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Portfolio management is moving toward a more flexible approach that is capable of capturing dynamics in risk and return expectations across an array of asset classes (Li and Sullivan [2011]). The change is being driven, in part, by the observation that risk premiums vary as investors cycle between risk aversion and risk adoration and that the decision to invest—whether to take risk and how much—is the most important investment decision (Xiong and Idzorek [2011]). Certainly, managers should take risks, but only if the returns appear to represent fair compensation. This all suggests that the traditional strategic approach of fixed asset allocation is outmoded. The challenge of portfolio choice is much more than merely selecting for inclusion uncorrelated asset classes that constitute significant economic exposure and then specifying a fixed proportion of each.[1]

Our effort facilitates this much needed dynamic flexibility to the asset allocation process. We propose a model of portfolio selection with heavy tails and dynamic return correlations. The powerful intuition behind our approach is that proper portfolio construction is an ongoing, dynamic process that integrates time-varying risks of the various asset classes within the investor's portfolio. We develop a dynamic asset allocation framework that determines an investor's optimal portfolio in accordance with changing global market environments and market conditions. Specifically, we consider how global return, variance, and covariance characteristics vary across time and states of global markets for a diversified portfolio of asset classes. We then use this dynamic information to consider the asset allocation implications in a practical setting. Our novel approach builds on the regime-switching framework of Ang and Bekaert [2002, 2004] and Kritzman and Page [2012], among others, and provides a framework that illuminates the changing nature of global market risks and directs accordingly asset allocation and risk decisions.

We argue that it is imperative for managers to monitor and react to changes in the macroenvironment on an ongoing basis. Our effort provides one such useful framework: a genuine barometer for monitoring risk dynamics across our global financial system and reacting to those market conditions.

THE FRAMEWORK FOR DYNAMIC ASSET ALLOCATION

The framework we offer has important implications for portfolio risk management and asset allocation decisions. It takes into account skewness and kurtosis, moments of the return distribution beyond mean and variance, as well as persistence in volatility, or volatility clustering. It also accounts for dynamic correlations of risky asset returns, which tend to increase during times of market turbulence, or return dependence. In other words, our nonlinear model framework is more dynamic and less restrictive than traditional, static methods that depend on returns following a Gaussian process. One practical application of our approach is that it provides a monitoring device regarding market instability and portfolio vulnerability. Furthermore, we demonstrate that investors can act before the iceberg is under the ship's keel. The result is a high-frequency, dynamic technique that allows investors to proactively monitor and manage portfolio risk via real-time asset allocation decisions.

We dynamically and proactively determine asset weightings as conditioned on changing market volatility and covariances. Asset allocation is further accomplished in accordance with one of two possible states of the world—normal risk (normal uncertainty: normal return volatility and correlations) and high risk (high uncertainty: high volatility and correlations). Behind the two states lies a mechanism driven by factors determined to possess predictive power of the degree of economic and market uncertainty governed by *forward* transition probabilities in which the regime variables are used to fit a Markov regime switching process; see Ang and Bekaert [2002, 2004]. Our regimes correspond to market dynamics and the non-normal return distributions that characterize markets; see, for example, Xiong and Idzorek [2011] and Sullivan, Peterson, and Waltenbaugh [2010]. We do not model changes in expected returns, which are known to be particularly difficult and often lead to models biased by hindsight and model over-fitting.

At a high level, the strategy we propose consists of three main, overarching parts. In the first part, we estimate the conditional value at risk (CVaR) for a market representative portfolio (Kaya, Lee, and Pornrojnangkool [2011]). The estimated CVaR then serves as a critical input into our second part, a forecast of market risk, which models the probability that markets are in or about to enter a turbulent financial period. This information then enables the third part, which is to proactively adjust the portfolio asset allocation in accordance with the market-risk-regime forecast obtained in the second part.

We begin by applying extreme value theory (EVT), which allows us to model fat-tailed return distributions for a host of asset classes with particular attention to volatility clustering and extreme co-movements across various markets; see, for example, Sullivan, Peterson, and Waltenbaugh [2010].[2] The asset classes included in our framework are global equity, U.S. investment-grade bonds, U.S. high-yield bonds, commodities, and U.S. real estate investment trusts. Our base-case portfolio asset allocation, described in Exhibit 1, is constructed based on weights typically found in institutional portfolios and close to the global capital market weights.

We employ conditional value at risk (CVaR) to facilitate forward-looking scenario-based outcomes outside the range of historical observations.[3] A two-state Markov switching model is applied to identify regimes in the forward-looking market downside risk measure, CVaR. The CVaR then forms the basis of our dynamic risk and asset allocation framework by providing an indicator of downside risk across markets and for optimization in portfolio construction.

Altogether, we build an effective regime-dependent investment strategy based on market downside risk and asset class co-movements across time. To accomplish this task, we follow a dynamic asset allocation framework under a mean CVaR optimization approach with varying target CVaR according to market regimes. The end result is an implementable tail risk management process in accordance with the increasingly interconnected and dynamic risks observed in markets.

EXHIBIT 1
Portfolio Assumptions

Index	Global Equity (MSCI ACWI)	Commodities (SPGSCI)	Real Estate (DW REITs)	High Yield (MLHY)	Investment Grade (Barclay Agg.)
Policy Allocation	45%	10%	10%	15%	20%
Portfolio Bounds	30%–70%	5%–15%	5%–15%	7%–23%	10%–40%
Expected Returns	7%	6.5%	7%	6%	4%

DATA AND MODEL SETUP

Exhibit 2 provides an overview of the five asset classes included in our analysis along with summary statistics. All asset classes are represented by indices in the following way: global equities by the Morgan Stanley Capital International ACWI Index

Exhibit 2

Asset Classes, Indices, and Summary Statistics of Daily Returns, February 1, 1996–October 10, 2011

Index	Global Equity (MSCI ACWI)	Commodities (SPGSCI)	Real Estate (DW REITs)	High Yield (MLHY)	Investment Grade (Barclay Agg.)
Mean	0.02%	0.03%	0.06%	0.03%	0.02%
Std. Deviation	1.04%	1.50%	1.90%	0.29%	0.26%
Median	0.07%	0.03%	0.03%	0.05%	0.03%
Min	–7.10%	–8.76%	–19.76%	–4.73%	–2.04%
Max	9.31%	7.48%	18.98%	2.78%	1.71%
1st Percentile	–2.98%	–4.08%	–6.30%	–0.87%	–0.64%
99th Percentile	2.68%	3.62%	6.24%	0.73%	0.66%
5th Percentile	–1.61%	–2.40%	–2.32%	–0.36%	–0.40%
95th Percentile	1.53%	2.37%	2.16%	0.36%	0.41%
10th Percentile	–1.09%	–1.70%	–1.33%	–0.21%	–0.27%
90th Percentile	1.07%	1.82%	1.39%	0.24%	0.32%
Skewness	–0.24	–0.16	0.44	–2.74	–0.27
Kurtosis	7.50	2.38	21.21	42.55	3.11

(MSCI ACWI), commodities by the Goldman Sachs Commodity Index (SPGSCI) total return index, U.S. real estate by the Dow-Jones Wilshire REIT (DW REIT) total return index, U.S. high-yield bonds by the Merrill Lynch High Yield Master II (MLHY II) total return index, and U.S. investment-grade bonds by the Barclays Capital Aggregate Bond Index (Barclays Agg.) gross return index. All summary statistics are based on daily data (not annualized) from February 1, 1996, to October 10, 2011. In reviewing Exhibit 2, we draw the reader's attention to the negative skewness observed for almost every asset class (except REITS) and the excess kurtosis across all asset classes, especially for REITs and high-yield bonds.

Consistent with prior research, further examination of the data reveals that autocorrelation is present in the return series, especially for day $t + 1$. This can be seen visually for MSCI ACWI by the autocorrelation functions of the log of daily returns and the square of log returns, or variance, shown in Exhibit 3, Panel A. We return to address these issues, which motivate our analysis, later in the article.

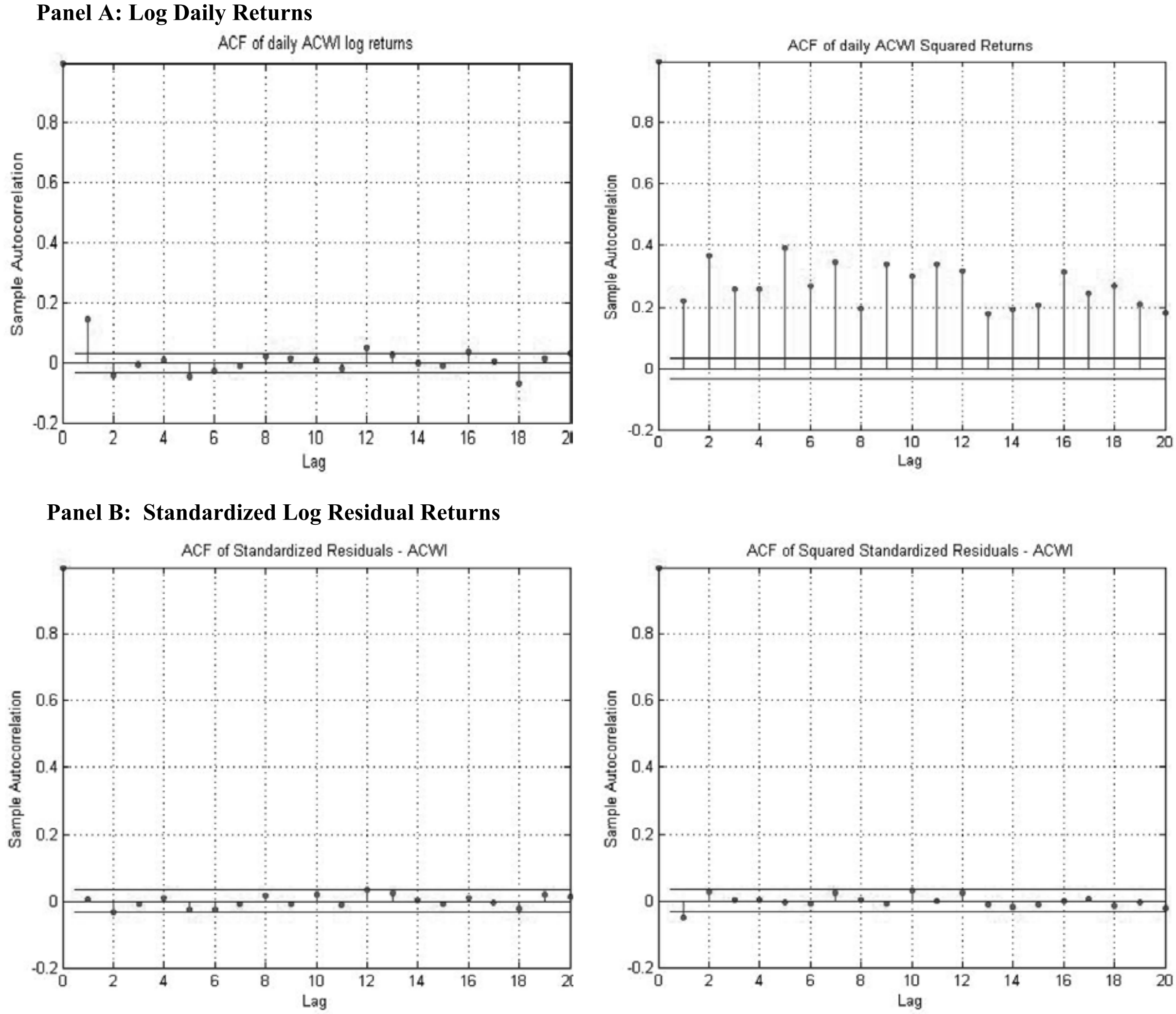
FORECASTING MARKET RISK

For the first phase in our three-part framework—a daily forecast of the risk of the overall portfolio—the model we employ for the joint fat-tailed distribution of returns and the subsequent calculation of CVaR involves the following five main steps:

1. *Return Filtering.* We filter each daily return series using AR(1)/GJR-GARCH(1,1) process to remove serial correlation and standardize the return residuals using the Student's *t* distribution to account for fat tails.
2. *Marginal Distribution Modeling.* We employ a peaks-over-threshold method to estimate the marginal semi-parametric empirical CDF of the filtered standardized residuals from Step 1; see, for example, Focardi and Fabozzi [2004] and Tsay [2005]. We use a non-parametric Gaussian kernel to derive the interior portion of the distribution and a parametric GPD to estimate the left-hand and right-hand tails.
3. *Extremal Dependence Modeling.* We transform the standardized return residuals from Step 1 into a uniform distribution (in which return residuals all have the same probability of occurring) using the semi-parametric empirical CDF derived in Step 2. We then fit a *t*-copula to the transformed data to allow for joint "fat" tails and model dependence of returns in the extrema; that is, we transform the returns so as to be independent from one day to the next but within any day possess the dependence, and specifically the rank correlation, of the copula from which they are drawn. In this way, the covariance of the returns among all the indices is maintained, yet individually randomized, for the simulation to be performed in the next step.
4. *Return Simulation.* Given the parameters derived from the *t*-copula joint distributions obtained for each index, we simulate the joint dependent features 21 times to correspond to a one-month trading period. We then run the simulation for all indices 10,000 times. To then arrive at a 21-day risk estimate for each index, we transform the simulated uniform residual return variates into standardized variates via the inversion of the semi-parametric marginal CDF for each index. Last, we reintroduce the autocorrelation and volatility clustering observed in the original index using parameters obtained from Step 1 to arrive at the simulated 21-day forecasted daily returns for all five asset classes.

EXHIBIT 3

Autocorrelations of Daily Returns and Squared Returns: ACWI



5. *Risk Forecasting.* We then combine the individual index risk estimates to arrive at a forecasted 21-day aggregate portfolio risk using the policy allocation shown in Exhibit 1 as the baseline portfolio. The average 21-day portfolio loss in the worst 5% scenarios based on the 10,000 simulations becomes the portfolio 95% CVaR. This CVaR is then used as the across-market tail risk indicator in the second and third part of our three-part framework—regime dependent dynamic asset allocation. Expected returns are also shown in Exhibit 1, and do not change for any regime environment.

We now discuss in more detail the five steps just outlined that are used to arrive at our dynamic, high-frequency estimate of portfolio risk using CVaR and extreme value theory. Modeling the tails of a distribution using EVT requires the observations to be approximately independent and identically distributed (i.i.d). As a consequence, we first filter our return series with the aim of the filtering process to produce approximately i.i.d observations. To accomplish this objective, for each return series we fit a first-order autoregressive model AR(1) to the conditional mean of the daily log returns using Equation (1) and an asymmetric GJR-

GARCH(1,1) (Glosten, Jagannathan, and Runkle [1993]) to the conditional variance using Equation (2),

$$r(t) = c + \theta r(t-1) + \epsilon(t) \tag{1}$$

$$\sigma^2(t) = \kappa + \alpha\sigma^2(t-1) + \phi\epsilon^2(t-1) + \psi[\epsilon(t-1) < 0]\epsilon^2(t-1) \tag{2}$$

$$z(t) = \frac{\epsilon(t)}{\sigma(t)} i.i.d.t(v)$$

With this model, we address the so-called leverage effect whereby a negative association has been observed to exist between shocks to asset returns and future volatility (Black [1972]). Specifically, the last term of Equation (2) incorporates asymmetry into the variance through the use of a binary indicator that takes the value of one, which predicts a higher volatility for the subsequent day if the prior residual return is negative, and takes on a value of zero otherwise. We then standardize the residuals by the corresponding conditional standard deviation as commonly done for such exercises. Finally, the standardized residuals are modeled using the standardized Student's *t* distribution in order to capture the well-known fat tails in the distribution of returns.

The result of this process is shown in Exhibit 3, Panel B, which plots the autocorrelations of the standardized residuals for the MSCI ACWI return series. As seen from Exhibit 3, Panel B, the filtering process we employ results in approximately i.i.d. observations and thus volatility clustering has been eliminated by the filtering process. The resulting standardized residual returns approximate a zero-mean, unit-variance, i.i.d. series. This allows us to employ EVT estimation of the tails from our sample cumulative distribution function (CDF).

Because EVT allows only for estimation of the tails of the distribution, we combine these tail distributions with a model for the remaining internal part of the distribution. To accomplish this task, we move to Step 2 and follow the peaks-over-thresholds approach (McNeil and Saladin [1997]) and define upper and lower thresholds as that set of minimum residual returns (we use the 90th percentile) found in each of the left-hand and right-hand tails. The result is a partition of the standardized residuals into three regions: the lower tail, the interior, and the upper tail. A nonparametric Gaussian kernel CDF is used to estimate the interior of the distribution. Using EVT, we then fit those extreme residuals in each tail beyond the thresholds. In particular, we use a parametric generalized Pareto distribution (GPD) estimated by maximum likelihood. The CDF of the GPD is parameterized using Equation (3), with exceedances (y), tail index parameter (zeta), and scale parameter (beta),

$$F(y) = 1 - \left(1 + \frac{\zeta y}{\beta}\right)^{-\frac{1}{\zeta}}, y \geq 0, \beta > 0, \zeta > -0.5 \tag{3}$$

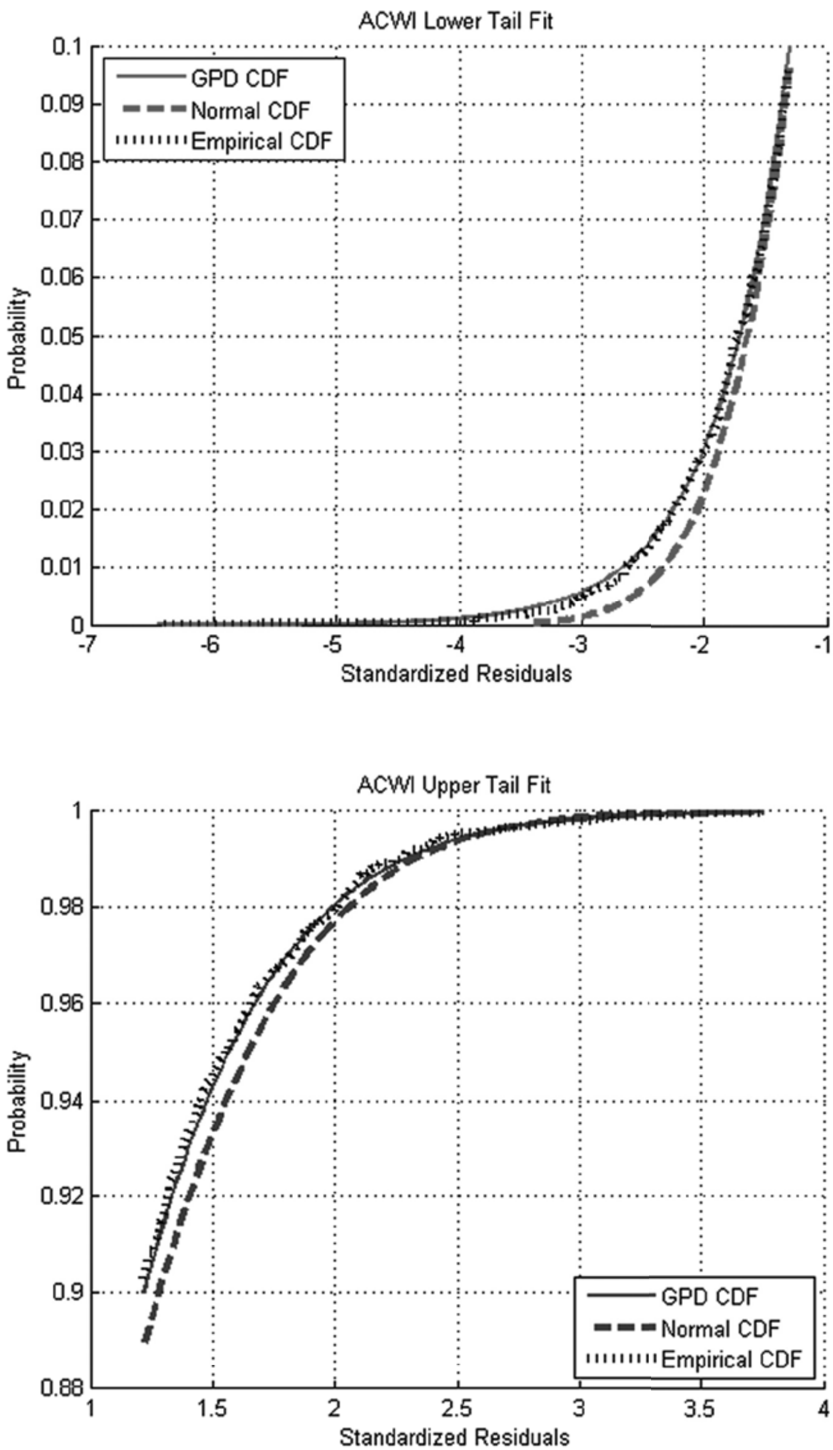
Exhibit 4 shows a visual representation of the upper and lower tails of the return distribution for ACWI. It shows that our GPD approach far better accommodates the fat tails observed historically in the return distribution. As can be seen from Exhibit 4, the GPD curve much more closely approximates the historical, or empirical, return distribution, and as such, allows for a more accurate representation of the reality of fat tails.

With our fat-tailed conditional distribution of returns in place, we can now turn our attention to the next important element in risk modeling, Step 3, or how asset class returns move together in the extremes. For our dependence modeling of returns in the extrema, we consider asset return covariances via the joint distribution of returns using copula theory (Focardi and Fabozzi [2004]). With copulas, we are able to model the observed increased co-dependence of asset class returns during periods of high market volatility and stress. Empirically, not only do individual asset classes have "fatter" tails than are allowed in a normal Gaussian distribution, but combinations of asset classes also exhibit a higher incidence of joint negative returns in times of market stress; that is, risky asset returns across asset classes abruptly decline in unison. By way of example, as shown in Exhibit 5, both MSCI ACWI and GSCI have occasionally realized simultaneous loss events of four standard deviations or more. A bivariate normal distribution would therefore provide a poor representation of the dynamics of these joint jumps in asset class returns observed in recent decades. A more realistic approach is needed.

To account for the incidence of returns abruptly moving in unison, we employ copula theory, which accommodates interrelated and extreme dependencies of returns. More specifically, copulas allow for the modeling of fat tails even when asset class returns present a high degree of co-movement, as has been observed historically. We choose to employ the *t*-copula because this particular copula enables us to better capture the effects

Exhibit 4

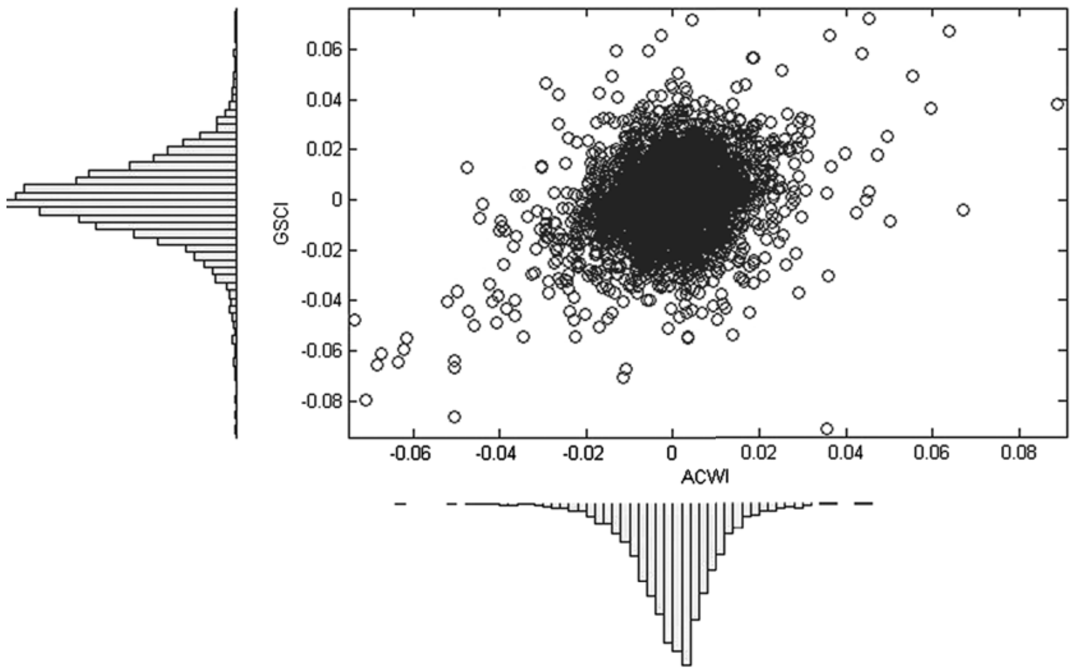
ACWI Lower- and Upper-Tail Fit



of fat tails and allocate nonzero probabilities to observations that may occur outside of the range of historical returns. By adjusting the copula's degrees-of-freedom parameter, we can extrapolate our multivariate fat-tailed distribution so that it is consistent with the observed empirical data. Having estimated the three regions of each marginal semi-parametric empirical CDF, we transform them to uniform variates and then fit the *t*-copula to the transformed data.

We can now move to Step 4 and generate our scenario-based forward-looking projections of downside risk across markets using Monte Carlo simulations. Given the parameters of the *t*-copula from Step 3, we simulate the joint dependent features 21 times for all five indices and repeat the simulation 10,000 times. We simulate 21 observations in order to correspond to a one-month trading period. Then, via the inversion of the semi-parametric marginal CDF of each index, we transform the simulated joint dependent features to standardized residuals in order to be consistent with those obtained from the AR(1)/GJR-GARCH(1,1) filtering process in Step 1. In other words, we transform the returns so as to be independent in time but at any point in time possess the dependence, and specifically the rank correlation, of the copula from which they are drawn. In this way, the covariance of the returns among all indices is maintained, yet individually randomized, for simulation purposes. It also maintains the autocorrelation and volatility clustering observed in the historical returns for each index. This allows us to move to Step 5 whereby we aggregate the portfolio and project a 21-day-forward downside risk for the aggregate portfolio. This downside risk is measured as the 95% CVaR and is the

EXHIBIT 5
Scatter Plot of ACWI vs. GSCI Log Daily Returns



average portfolio loss in the worst 5% scenarios based on 10,000 Monte Carlo simulations.[4]

To generate the time series of our 21-day look-ahead portfolio risk forecast, we repeat the previous steps and forecast the portfolio 95% CVaR under an expanding-window approach. We choose 21 days because the period corresponds to a trading month. To avoid look-ahead bias, we incorporate only the market information available at the time the model forecast is generated. The result of our risk forecast effort is shown in Exhibit 6 as represented by our 21-day-forward combined portfolio CVaR for the base portfolio. As can be seen from Exhibit 6, our portfolio risk estimate is highly responsive to actual market dynamics.

FORECASTING MARKET RISK ENVIRONMENTS

In the next part of our framework, we estimate the probability that the market environment is already in or about to enter a turbulent state and use this information to inform our asset allocation decision. Here, our asset allocation is determined in accordance with one of two possible states of the world: normal risk (normal uncertainty: normal return volatility and correlations) and high risk (high uncertainty: high volatility and correlations, low returns). The two market states are governed by a *forward* transition probability forecast of markets being in a high-risk or "event" regime, as derived from our earlier CVaR forecast. Specifically, our CVaR forecast is used as the regime variable to estimate the likelihood that markets will be in a high-risk state within the next 21 days, with the probability estimate falling between 0% and 100%. This is accomplished via fitting our 21-day CVaR forecast to a two-state Markov regime-switching process; see Ang and Bekaert [2002, 2004]. In this way, our regimes correspond to market dynamics and the non-normal return distributions that characterize markets.

Exhibit 7 reveals the meaningful presence of a normal regime and an event regime in our time-series forecast of market downside risk. This is evidenced by the substantial change in both the mean and the standard deviation of our CVaR regime variable. Over the estimation period, the high-risk event regime shows an average 21-day CVaR (95%) of –14.22% with a standard deviation of 5.69%, as compared to a higher average CVaR of –6.12% with a lower standard deviation of

EXHIBIT 6
Historical 21-Day Forward-Looking 95% CVaR(Log), 1/3/2000–10/10/2011

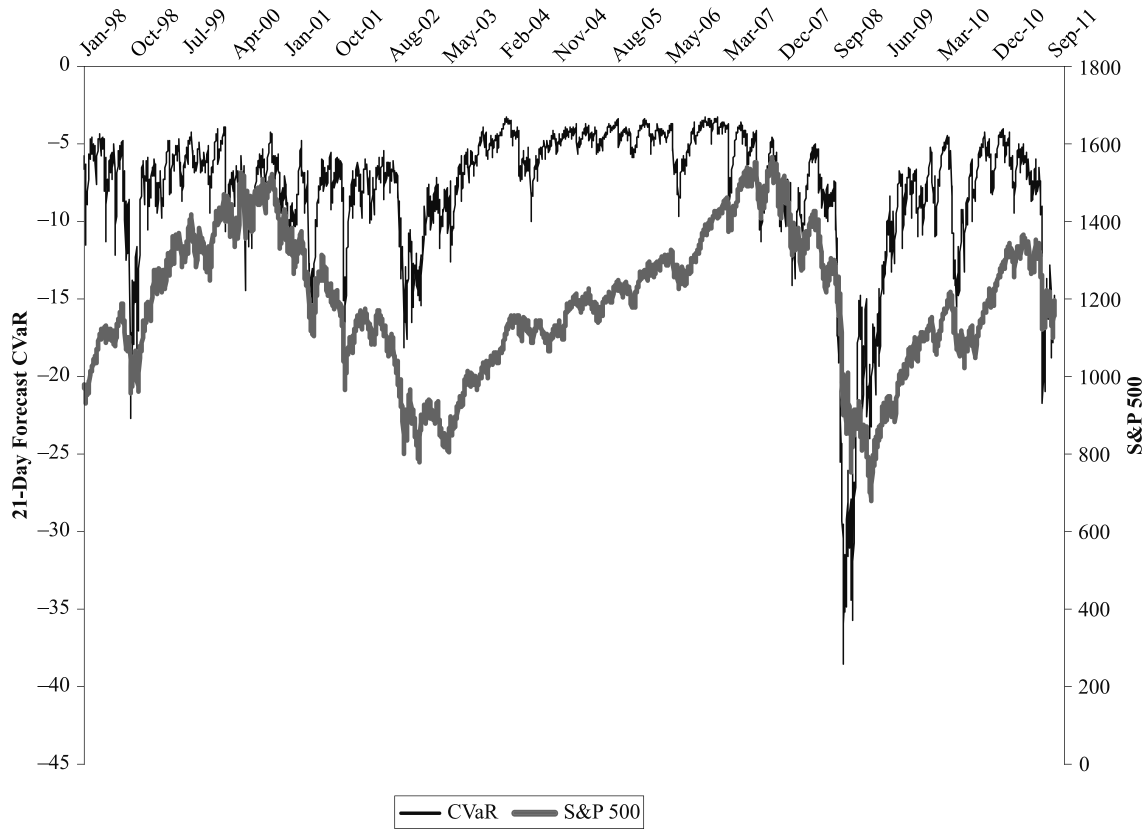


EXHIBIT 7
Markov Switching Model Perfect-Insights Estimation

	Regime 1 (Normal Risk)			Regime 2 (High Risk)		
	Persistence	Mu	Sigma	Persistence	Mu	Sigma
Market Downside Risk	99.14%	–6.12%	1.63%	96.29%	–14.22%	5.69%

1.63% for the normal regime. The regime model we employ is equipped to adapt over time to changing market conditions in real time.

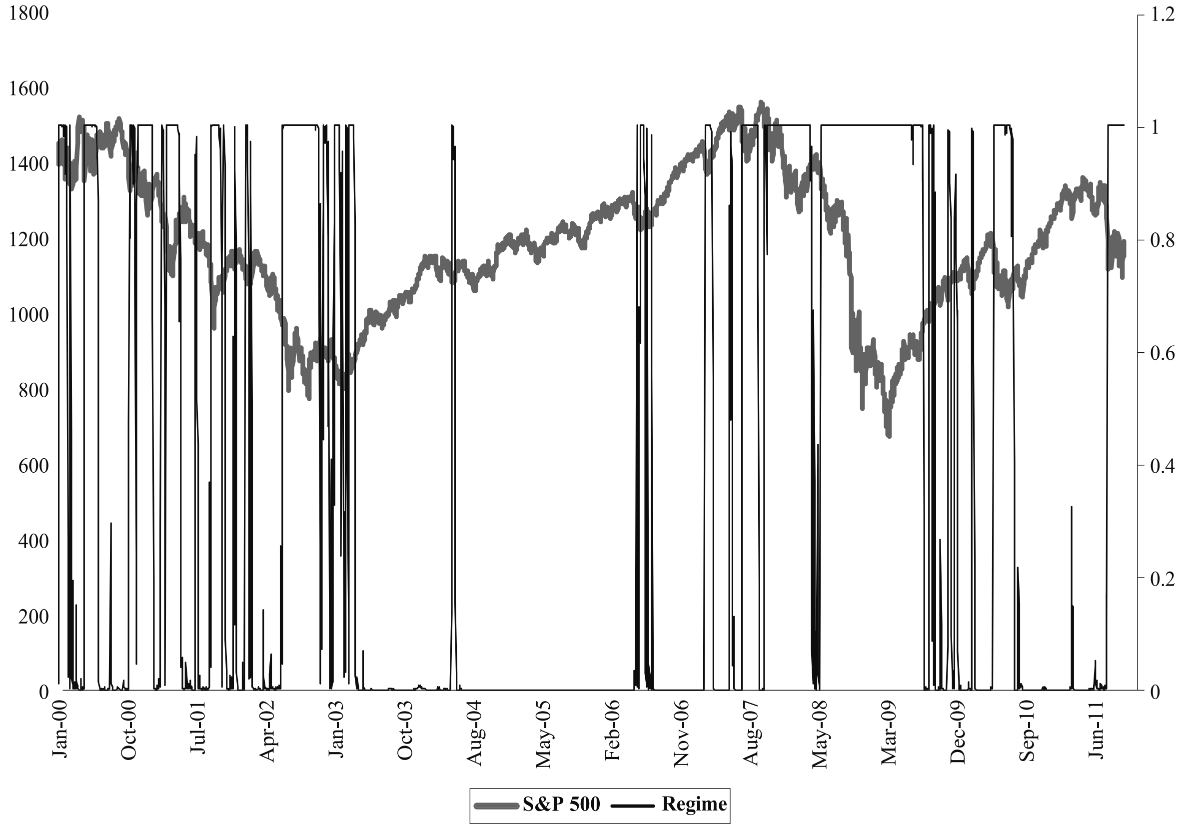
Exhibit 8 shows the time-series results of the resulting forecast of the probability that the markets are in, or about to be in, a high-risk state. For our purposes, we identify a high-risk-event regime period when our Markov model suggests that the probability of being in a high-risk regime is greater than 50% over the 21-day forecast horizon.[5] To estimate our model, we use an expanding-window approach with our first estimate in January 3, 2000, using data from February 1, 1996, to January 2, 2000. We generate each new forecast daily by simply adding new observations and re-estimating the model with the new observations as the data become available. The results, shown in Exhibit 8, Panel A, highlight that our Markov switching model succeeded in meaningfully partitioning the market into two regimes. Exhibit 8, Panel B, identifies the specific dates for which the market is predicted to be in a high-risk-event regime (again, defined as an event probability of at least 50%).

A further understanding of the impact of our regime risk model on asset class performance can be inferred from the data presented in Exhibit 9. Here, we summarize the risk and return statistics for each of our

Exhibit 8

Expanding-Window Approach, 1/3/2000–10/10/2011

Panel A: Probability of the Event Regime



Panel B: High-Risk-Regime Periods

Start	4-Jan-00	9-Feb-00	18-Feb-00	14-Apr-00	11-Oct-00	10-Nov-00	20-Feb-01	9-Mar-01	6-Jul- 01	6-Sep-01
End	8-Feb-00	10-Feb-00	23-Feb-00	9-Jun-00	6-Nov-00	18-Jan-01	6-Mar-01	9-May-01	17-Jul-01	24-Oct-01
Trading Days in Between	24	1	2	38	18	45	10	42	7	31
Start	29-Oct-01	10-Dec-01	13-Dec-01	29-Jan-02	19-Feb-02	26-Jun-02	26-Nov-02	5-Dec-02	9-Dec-02	27-Dec-02
End	8-Nov-01	11-Dec-01	21-Dec-01	13-Feb-02	26-Feb-02	21-Nov-02	27-Nov-02	6-Dec-02	26-Dec-02	2-Jan-03
Trading Days in Between	8	1	6	11	5	104	1	1	12	3
Start	21-Jan-03	24-Jan-03	24-Feb-03	10-Mar-03	24-Mar-03	10-May-04	19-May-06	2-Jun-06	6-Jun-06	6-Jul-06
End	23-Jan-03	21-Feb-03	28-Feb-03	19-Mar-03	15-Apr-03	25-May-04	23-May-06	5-Jun-06	30-Jun-06	7-Jul-06
Trading Days in Between	2	19	4	7	16	11	2	1	18	1
Start	21-Jul-06	25-Jul-06	27-Feb-07	7-Jun-07	12-Jun-07	26-Jul-07	19-Oct-07	9-May-08	6-Jun-08	27-Jul-09
End	24-Jul-06	26-Jul-06	30-Mar-07	8-Jun-07	15-Jun-07	1-Oct-07	2-May-08	12-May-08	24-Jul-09	28-Jul-09
Trading Days in Between	1	1	23	1	3	46	134	1	285	1
Start	17-Aug-09	3-Sep-09	9-Sep-09	30-Oct-09	27-Nov-09	4-Feb-10	4-May-10	18-Mar-11	4-Aug-11	
End	2-Sep-09	4-Sep-09	10-Sep-09	12-Nov-09	9-Dec-09	17-Feb-10	26-Jul-10	21-Mar-11	10-Oct-11	
Trading Days in Between	12	1	1	9	8	8	57	1	46	

five asset classes during the study period from January 3, 2000, to October 10, 2011. A comparison of Panel A (event days) and Panel B (full period) in Exhibit 9 shows that during the event periods the median returns of all risky assets are lower and the standard deviation of returns are all higher versus the full period. These results suggest that the model assisted in anticipating turbulent periods.[6] Furthermore, extreme returns are shown to be a dominant presence during forecasted event regimes. This can be seen from the percentiles, for example, the 5th percentile and 95th percentile are much further apart for the event-regime daily return distributions versus the full period.

DYNAMIC ASSET ALLOCATION

We now discuss the third, and final, part of our model: incorporating our forecast of market turbulence into an effective dynamic asset allocation framework. Our portfolio construction process responds to market dynamics by adjusting the overall portfolio asset allocation in accordance with our regime-based risk fore-

EXHIBIT 9
Summary Statistics of Daily Returns, 1/3/2000–10/10/2011

Panel A: Event Days

Index	Global Equity MSCI ACWI	Commodities SPGSCI	Real Estate DW REITs	High Yield MLHY	Investment Grade Barclay Agg.
Mean	0.01%	–0.02%	0.05%	0.00%	0.03%
Std. Deviation	1.59%	1.89%	3.32%	0.47%	0.30%
Median	0.08%	0.03%	0.03%	0.03%	0.04%
Min	–7.10%	–8.76%	–19.76%	–4.73%	–2.04%
Max	9.31%	7.48%	18.98%	2.78%	1.71%
1st Percentile	–4.90%	–5.35%	–9.27%	–1.62%	–0.70%
99th Percentile	4.41%	4.93%	11.12%	1.12%	0.79%
5th Percentile	–2.50%	–3.13%	–5.31%	–0.74%	–0.47%
95th Percentile	2.31%	2.81%	5.00%	0.64%	0.47%
10th Percentile	–1.75%	–2.26%	–3.00%	–0.46%	–0.34%
90th Percentile	1.66%	2.12%	3.17%	0.44%	0.37%
Skewness	–0.14	–0.28	0.33	–1.45	–0.18
Kurtosis	6.50	5.13	9.66	17.79	6.53

Panel B: Full Period

	MSCI ACWI	SPGSCI	DW REITs	MLHY	Barclay Agg.
Mean	0.01%	0.03%	0.07%	0.03%	0.03%
Std. Deviation	1.13%	1.62%	2.20%	0.33%	0.26%
Median	0.07%	0.05%	0.09%	0.05%	0.03%
Min	–7.10%	–8.76%	–19.76%	–4.73%	–2.04%
Max	9.31%	7.48%	18.98%	2.78%	1.71%
1st Percentile	–3.55%	–4.37%	–7.37%	–0.96%	–0.64%
99th Percentile	2.90%	3.84%	7.33%	0.84%	0.66%
5th Percentile	–1.74%	–2.62%	–2.88%	–0.42%	–0.41%
95th Percentile	1.61%	2.54%	2.61%	0.42%	0.42%
10th Percentile	–1.21%	–1.86%	–1.71%	–0.25%	–0.28%
90th Percentile	1.14%	2.00%	1.65%	0.28%	0.33%
Skewness	–0.21	–0.19	0.38	–2.51	–0.21
Kurtosis	9.86	5.02	18.63	38.30	6.13

cast and mean-CVaR optimization.[7] As mentioned, we employ a risk-on or risk-off approach as driven by our model prediction for either a normal or high-risk state, respectively. Importantly, the dynamic portfolios we construct facilitate a direct evaluation of the risk present in markets with an eye toward mitigating the impact of abrupt downside events frequenting markets via dynamic asset allocation.

Specifically, we solve Equation (4) to obtain the weights of a portfolio that maximizes expected return while targeting the CVaR to a desired level; see Rockafellar and Uryasev [2000]. Expected returns and benchmark portfolio weights are shown in Exhibit 1. This approach allows us to incorporate our copula-driven fat-tailed simulation scenarios into a portfolio allocation optimization problem. Furthermore, as we will see, this allows the optimal portfolio allocation to be determined in accordance with our market regime prediction. Specifically, the fixed expected return vector is represented by μ, and w is the set of weights that belongs to the space X. We examine both an unconstrained portfolio with no shorting and no leverage (weights must be between 0% and 100%) and a constrained portfolio with bounds as shown in Exhibit 1. The CVaR target constraint is represented by $\tilde{\theta}_{95\%}(W)$ and is the resulting forward-looking CVaR at a 95% confidence level as estimated given the set of weights, w, with a target CVaR level of γ,

$$\begin{gathered} \textit{Maximize } \mu w \\ \textit{Subject to } \tilde{\theta}_{95\%}(W) \leq \gamma,\ w \in X \end{gathered} \tag{4}$$

We next demonstrate the approach by backtesting the model outcomes and combining all three parts of our process over time. We explore both unconstrained and constrained portfolio weighting schemes as shown in Exhibit 1. As mentioned, we reduce the effect of any hindsight bias on our results by using static, unadjusted expected returns. Our main focus in this article is to show the meaningful impact that can be had on portfolio performance by adjusting "only" the portfolio asset allocation in accordance with dynamic forecasts of market risk as captured by changing variances and covariances across asset classes over time. To this end, we forecast risk and then we rebalance the portfolio according to pre-specified rules discussed later. For all results, we use static expected returns and the policy portfolio as the benchmark portfolio, as shown in Exhibit 1.

Exhibit 10 shows the results from our portfolio construction process. In Panel A, we employ a set monthly rebalancing rule whereby we rebalance the portfolio every 21 days. We compare the performance of the benchmark portfolio to an unbounded portfolio construction process (weights must be between 0% and 100%, which disallows shorting and leverage), all rebalanced each 21 days. The two unbounded portfolios are optimized portfolios based on our CVaR estimate, as discussed previously. We show results for several static target levels of CVaR, and we then allow the target level of CVaR to switch over time between a high- and low-risk level in accordance with our dynamic regime forecast.

In the first row of Exhibit 10, Panel A, we show the performance of the benchmark portfolio. We compare outcomes for our unbounded portfolios which allow the weights for each of our five asset classes to vary between 0% and 100% over the study period. First, we show the performance of overall portfolios created by imposing a series of constant maximum-allowable level of mean CVaRs. Here, we report the results for five constant target CVaR levels ranging from lowest risk (3% CVaR) to highest risk (7% CVaR). This allows a comparison of how various CVaR limits reflect changing risk conditions as estimated solely by our CVaR model, while temporarily ignoring our market-risk-regime forecast in constructing portfolios. As expected, portfolio drawdowns and volatility rise with each higher level of allowable risk along with higher realized total returns. Importantly, all mean CVaR–optimized portfolios provide improved risk–return profiles, each outperforming the benchmark portfolio showing positive alphas along with higher corresponding Sharpe ratios of around 0.65.

Next, we add Step 3 to our process by incorporating the signal derived from our Markov-switching risk model that identifies the current market state as being in either a high-risk or low-risk environment. We estimate our model under the expanding-window approach with daily data beginning on February 1, 1996, with our first-regime risk estimate occurring on January 3, 2000. If the risk model output suggests that the current environment is low-risk (high-risk) measured as less (more) than a 50% likelihood of being in a high-risk state, then we implement a risk-on (risk-off) strategy and optimize portfolio weights allowing for a CVaR risk target of 7% (3%), respectively. As before, expected

Exhibit 10

Optimization Performance Summary

Panel A: Unbounded Optimization

		Info Ratio	Alpha	Tracking Error	Sharpe Ratio	Return/ MaxDD	MaxDD	Worst 21-Day Loss	Annual Return	Annual Volatility	Hit Ratio
	Benchmark				0.24	0.11	42.83%	15.99%	4.84%	14.26%	
No leverage, no shorting	**Constant Target CVaR**										
	3%	0.23	2.91%	12.56%	0.67	0.39	19.73%	13.00%	7.75%	8.32%	49.65%
	4%	0.48	4.25%	8.78%	0.67	0.33	27.62%	15.52%	9.09%	10.56%	53.90%
	5%	0.65	4.81%	7.36%	0.63	0.29	33.22%	16.34%	9.65%	12.45%	56.03%
	6%	0.78	5.47%	7.05%	0.61	0.27	37.71%	16.97%	10.31%	14.11%	65.96%
	7%	0.79	5.68%	7.20%	0.59	0.26	40.90%	17.53%	10.52%	15.29%	63.83%
	Regime-Based Target CVaR (Same Rebalancing Conditions)										
	3%, 7%	0.44	4.90%	11.09%	0.68	0.49	19.73%	14.84%	9.74%	11.39%	60.28%

Panel B: Bounded Optimization

Regime-Based Target CVaR (Same Rebalancing Conditions)

		Info Ratio	Alpha	Tracking Error	Sharpe Ratio	Return/ MaxDD	MaxDD	Worst 21-Day Loss	Annual Return	Annual Volatility	Hit Ratio
Bound	3%, 7%	0.37	1.40%	3.74%	0.37	0.18	35.36%	13.20%	6.24%	12.54%	59.57%

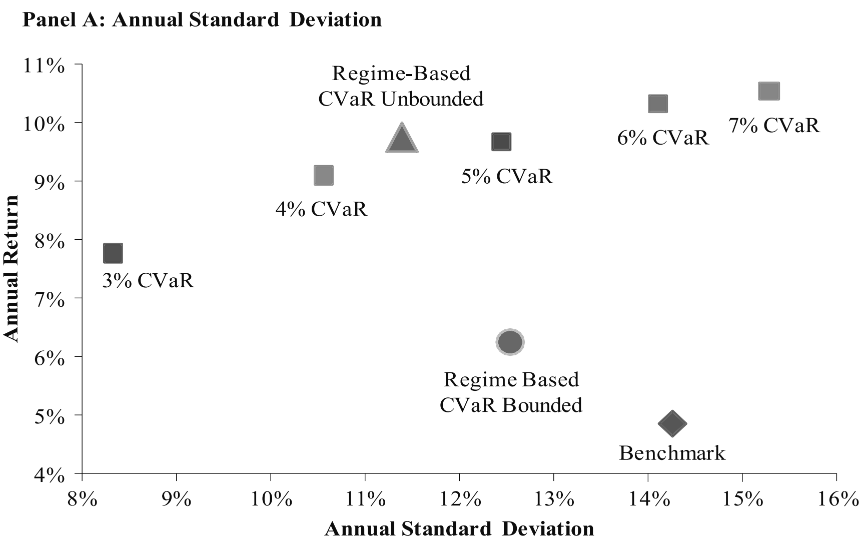
return assumptions are the same for each state, and the optimization techniques are the same as those used in the constant target CVaR process. The only difference is that we now allow the target portfolio to change its risk profile to either risk-on (7% CVaR) or risk-off (3% CVaR) in order to reflect our dynamic forecast of market risk.

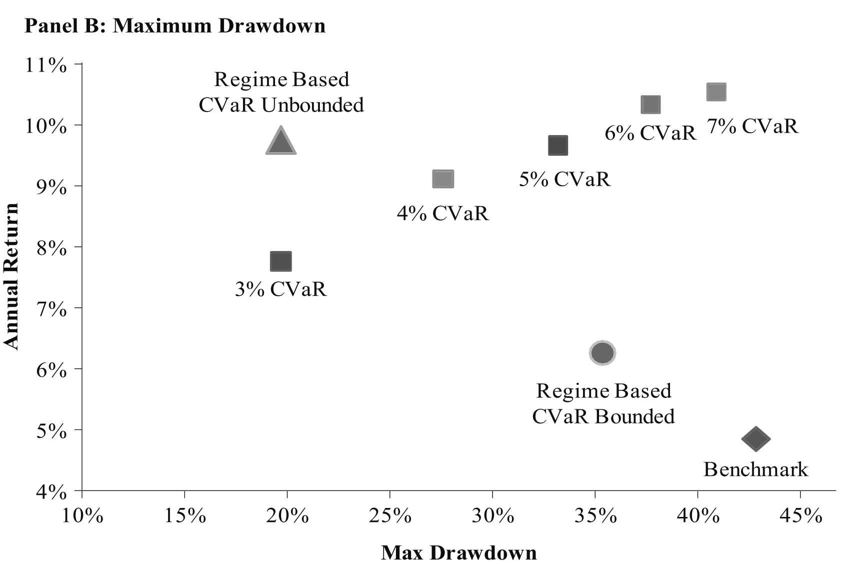
In this approach, we simply use the same rebalancing conditions as the constant CVaR process (i.e., same rebalancing dates and the constant 21-day rebalancing period). On rebalancing days, we choose the target CVaR for the upcoming 21-day period based on the prior day's market risk regime signal. Results show that incorporating the two-state market risk forecast meaningfully improves results over the benchmark and the constant CVaR approach. With this risk-on/risk-off framework, we are better able to capture a meaningful part of the upside that markets have to offer while also reducing the downside. This approach represents considerable improvement over the rebalanced static benchmark and also the various static levels of CVaR. As evidence, consider that for the risk-on/risk-off model the Sharpe ratio rises to 0.68 whereas the maximum drawdown is 19.73%, about half that of the benchmark. We note that this max drawdown is equivalent to that calculated under the 3% CVaR portfolio, but now captures much of the upside afforded by the risk-on days.[8]

Exhibit 11 summarizes our results by plotting the risk–return relationships of the various portfolios for which we define risk in terms of standard deviation and the more relevant definition of maximum drawdown. Overall, results reflect the view that our CVaR tail-risk framework offers a highly relevant risk measurement approach for investors. All CVaR-related portfolios dominate the rebalanced static benchmark. Even the 3% CVaR, our lowest static risk portfolio, offered an appreciable excess return with far less risk than the benchmark. Adding a regime-based risk-on/risk-off dynamic process enhances the performance even further. The regime-based optimization process outperforms the constant-risk target allocations with significantly improved return/maximum drawdown ratios.

Overall, our high-frequency signal-triggered rebalancing framework offers a high degree of sensitivity of portfolio performance to market risk regime changes. Put differently, our flexible approach offers meaningful improvement in port-

EXHIBIT 11
Portfolio Risk and Return Comparison





folio performance. We discuss the bounded portfolio process later in the article. We offer these approaches as examples to allow readers a robust comparison of how various regime-based strategies might perform over time. Finally, with each of our approaches, we are sensitive to keeping transaction costs associated with a high frequency of signals low, an issue facing many such dynamic frameworks.

Exhibit 12 shows the corresponding portfolio exposures of our five asset classes over time from our unbounded risk-on/risk-off regime-based approach that rebalances every 21 days. The exhibit shows the wide variation in asset allocation weightings generated by this approach. Many will consider the unbounded model presented here too demanding because it dictates dramatic shifts in portfolio asset allocation over time.

We appreciate the concern associated with such wide swings in asset weightings associated with portfolio rebalancing. To mitigate the wide swings in asset allocation associated with our unbounded approach, we next impose constraints on the range of allowable portfolio weights. Specifically, we test a constrained portfolio that allows the range of portfolio weights to vary only as much as indicated in Exhibit 1. This bounded portfolio framework we test follows the 21-day rebalancing approach discussed earlier. Results show that performance versus a rebalanced static-weighted benchmark can benefit from our dynamic risk-modeling framework even when imposing target portfolio constraints as typically done by many investors. As shown in Exhibit 11, Panels A and B, the bounded model also offers considerable improvement in both

Exhibit 12

Optimal Portfolio Weights: Regime Based (3% and 7% Target CVaR)

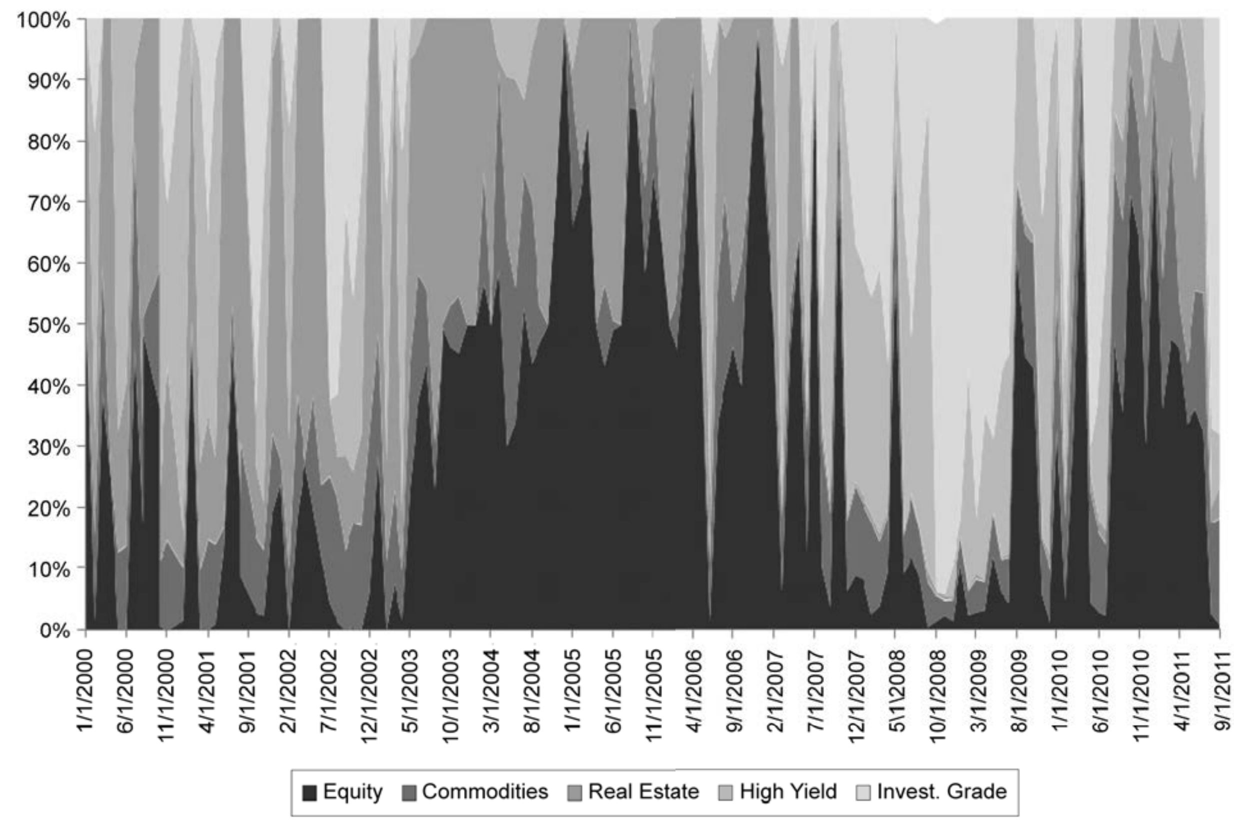
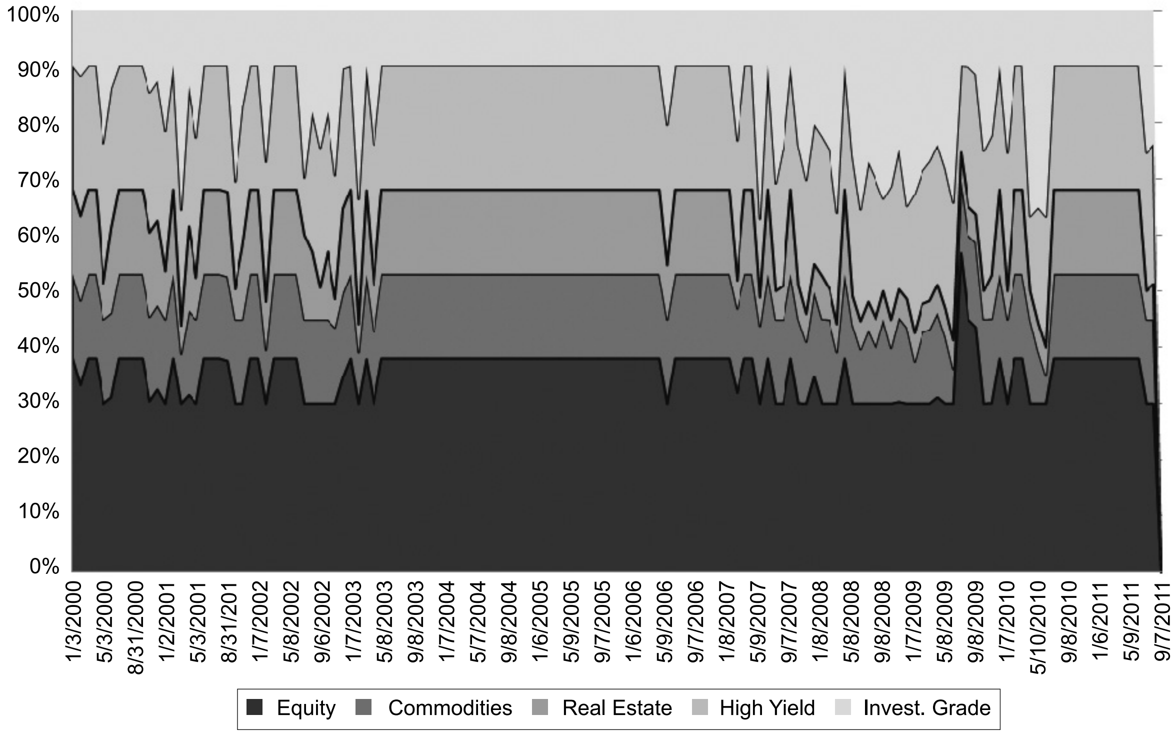


Exhibit 13

Optimal Portfolio Weights: Bounded Portfolio



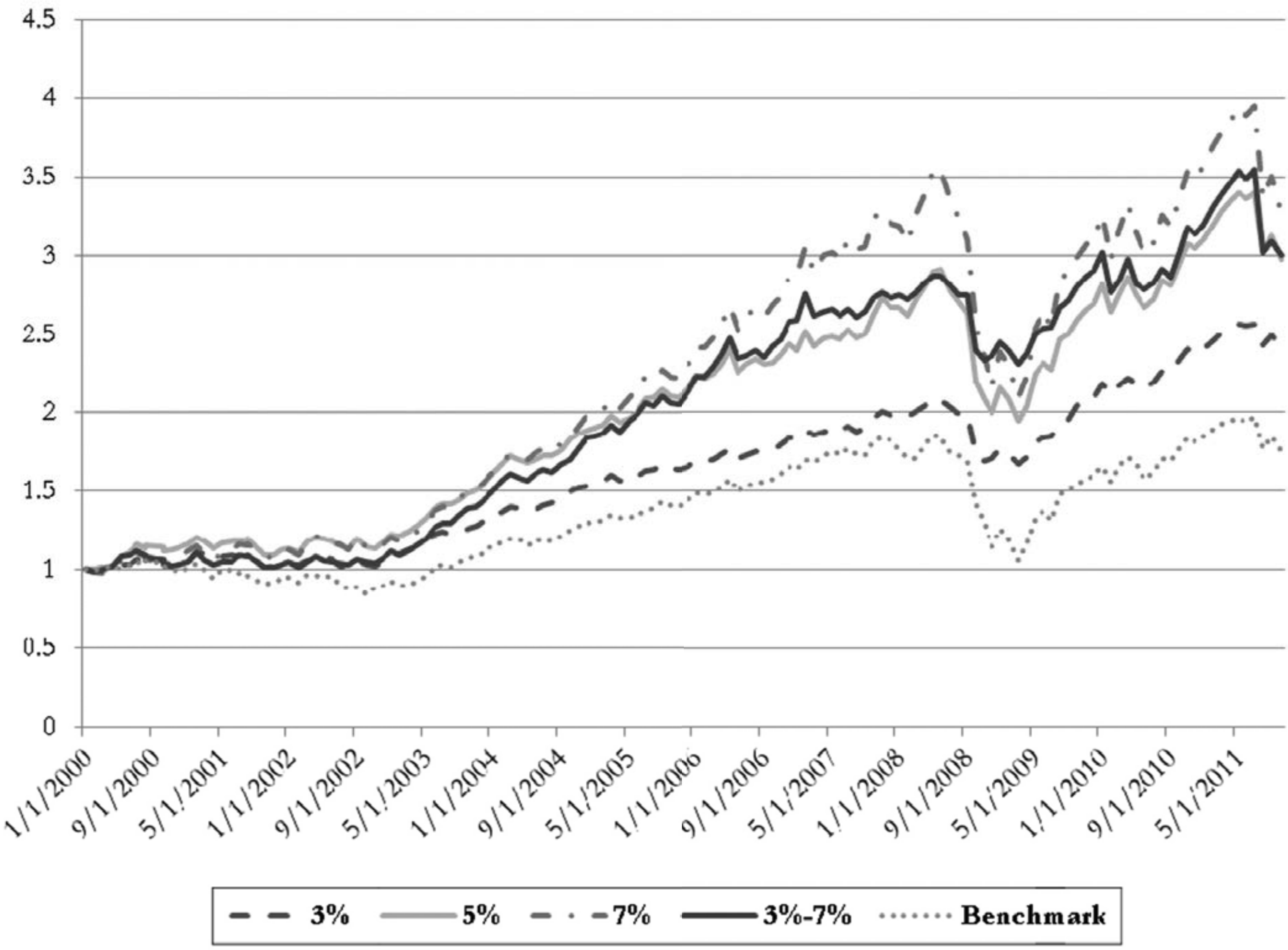
risk and return versus the rebalanced static benchmark. Exhibit 13 shows the corresponding portfolio exposures over time for each of the five asset classes associated with the bounded risk-on/risk-off regime-based approach. As expected, it differs markedly from Exhibit 12. We note that during the global financial crisis of 2008–2009, given the minimum allowable allocation to risky assets, the model is unable to consistently achieve the desired 3% CVaR associated with a risk-off regime. This simply means that we are not always able to obtain the portfolio risk limits imposed when using a constrained approach with sizable minimum-allowable allocations to risky assets.

Exhibit 14 shows the total cumulative returns to the various rebalancing approaches: benchmark; static allocations with 3%, 5%, and 7% constant target CVaR; and our regime-based allocation that switches between 3% and 7% target CVaR under the same rebalancing conditions as the static allocations. This exhibit offers visual evidence that our regime-based risk framework offers investors a meaningful approach to portfolio construction in the presence of fluctuating market risk.

CONCLUSIONS

We propose a dynamic portfolio construction model that accounts for the reality of heavy tails and dynamic return correlations as witnessed in markets. The powerful framework behind our portfolio construction is a dynamic process that integrates high-frequency information to capture the time-varying risks of asset classes within the investor's portfolio. We use our dynamic risk information to adjust optimal asset allocation across time and market states using only information known at the time of model implementation. We find that ongoing monitoring of markets using our market risk barometer and corresponding asset allocation framework offers investors the promising opportunity to improve portfolio performance in challenging market environments.

EXHIBIT 14
Cumulative Portfolio Returns



ENDNOTES

We thank Michael Barry, John Hall, Xi Li, James Xiong, and the team at Georgetown University Investment Office for their valuable comments and assistance.

[1]For further discussion on this topic, see Sullivan [2008].

[2]Readers are referred to Embrechts, Klüppelberg, and Mikosch [1997] for a comprehensive treatment of extreme value theory.

[3]CVaR measures the expected loss during a given period at a certain confidence level. As a better alternative to VaR, it incorporates both the possibility and expected magnitude of loss. Moreover, it is coherent and convex and can readily be incorporated into a discrete optimization process in risk management (Rockafellar and Uryasev [2000]). For example, a 95% 21-day CVaR of 20% means the investor expects to lose 20% within the 5% worst-case scenarios in a month. CVaR is known as mean excess loss for continuous distributions and is defined as the weighted average of VaR and losses strictly exceeding VaR for discrete distributions.

[4]We note that our 95% CVaR results are similar when we measure CVaR at a 99% confidence interval but require fewer iterations to converge.

[5]Although the 50% threshold may seem somewhat arbitrary, we note that given the binary nature of our high-market-risk probability forecast, our results are not dependent on our chosen threshold.

[6]Furthermore, the maximum and minimum daily returns always occurred in the high-volatility-event regime suggesting that investors might benefit from a regime model that can correctly distinguish a third regime for high-return periods.

[7]An extensive literature on advanced portfolio optimization techniques exists, including, for example, Fabozzi et al. [2007] and Rachev, Stoyanov, and Fabozzi [2008].

[8]Our conclusions are unaffected when backtesting other rebalancing definitions.

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