.

Under the models' assumptions, derivative securities can be replicated by portfolios of Treasuries. Opinions about forward rates do not matter. Derivative securities are priced at the cost of their Treasury replication portfolio (at least for a zero OAS derivative).

The assumption that prepayments are an accurate deterministic function of interest rates is what allows these Treasury portfolio replications to occur. This assumption definitely bears further study.

ENDNOTES

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[1]It is of course possible that for some type of model the

criticism is valid. Our article relates to most common OAS models.

[2]Non-technically, the term, "on average realized," can be taken to mean that the distribution of future spot rates is in some sense centered around current forward rates.

[3]Taking risk-neutralized expectations and discounting at the risk-free rate is known as using a martingale pricing operator. See Ross [1978] or Harrison and Kreps [1979] for further (and deeper) analysis.

[4]This discussion follows Black, Derman, and Toy [1990] and Cox, Ross, and Rubinstein [1979].

[5]Formally, there are two qualifications to these statements. First, under this version of the expectations hypothesis, expected future rates are not exactly equal to current forward rates because of Jensen's inequality. Second, risk neutrality does not always imply the expectations hypothesis. In more complex models it is more accurate to say that we work in a "risk-adjusted" world rather than a "risk-neutral" world (see Cox, Ingersoll, and Ross [1979]). For the simple model we employ in this article, and for convenience, we use the term "risk-neutral," and we take the expectations hypothesis to mean that forward rates are on average realized.

[6]Again, the fact that the expected rate does not exactly equal the forward rate of 12.56% is a consequence of Jensen's inequality; it does not alter whatsoever the spirit of our arguments. In general, the term "expectations hypothesis" can refer to the assumption that expected returns are equal across securities or the assumption that forward rates are expected to be realized. These are strongly related yet distinct hypotheses. Cox, Ingersoll, and Ross [1985] and Jarrow [1981] discuss this issue further. We ignore the distinction.

[7]This situation is analogous to negative beta securities having negative expected excess returns under the Capital Asset Pricing Model.

[8]Of course, any of the model's other assumptions may be at fault. For instance, it is possible that the market is using a different prepayment model. This is a different issue, and problem, from the one we address.

[9]Once again, we stress that other assumptions of the model may indeed be flawed.

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