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**Pathetic Protection: *The Elusive  
Benefits of Protective Puts***

Roni Israelov



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# Pathetic Protection: *The Elusive Benefits of Protective Puts*

RONI ISRAELOV

**RONI ISRAELOV**  
is a principal at AQR  
Capital Management in  
Greenwich, CT.  
[roni.israelov@aqr.com](mailto:roni.israelov@aqr.com)

Any investor would like to maximize upside participation while mitigating losses. These unsurprising preferences have given rise to a liquid insurance market in the form of equity index options. A put option, when combined with an equity position, is designed to limit losses while maintaining unbounded gains. A call option is designed to achieve the same outcome standalone, effectively bundling a long put option with a long equity position.<sup>1</sup>

Exhibit 1 plots a familiar example payoff diagram for a protective strategy, one that purchases a put option for \$1 at a \$95 strike price.<sup>2</sup> The protected strategy's *minimum* value at expiration is \$94, and it moves one for one with the index when the index is above the \$95 strike price. If the index value is \$100 when the investor purchases the put option, the protected portfolio cannot lose more than \$6 over the option's holding period.

*This option property is so clear and straightforward that protective put options are often the gold standard against which other tail protection strategies are measured.*

Great attention has been paid to the cost of protective put options.<sup>3</sup> But what about

their benefits? Are they an effective tail hedge?<sup>4</sup> For those who have time and time again seen payoff diagrams such as that shown in Exhibit 1, this may seem a ridiculous question. Put options are the gold standard after all. In this article, I ask this question and demonstrate that the protection put options provide is often, well, pathetic.

Even if crash risk is not priced—that is, there is no volatility risk premium—the protective benefits of put options are uninspiring. Add in some volatility risk premium and buying

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insurance. Gârleanu, Pedersen, and Poteshman (2009) show how natural demand for put options can give rise to a volatility risk premium in their demand-based option pricing model. Also see Bakshi and Kapadia (2003), Ilmanen (2012), Israelov and Nielsen (2015a,b), and Israelov, Nielsen, and Villalon (2017).

<sup>4</sup>Figlewski, Chidambaran, and Kaplan (1993) evaluate, via simulation, how buying monthly put options alters the distribution of annual returns. In their simulations, options do not price crash risk (i.e., there is no volatility risk premium). Investigating fixed strike, fixed percentage strike, and ratcheting strike strategies, they conclude that buying put options does not significantly improve the left-tail of one-year returns. My analysis focuses on peak-to-trough drawdowns over different holding periods and on single-day market crashes rather than one-year returns. For example, I find that buying 20-day put options does not significantly improve peak-to-trough drawdowns over 20-day holding periods. I also analyze the impact of the volatility risk premium on hedging efficacy, both in simulation, and in a real-world implementable protection strategy, as proxied by the Cboe S&P 500 5% Put Protection Index.

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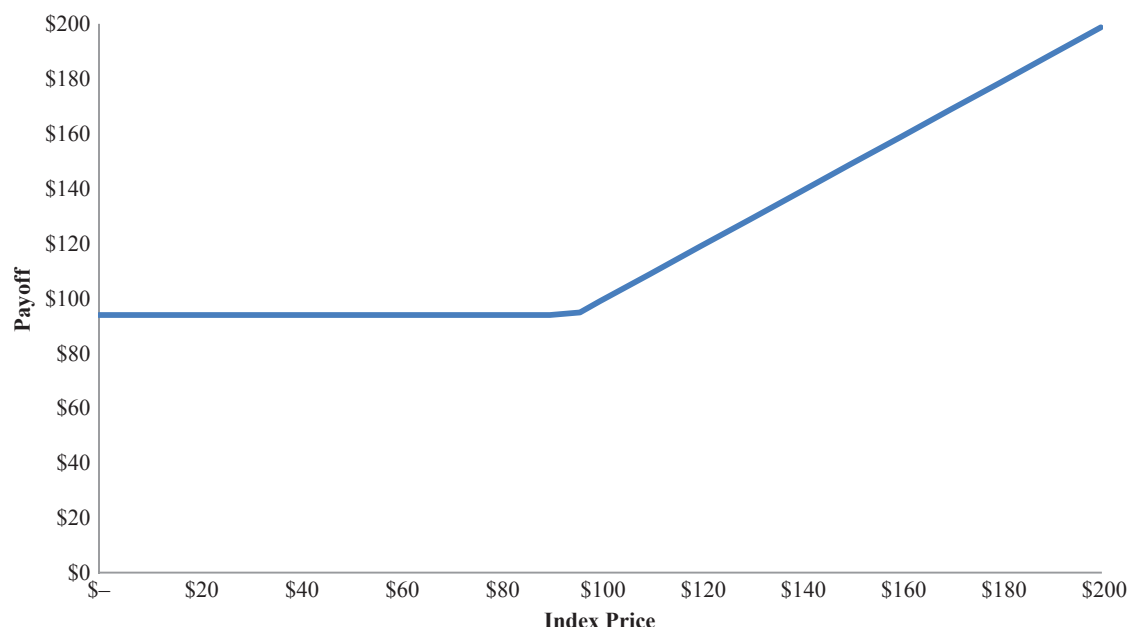
<sup>1</sup> This relationship holds due to the no-arbitrage restriction commonly referred to as put-call parity.

<sup>2</sup> For parsimony, the put option is financed using the equity as collateral.

<sup>3</sup> Equity index options are typically priced in equilibrium with a volatility risk premium that compensates option sellers for underwriting financial

## EXHIBIT 1

### Illustrative Payoff Diagram for a Protected Strategy



Note: Illustrative payoff diagram purchases a \$95 strike put option for \$1 when the index price is \$100.

Source: AQR. For illustrative purposes only.

put options often does more harm than good. Portfolios protected with (expensive) put options have *worse* peak-to-trough drawdown characteristics *per unit of expected return* than portfolios that have instead simply statically reduced their equity exposure in order to reduce risk.<sup>5</sup> This means investors who reduce their positions will likely achieve better outcomes than those who purchase protection. For example, I find that the strategy that invests 40% in equity and 60% in cash has delivered similar returns as the protected strategy, but with less than half the volatility and significantly improved peak-to-trough drawdowns.

How is it possible that an option with such a well-defined limited loss profile, as shown in Exhibit 1, can fail us? A put option's protective armor is nearly impenetrable over drawdowns that coincide with its option expiration cycle. Unfortunately, equity drawdowns have lives of their own that may not conveniently coincide with option expiration cycles. In these cases, the put option's protective armor is easily penetrated.

<sup>5</sup>Note that static divestment differs from the option-replicating, dynamic trading strategy that is typically referred to as "portfolio insurance."

Buying an equity put option reduces equity exposure. In that regard, it is similar to divestment. Both actions reduce risk and consequently expected return. Where the two approaches differ is that the put option introduces time-varying equity exposure, which is intended to help reduce tail exposure. And the put option may price crash risk (volatility risk premium), which may further reduce realized returns.

Time-varying equity exposure adds risk,<sup>6</sup> and a put-protected equity position is more volatile than an equity position that is sized to match the beta of the put protected portfolio. The protected position is less negatively skewed than the divested position, but its increased volatility is unhelpful, and unlike the divested position, the protected position is subject to path-dependent outcomes.<sup>7</sup>

<sup>6</sup>See Israelov and Nielsen (2015a).

<sup>7</sup>The path-dependent outcome may at times be desirable because the protection is naturally de-levering the equity exposure during a drawdown. But then the put option expires and the equity exposure is reset up to a higher level, even though the drawdown may continue to worsen.

In this article, I test the hedging properties of put protection strategies and compare them with the straightforward risk-reducing alternative that statically divests the equity position. I begin by showing in the first section, through a hypothetical illustration, how path-dependent outcomes can circumvent a put option's intended downside protection. The culprit is the misalignment of the option protection cycle with the draw-down period.

Having shown how put protection can fail to protect, albeit in a contrived illustration, I continue by testing, in the second section, a real-world implementable protection strategy as proxied by the Cboe S&P 500 5% Put Protection Index. The primary goal of a protection strategy is to allow investors to earn their desired returns with improved peak-to-trough drawdowns. I show that a divested equity strategy has significantly outperformed the put protection index in this regard. A secondary motivation for protection strategies is to achieve greater upside participation. I test this by measuring trough-to-peak "drawups" and find that the put protection strategy is successful in this regard. These two results are easily reconciled. Protecting with put options has led to a significantly lower Sharpe ratio than has divesting. The same return is earned with more volatility, leading to both larger downside and larger upside outcomes despite the strategy's asymmetric exposure to equity markets.

The real world is messy. Equity prices are not log-normal, they may exhibit periods of trend or reversal, volatility is stochastic, there is a volatility risk premium, and the volatility risk premium is also stochastic. We learn about protective puts in an idealized setting. It is worth testing their hedging efficacy in a similarly idealized environment. I turn to Monte Carlo simulations to do so. In the third section, I begin by testing the super-idealized scenario in which crash risk is unpriced to demonstrate how damaging misalignment of the option premium cycle and the drawdown period can be, expanding on the contrived illustrative example presented earlier. I then revisit my peak-to-trough and trough-to-peak analyses in the simulated environment and find the benefits to protecting versus divesting to be marginal at best.<sup>8</sup>

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<sup>8</sup>Evaluating the impacts that non-lognormal prices, price trends and reversals, stochastic volatility, and stochastic volatility risk premia have on the hedging efficacy of put options is out of

We know that crash risk is priced—there exists a volatility risk premium and options tend to be expensively priced. I continue my analysis by incorporating this important reality into my simulations in the fourth section. The simulations provide additional evidence supporting the findings from the analysis that used the Cboe 5% Put Protection Index. When options are richly priced, protecting is a much riskier approach to earning a unit of return than divesting. Protecting has both more painful peak-to-trough drawdowns as well as more enjoyable trough-to-peak rallies.

There are many possible constructions of a protective put overlay due to the large universe of options available that span both the strike and maturity dimension—too many to fully consider within the scope of this article. I look into the role that option maturity plays in protection efficacy in the fifth section. I find that the quality of protection improves (or more accurately, is less bad) when option maturity is most closely aligned with the length of the peak-to-trough drawdown cycle. For example, monthly options do a less poor job of protecting against drawdowns that last about a month than those that last about a year. Unfortunately, investors cannot know *ex ante* how long future peak-to-trough drawdowns will last, but understanding the drawdown horizons that they are most concerned about rather than the horizons that are more likely to occur can offer some guidance.

Finally, and importantly, I consider protection efficacy against sudden (one-day) equity crashes in the sixth section. Options are convex instruments, and they naturally and automatically reduce equity exposure as markets crash. Static (and dynamic) divestment strategies do not. I find that protection against extreme market crashes, even when options are realistically pricing crash risk, is where buying options shines, on average, against divesting. However, path dependence continues to play a role and the crash protection benefits of vanilla options are uncertain. Those who specifically desire crash protection and are willing to reduce their expected returns to pay for them may be better served by considering more complex convex instruments that have

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the scope of this article. Future work in this area could investigate the impacts these characteristics have on the effectiveness of option hedges.

less path-dependent exposures, such as variance swaps or option cliquets.<sup>9</sup>

Ultimately, investors who are evaluating put protection against divestment must determine what they are most concerned with: the infrequent sudden extreme market crash or the more common protracted drawdown.

## A HYPOTHETICAL ILLUSTRATION OF FAILURE TO PROTECT

The most liquid equity index options typically expire on the third Friday each month. As previously described and depicted in Exhibit 1, an investor who purchases a 5% out-of-the-money put option for a price of 1% of net asset value (NAV) on the third Friday of the month and holds it until expiry will have a maximum loss of 6% over that precise holding period.

Let us consider what happens for an investor who serially protects their portfolio, when their option expires in the midst of an equity drawdown that begins at month end and ends one month later.

Exhibit 2 illustrates the scenario. Options expire mid-month at times  $t$  (in months) =  $-0.5$ ,  $0.5$ ,  $1.5$ , and so on. At the beginning of the scenario (time  $t = -0.5$ ), the stock price is \$100. An investor with a NAV of \$100 purchases one share of stock and finances the purchase of a \$1 put option that is \$5 out of the money (strike = \$95).<sup>10</sup> One month later (time  $t = 0.5$ ), the stock is down \$5 to \$95 and the option expires worthless. The investor repeats the protective put process, buying another \$5 out of the money put option for \$1. Again, at option expiration (time  $t = 1.5$ ), the stock price is down \$5 and the option expires worthless. Over the two option expiration cycles, highlighted in blue and red in the top panel of Exhibit 2, the stock was down 10% and the portfolio was down 12%.

Our investor is particularly interested in the tail risk of calendar month returns. This period, from time  $t = 0.0$  to  $t = 1.0$ , is highlighted in purple in the lower

panel of Exhibit 2. Over this period, the stock price declined 14.6%, from \$103 at time  $t = 0.0$  to \$88 at time  $t = 1.0$ . The first put option did little to help during the first half of the drawdown because it was \$8 out-of-the-money at the end of the month (time  $t = 0.0$ ) with a price of \$0.25, bringing the investor's NAV to \$102.25, and then expired worthless at time  $t = 0.5$ . The second put option offered some protection because it was \$2 in the money at the end of the following month (time  $t = 1.0$ ) with a price of \$3. Over this calendar month, the investor's NAV dropped by 12.9%.

Even though the investor was always protected with a put option and purchased monthly put options 5% out of the money, she lost nearly 13% over the calendar month, and the put option only protected 11% of the stock's losses over the coinciding period. This example illustrates that the path dependence of the stock's returns in relation to the initiation and expiration dates of the option position clearly plays a large role in determining the effectiveness of protective puts.

## THE Cboe S&P 500 5% PUT PROTECTION INDEX

To test real-world hedging efficacy, I investigate the Cboe S&P 500 5% Put Protection Index (PPUT), which systematically purchases monthly put options that are 5% out of the money.<sup>11</sup> The strategy uses short-dated, renewing put options to reduce downside risk. The analysis begins on July 1, 1986 and ends on May 19, 2016.

I compute 21-day overlapping returns in excess of three-month LIBOR for the PPUT and the S&P 500 Total Return Index (SPX) and report the regression of the former on the latter:<sup>12</sup>

<sup>11</sup> Chicago Board Options Exchange describes the Cboe S&P 500 5% Put Protection Index as follows: The Cboe S&P 500 Put Protection Index (PPUT) is a benchmark index designed to track the performance of a hypothetical risk-management strategy that consists of a long position indexed to the S&P 500 Index (SPX Index) and a rolling long position in monthly 5% out-of-the-money (OTM) SPX put options.

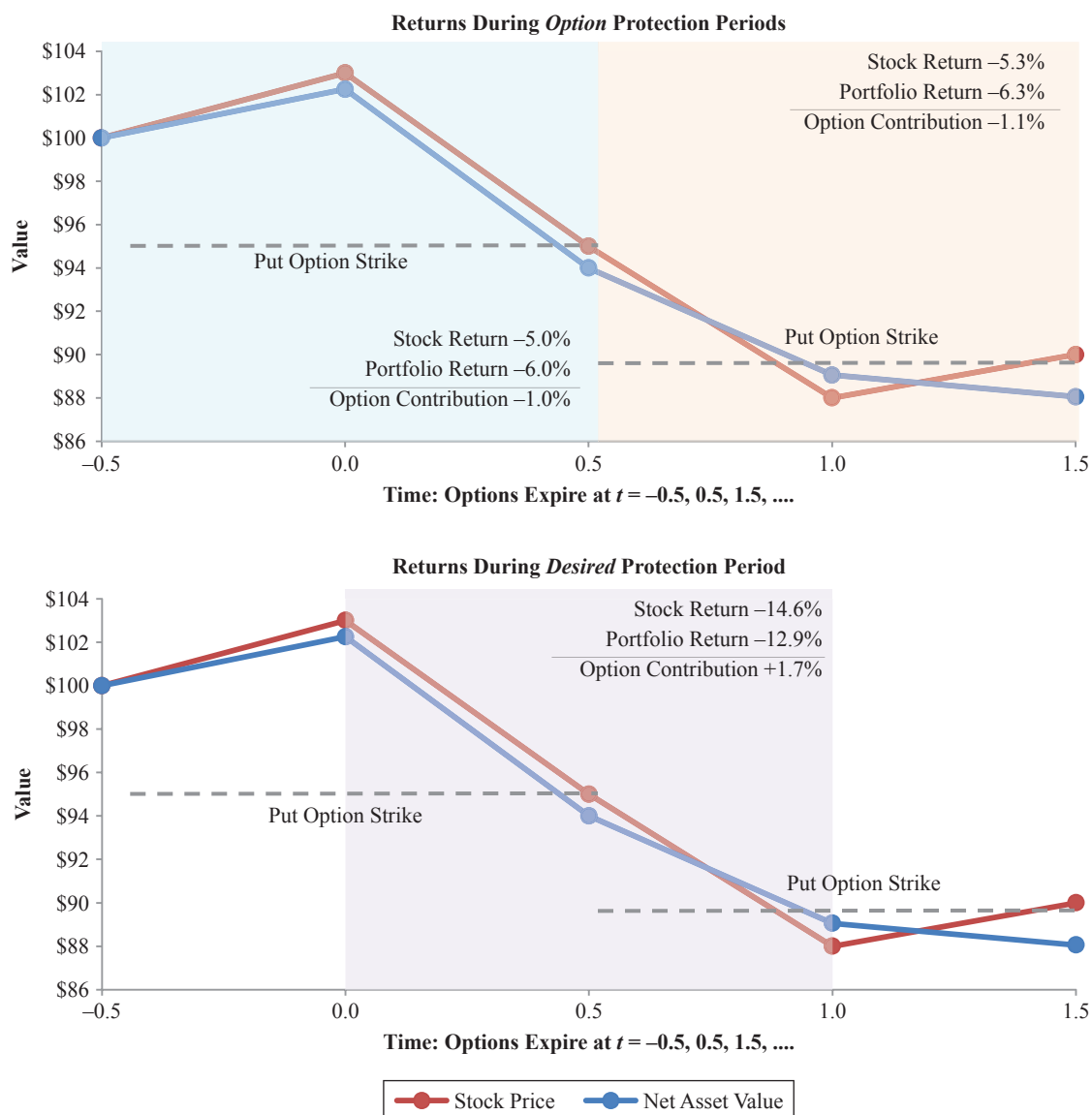
<sup>12</sup> In order to deal with nonsynchronicity, I regress using 21-day overlapping returns and report Newey–West adjusted  $t$ -statistics. The regression may be thought of as identifying the “passive equity” exposure identified in Israelov and Nielsen (2015a). The regression residual (combined with the intercept) is the return attributable to the combined “long volatility” and “dynamic equity” exposures.

<sup>9</sup> A cliquet option is a basket of forward start options. The strike of each forward start option is determined when the preceding option expires. An example would be a 10% out-of-the-money six-day cliquet. The buyer of this option is protecting against one-day crashes of greater than 10% over a period of six months.

<sup>10</sup> For the purpose of this illustrative example, the put option is priced with an approximately 22.5% annualized volatility and the financing rate is 0%.

## EXHIBIT 2

### A Hypothetical Illustration of “Failure” to Protect



Source: AQR. For illustrative purposes only.

$$r_{\text{protected},t} = -15 \text{ bps} + 0.74r_{\text{equity},t} \quad R^2 = 0.85$$

The long put options reduce the portfolio's equity exposure by about a quarter and have  $-1.8\%$  of annualized alpha (the  $-15$  basis points in the regression is a monthly alpha) with a  $-2.0$   $t$ -statistic. This alpha is consistent with the findings of Israelov and Nielsen (2015b), who report a  $-2.0\%$  annualized return for

owning delta-hedged 5% out-of-the-money put options over the period from March 1996 through June 2014.

Over the sample period, SPX realized  $5.8\%$  annualized geometric returns in excess of cash versus  $2.5\%$  for PPUT. Often times, protected strategies are compared with their fully invested unprotected counterparts. But comparing the drawdown characteristics of a protected strategy to another strategy that has 130% higher average returns can lead to incorrect conclusions.



Investing 36.5% in SPX and holding 63.5% in cash provided the same 2.5% compound annualized excess return as PPUT.<sup>13</sup> This is such an astounding result that it bears repeating: *Investing 36.5% in the S&P 500 Index and holding 63.5% in cash provided the same 2.5% compound annualized excess return as the Cboe S&P 500 5% Put Protection Index.*

To keep the analysis apples to apples in terms of realized returns, I compare the protected strategy with one that invests 36.5% of NAV in SPX and 63.5% of NAV in cash. Throughout this article, I refer to this portfolio as the *divested* portfolio.

Exhibit 3 scatter plots the daily returns of the Cboe S&P 500 5% Put Protection Index against those of the S&P 500 Index. The protection strategy's reduced equity exposure and convexity is visible. Both its losses and its gains are smaller in magnitude than those of the S&P 500. That may appear promising, but remember that the protected strategy has less than half the average return of the equity index. The bottom panel scatter plots the daily returns of the put protection index against the divested portfolio. The outcome has flipped. The protection strategy's losses and gains are generally greater in magnitude than those of the divested strategy, despite having the same compounded return.

## Downside Protection

Tail protection strategies are most effective if they can meaningfully reduce peak-to-trough drawdowns. I now investigate how well protective put options achieve this objective. I compute peak-to-trough drawdowns over rolling overlapping windows of the following sizes: 5, 10, 20, 63, 125, and 250 business days.

Exhibit 4 reports peak-to-trough drawdowns at the 1st, 5th, 10th, 25th, and 50th percentiles, and Exhibit 5 plots the empirical probability density functions. I report results over the different window lengths for the *protected equity* portfolio and the daily-rebalanced *divested equity* portfolio. Having sized the two approaches to provide the same expected return, I can fairly compare their drawdown characteristics. Note that this is an *ex post* performance analysis.

The divested portfolio nearly universally has better drawdown characteristics than does the protected portfolio. For example, the worst 1% peak-to-trough

drawdowns over a 20-day period are -9.6% for the protected portfolio versus -6.6% for the divested portfolio. Arguably, investors should be more concerned about longer-term drawdowns. Over 250-day windows, the results are even worse for protection: -32.1% for the protected portfolio and -20.9% for the divested portfolio.

Exhibit 5 shows that PPUT is significantly more likely to have larger peak-to-trough drawdowns than the divested portfolios. This finding holds across the wide range of measurement windows I consider. Exhibit 6 plots the percentage of time that divesting had better peak-to-trough drawdowns than protecting. Over the shortest evaluation windows, divesting won 97% of the time. Over periods greater than about half a year, divesting won 100% of the time. This is a pathetic outcome for the put protection strategy.

## Upside Participation

The typical impetus for buying put options for tail risk protection is to preserve upside participation while reducing downside exposure. Previously, I showed that the protective put index has exacerbated drawdowns (per unit of earned return). How does the protected approach fare in terms of upside participation?

To test, I follow a similar framework as in the peak-to-trough analysis, except that I analyze trough-to-peak returns instead. Exhibit 7 reports trough-to-peak drawups at the 99th, 95th, 90th, 75th, and 50th percentiles and Exhibit 8 plots the probability density functions.

Things look brighter for the protective put index in terms of upside participation. The protected portfolio handily wins the race. The 99th percentile trough-to-peak equity rally over a 20-day period is 11.7% for the protected portfolio versus 5.4% for the divested portfolio. Over 250 days, the protected portfolio's 99th percentile rally is 36.7% versus 20.3% for the divested portfolio.

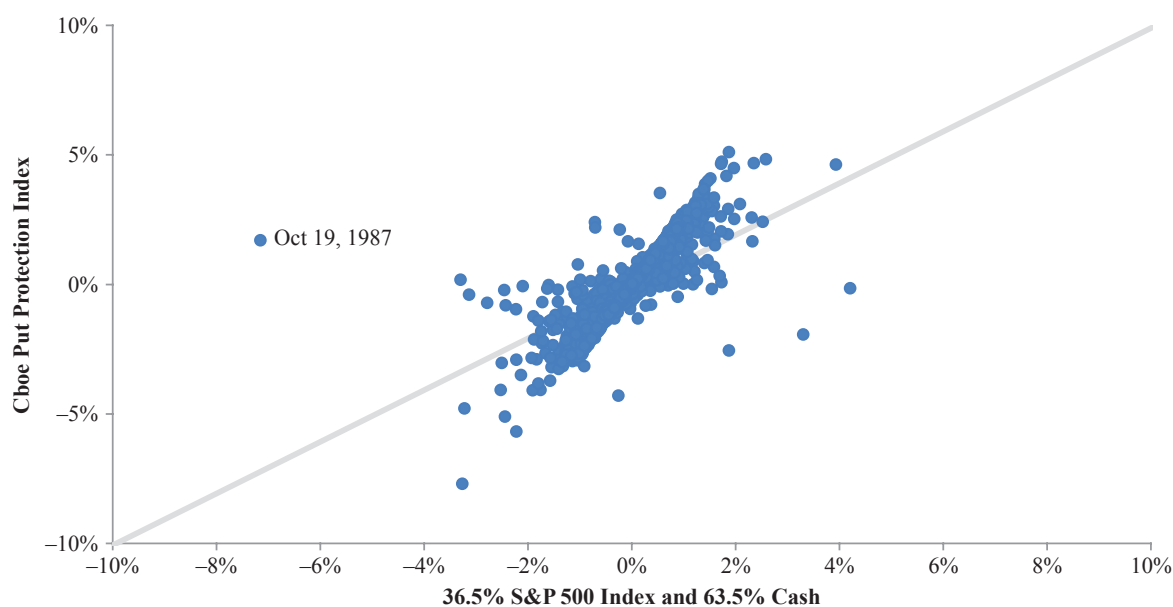
What drives these stark differences in downside risk and upside participation? Beta. The divested portfolio has half of the 0.74 beta of the protected portfolio. This 0.37 difference in beta nearly assures that the protected portfolio will underperform during drawdowns and outperform during rallies. Because of this large difference in equity exposure, the protected portfolio is significantly more volatile. Its annualized volatility is 13.5% versus 6.6% for the divested portfolio. The divested portfolio earns the same return as the protected portfolio with about half the volatility.

<sup>13</sup>This 36.5% allocation is lower than  $2.5/5.8 = 43\%$  because of the benefits of reduced volatility drag on compounded returns.



## EXHIBIT 3

### Cboe S&P 500 5% Put Protection Index Daily Returns



Note: Results shown over period July 1, 1986 to May 19, 2016.

Sources: Bloomberg and Chicago Board Options Exchange. For illustrative purposes only.

#### Ex Ante Divestment

The preceding analysis is *ex post*, and it strongly suggests that divestment is preferable to protection. But an investor needs to determine how much to divest

*ex ante*. How does an implementable divestment strategy fare relative to protection?

I consider the following simple illustrative approach. The divested strategy's exposure to the S&P 500 is equal to the expanding average *delta* of the

## EXHIBIT 4

### Peak-to-Trough Drawdowns (Cboe Put Protection Index vs. Divested S&P 500)

		Percentiles				
		1st	5th	10th	25th	Median
5-Day Drawdown	Protected	-5.1%	-3.5%	-2.9%	-1.9%	-1.0%
	Divested	-3.1%	-1.7%	-1.3%	-0.8%	-0.4%
	Improvement	-2.0%	-1.9%	-1.6%	-1.1%	-0.6%
10-Day Drawdown	Protected	-7.4%	-5.1%	-4.3%	-3.0%	-1.8%
	Divested	-4.4%	-2.5%	-1.9%	-1.2%	-0.7%
	Improvement	-3.0%	-2.6%	-2.3%	-1.8%	-1.1%
20-Day Drawdown	Protected	-9.6%	-7.4%	-6.3%	-4.5%	-2.9%
	Divested	-6.6%	-3.7%	-2.8%	-1.8%	-1.1%
	Improvement	-2.9%	-3.8%	-3.5%	-2.6%	-1.7%
63-Day Drawdown	Protected	-17.1%	-13.2%	-11.3%	-8.2%	-5.3%
	Divested	-12.4%	-7.3%	-5.1%	-3.3%	-2.1%
	Improvement	-4.8%	-5.9%	-6.3%	-4.9%	-3.2%
125-Day Drawdown	Protected	-22.8%	-19.0%	-16.4%	-10.9%	-7.3%
	Divested	-17.1%	-10.7%	-7.6%	-4.4%	-2.7%
	Improvement	-5.7%	-8.3%	-8.8%	-6.5%	-4.6%
250-Day Drawdown	Protected	-32.1%	-26.2%	-22.6%	-15.7%	-9.8%
	Divested	-20.9%	-13.5%	-12.3%	-6.9%	-3.6%
	Improvement	-11.3%	-12.7%	-10.3%	-8.8%	-6.2%

Note: Results shown over period July 1, 1986 to May 19, 2016.

Sources: AQR, Bloomberg, and Chicago Board Options Exchange. For illustrative purposes only.

PPUT.<sup>14</sup> The expanding average delta is a backward-looking estimate of the protected strategy's long-term expected exposure to equities.<sup>15</sup> This approach differs from the preceding analysis in that it does not seek to match expected returns because the expected equity and volatility risk premia returns were not known *ex ante*.

<sup>14</sup>The one-month, 5% out-of-the-money put option delta was calculated using Black-Scholes. We used the Cboe VXO Index (from Bloomberg) for implied volatility and three month USD LIBOR (from Bloomberg) as the interest rate. The dividend yield was assumed to be 2%. The VXO Index tracks the 30-day, at-the-money implied volatility of S&P 100 Index options. The more commonly used VIX Index tracks the 30-day implied volatility of S&P 500 options, averaged across a large set of strikes. Despite the difference in underlying equity indexes and strikes used to compute implied volatility, the two volatility indexes are 0.99 correlated. I use the VXO Index to allow for an extended sample of analysis. The VXO Index began in 1986 versus the VIX Index, which began in 1990.

<sup>15</sup>An alternative approach would have the divested strategy match the point-in-time estimated delta of the protected strategy at the time of the option roll. This approach would have the divested strategy match the protected strategy's expected short-term exposure to equity rather than its long-term exposure.

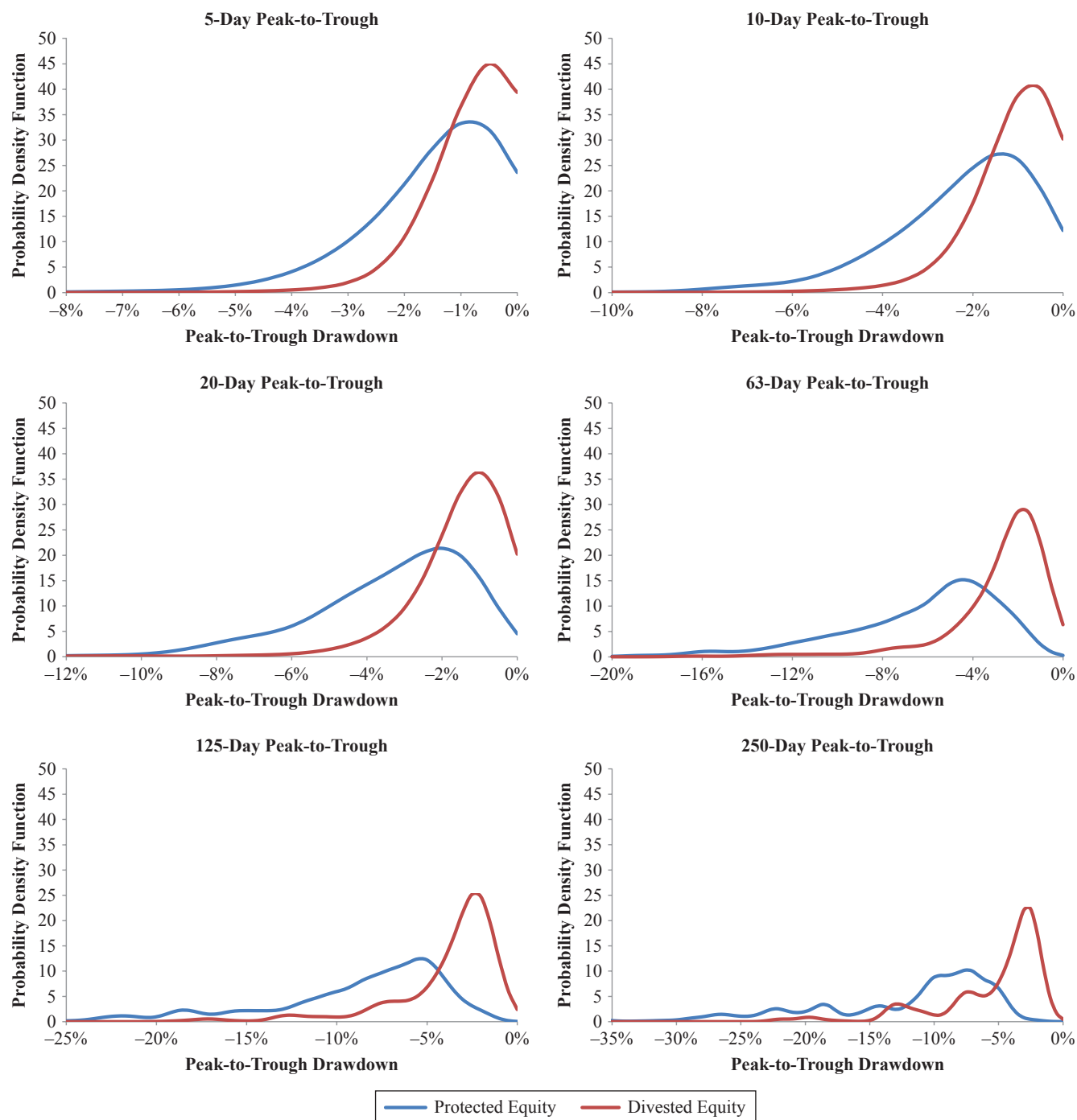
Exhibit 9 plots the divested portfolio's equity allocation. On average, it invests 84% of NAV in equities. This is about 10% higher than the put protection index's full sample beta to equities.

Exhibit 10 reports portfolio characteristics for the S&P 500, the PPUT, and the *ex ante* divested portfolio. The divested portfolio has realized 80% higher returns than PPUT (5.9% versus 3.2%), with about 10% higher volatility, leading to a 60% improvement in Sharpe ratio. The 5th percentile peak-to-trough drawdowns are between 10% and 20% lower for the protected portfolio versus the *ex ante* divested portfolio. The loss in realized returns is disproportionate to the reduction in tail risk when protecting. These *ex ante* implementable results are similar to the previously reported *ex post* performance characteristics and confirm that implementable divestment would have led to better outcomes than buying protection.

The *ex ante* divested strategy's equity investment is computed as the expanding window average delta of the PPUT, estimated as follows: The one-month, 5% out-of-the-money put option delta was calculated using Black-Scholes. I used the Cboe VXO Index for implied

## EXHIBIT 5

### Peak-to-Trough Drawdowns (Cboe Put Protection Index vs. Divested S&P 500)

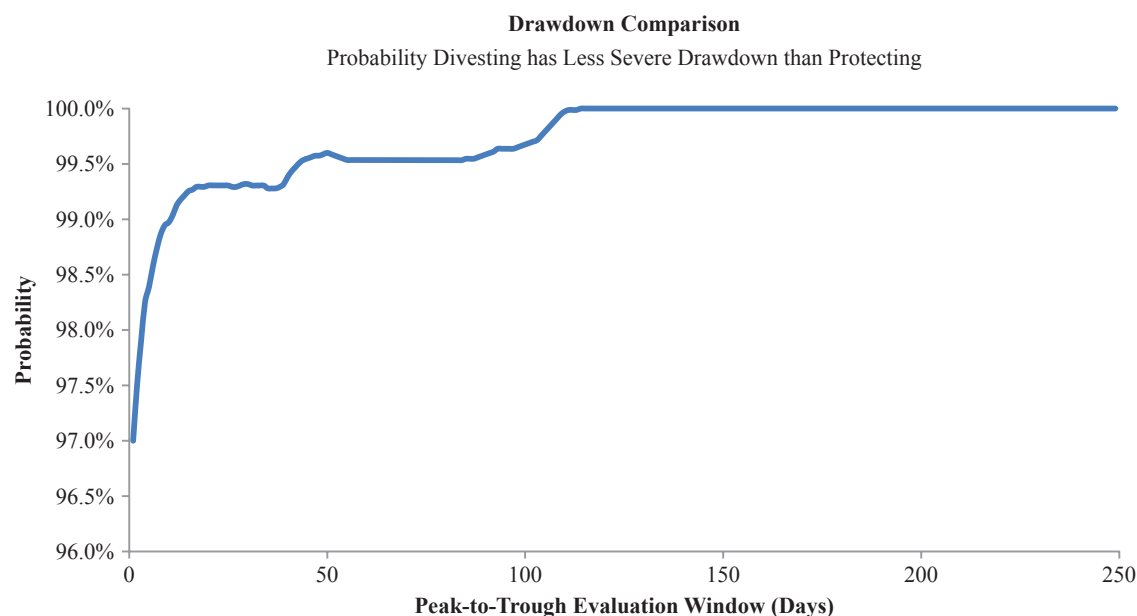


Note: Results shown over period July 1, 1986 to May 19, 2016.

Sources: AQR, Bloomberg, and Chicago Board Options Exchange. For illustrative purposes only.

## EXHIBIT 6

### Drawdown Comparison (Cboe Put Protection Index vs. Divested S&P 500)



Note: Results shown over period July 1, 1986 to May 19, 2016.

Sources: AQR, Bloomberg, and Chicago Board Options Exchange. For illustrative purposes only.

## EXHIBIT 7

### Trough-to-Peak Drawups (Cboe Put Protection Index vs. Divested S&P 500)

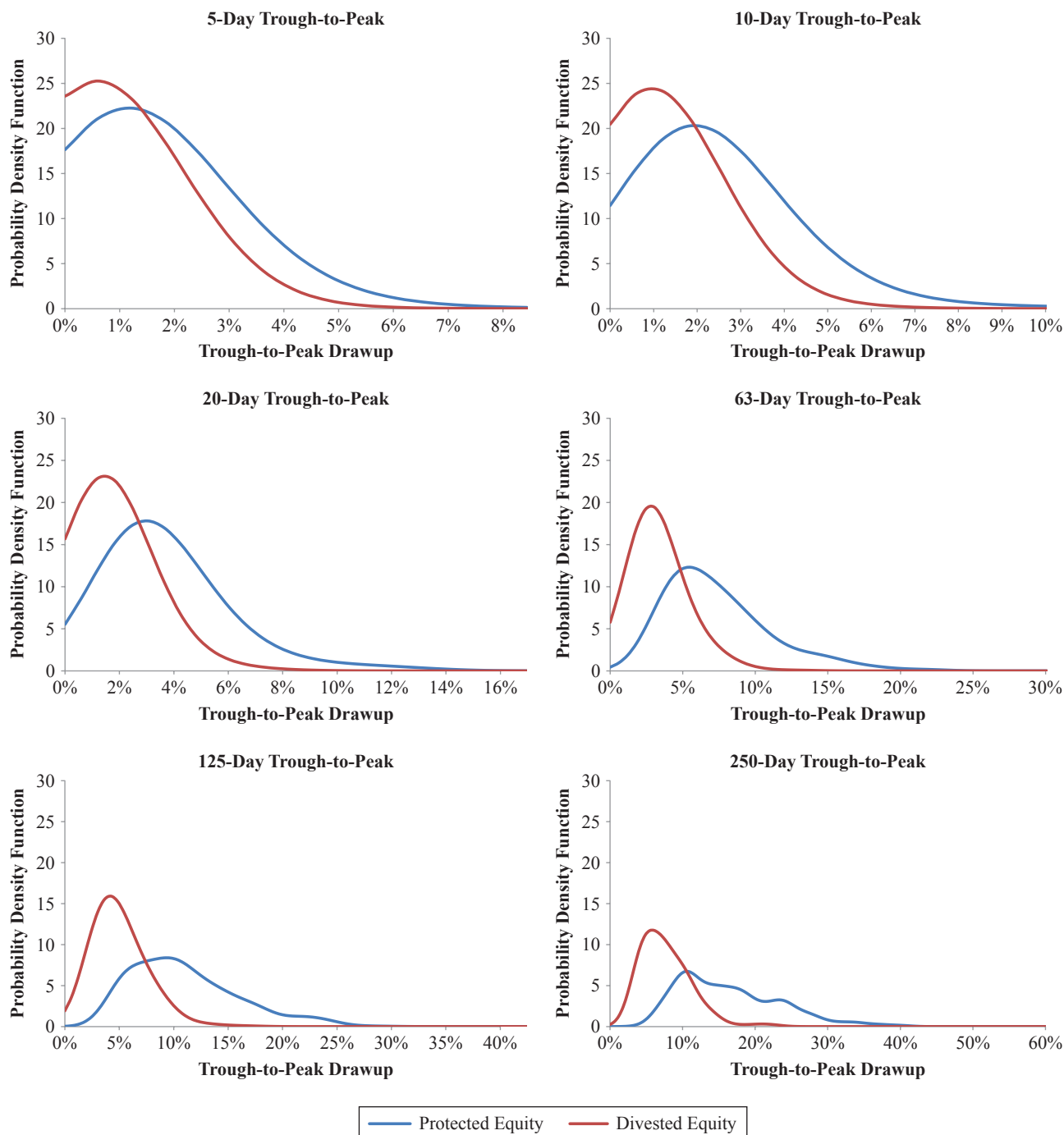
			Percentiles				
			99th	95th	90th	75th	Median
5-Day Drawup	Protected		5.2%	3.5%	2.8%	1.9%	1.1%
	Divested		2.7%	1.6%	1.3%	0.8%	0.5%
	Improvement		2.5%	1.9%	1.5%	1.1%	0.6%
10-Day Drawup	Protected		8.1%	5.1%	4.2%	3.0%	2.0%
	Divested		4.1%	2.4%	1.9%	1.3%	0.9%
	Improvement		4.0%	2.7%	2.3%	1.7%	1.1%
20-Day Drawup	Protected		11.7%	7.7%	6.2%	4.5%	3.2%
	Divested		5.4%	3.5%	2.8%	2.0%	1.4%
	Improvement		6.3%	4.2%	3.4%	2.4%	1.8%
63-Day Drawup	Protected		18.2%	14.6%	12.4%	9.1%	6.5%
	Divested		8.7%	6.7%	5.6%	4.0%	2.9%
	Improvement		9.6%	7.9%	6.8%	5.1%	3.6%
125-Day Drawup	Protected		24.6%	21.4%	17.8%	13.7%	10.2%
	Divested		12.7%	9.1%	8.2%	6.3%	4.5%
	Improvement		11.9%	12.3%	9.6%	7.4%	5.7%
250-Day Drawup	Protected		36.7%	28.8%	26.1%	21.1%	15.0%
	Divested		20.3%	13.6%	11.8%	9.7%	7.2%
	Improvement		16.3%	15.2%	14.2%	11.4%	7.8%

Note: Results shown over period July 1, 1986 to May 19, 2016.

Sources: AQR, Bloomberg, and Chicago Board Options Exchange. For illustrative purposes only.

## EXHIBIT 8

### Trough-to-Peak Drawups (Cboe Put Protection Index vs. Divested S&P 500)

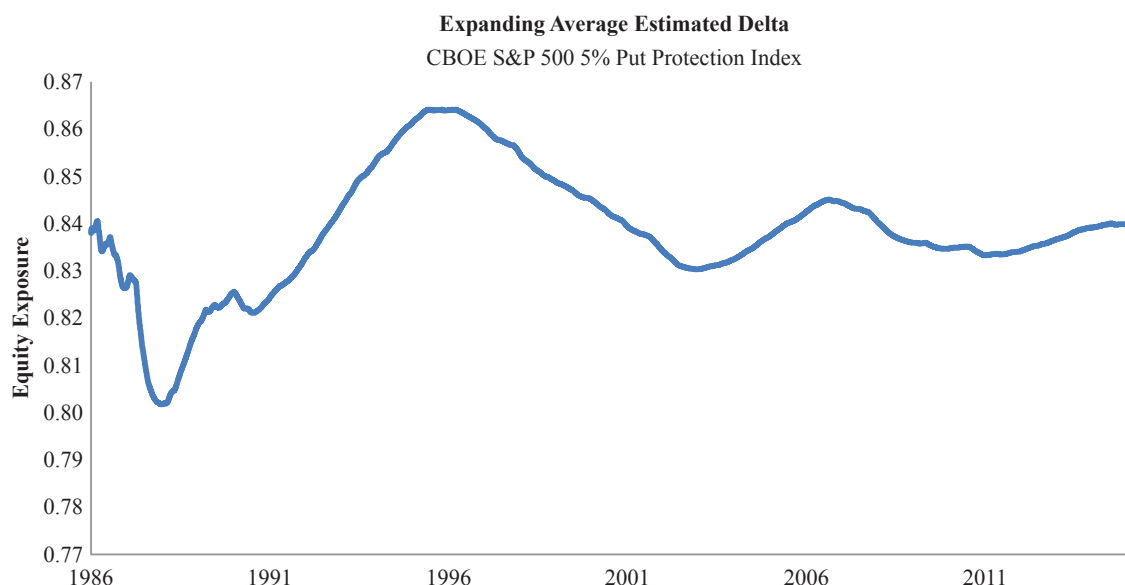


Note: Results shown over period July 1, 1986 to May 19, 2016.

Sources: AQR, Bloomberg, and Chicago Board Options Exchange. For illustrative purposes only.

## EXHIBIT 9

### Ex Ante Divested Portfolio Weights (Cboe Put Protection Index vs. Divested S&P 500)



Notes: Results shown over period July 1, 1986 to May 19, 2016. The one-month, 5% out-of-the-money put option delta was calculated using Black–Scholes. I used the Cboe VIX Index (from Bloomberg) for implied volatility and three-month USD LIBOR (from Bloomberg) as the interest rate. The dividend yield was assumed to be 2%.

Sources: AQR, Bloomberg, and Chicago Board Options Exchange. For illustrative purposes only.

## EXHIBIT 10

### Ex Ante Divested Portfolio Return Characteristics (Cboe Put Protection Index vs. Ex Ante Divested S&P 500)

	S&P 500 Index	Cboe PPUT Index	Ex Ante Divested
Annualized Excess Return (Geometric)	5.8%	2.5%	5.1%
Annualized Excess Return (Arithmetic)	7.0%	3.2%	5.9%
Annualized Volatility	18.0%	13.5%	15.1%
Sharpe Ratio	0.39	0.24	0.39
20 Day Peak-to-Trough (5th Percentile)	−9.9%	−7.4%	−8.2%
63 Day Peak-to-Trough (5th Percentile)	−19.1%	−13.2%	−16.3%
125 Day Peak-to-Trough (5th Percentile)	−27.3%	−19.0%	−23.4%
250 Day Peak-to-Trough (5th Percentile)	−33.7%	−26.2%	−28.9%

Note: Results shown over period July 1, 1986 to May 19, 2016.

Sources: AQR, Bloomberg, and Chicago Board Options Exchange. For illustrative purposes only.

volatility and three month USD LIBOR as the interest rate. The dividend yield was assumed to be 2%.

### SIMULATIONS—WITHOUT VOLATILITY RISK PREMIUM

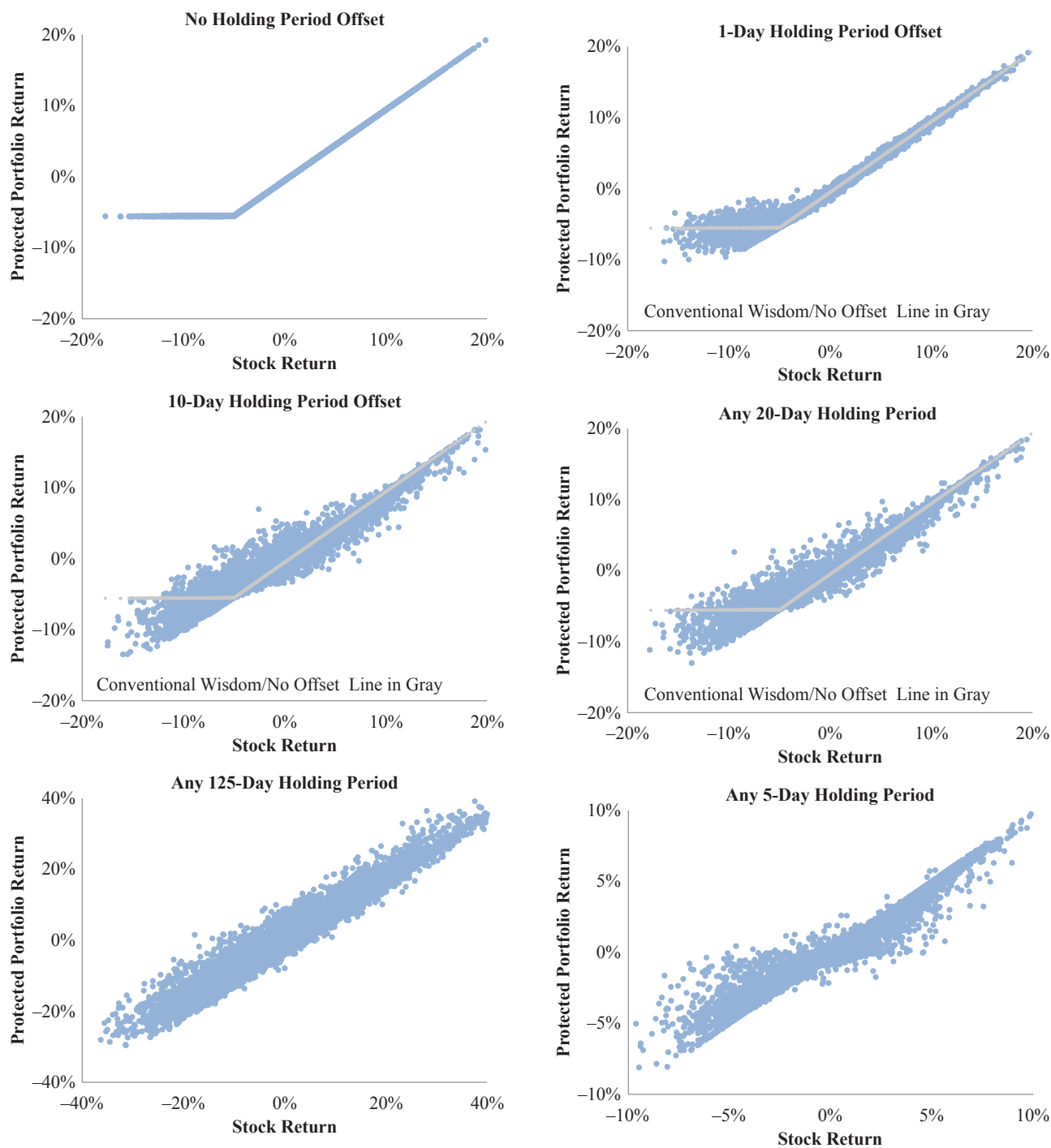
The protective put strategy leaves something to be desired in terms of its real-world downside risk

mitigation. I employ Monte Carlo simulations to help explain how the hedging breaks down. The simulations can be performed in a laboratory-like setting without much of the messiness that exists in actual markets. How do protective strategies fare when everything is clean and simple?

I draw equity prices from a lognormal distribution with 4% annualized excess of cash growth rate and

## EXHIBIT 11

### Simulated Protected Returns with Various Holding Period Offsets



Source: AQR. For illustrative purposes only.



20% annualized return volatility, not too dissimilar from historical realizations. The 4% annualized geometric return translates to a 6% arithmetic return, in line with the historical equity risk premium. Without loss of generality, the risk-free return and dividend yield are set to zero. Put options are purchased 5% out of the money with 20 business days until expiration and held until they expire. Then, the cycle repeats. I simulate one million daily returns.

Option prices are modeled according to Black–Scholes with *no* volatility risk premium—that is, implied volatility is 20%. This is an important departure from reality. It means that the simulated protective put strategy should have zero alpha to equities versus the PPUT’s realized  $-1.8\%$  annualized alpha. This allows me to test the protective put’s hedging efficacy in the best case (albeit unrealistic) scenario in which the market is not pricing crash risk.<sup>16</sup>

I report the regression of the simulated protected portfolio daily returns on coinciding equity returns:

$$r_{\text{protected},t} = 0.0 \text{ bps} + 0.83r_{\text{equity},t} \quad R^2 = 0.94$$

Buying the put option reduces equity exposure by 0.17 (the put option has an average delta of  $-0.17$ ). Given the one million observations, the 0.83 beta is a relatively precise estimate with a standard error of 0.0002. The simulation’s 0.83 beta is higher than the PPUT’s 0.74 beta. This can be attributed to the real-world, 5% out-of-the-money put options having more negative delta in periods of increased volatility relative to the 20% annualized volatility assumed in the simulations. For example, the put options have an average delta of  $-0.33$  if implied volatility is 40%. Also, implied volatility tends to increase when equities decline in value. This further reduces PPUT’s beta relative to the simulations. The intercept is near zero because option prices are simulated with no volatility risk premium.

In this case, the *divested* portfolio invests 78% of the NAV in equity and 22% of the NAV in cash to match the geometric return of the *protected* portfolio.

<sup>16</sup> Arguably, there are a number of reasons why real-world protection performance may be better than simulated performance. Equity prices are more negatively skewed than the lognormal distribution. Equities may trend and long options are long momentum. Implied volatilities tend to move inversely with equity returns, which may provide positive pressure on a put option’s price during equity losses, improving its downside hedging properties.

## Monthly Returns: A Visual Representation

Exhibit 11 scatter plots the 20-business-day returns of the protected portfolio against those of the underlying stock. The upper left panel shows the relationship when the option holding period matches the desired protection period—that is, buy monthly put options on the 20th of the month and protect monthly returns beginning on the 20th of the month. When investors picture a protected portfolio payoff diagram, such as the one depicted in Exhibit 1, this is likely what they envision. But when the option holding period does not perfectly align with the desired protection period, things begin to fall apart.

The upper right panel plots returns for an offset of just one day—that is, buy monthly put options on the 20th of the month but want to protect monthly returns beginning on the 21st of the month. To be clear, the protected returns on the  $y$ -axis are perfectly aligned with the unprotected returns on the  $x$ -axis. But the one-month returns are computed on the 21st of the month and options are purchased and expire on the 20th of the month. A small and seemingly immaterial misalignment of option holding period and desired protection period begins to reveal the gaps in the protective put’s armor. The middle left panel presents results when the option holding period and desired protection period are maximally misaligned, when the desired protection period begins halfway through the 20-day option holding period. Path dependence greatly diminishes the protected strategy’s protection.

Rather than focus on 20-day holding periods with specific offsets relative to the option cycle, investors may seek to protect returns over any 20-day holding period, irrespective of when it begins or ends. To this end, the middle-right panel plots rolling overlapping 20-day holding period returns (i.e., offsets of 0, 1, ..., 18, 19). The difference between this panel and the upper-left panel is not subtle. These protected returns do not even remotely resemble the protected payoff diagram we typically envision. Whatever protection that exists is well camouflaged in a sea of poor performance when considering all potential 20-day holding periods.

These scatterplots show how detrimental misalignment of the desired protection period and the option expiration cycle can be. Protection benefits can diminish further under another type of misalignment. Investors may serially purchase monthly put options because they primarily care about protecting monthly returns, but

## EXHIBIT 12

### Simulated Peak-to-Trough Drawdowns (no volatility risk premium)

		Percentiles				
		1st	5th	10th	25th	Median
5-Day	Protected	-5.3%	-4.1%	-3.5%	-2.5%	-1.6%
	Drawdown	-5.2%	-3.9%	-3.3%	-2.3%	-1.5%
	<i>Improvement</i>	-0.2%	-0.2%	-0.2%	-0.2%	-0.1%
10-Day	Protected	-7.3%	-5.8%	-5.0%	-3.9%	-2.7%
	Drawdown	-7.4%	-5.7%	-4.9%	-3.6%	-2.4%
	<i>Improvement</i>	0.1%	0.0%	-0.1%	-0.2%	-0.3%
20-Day	Protected	-9.8%	-8.0%	-7.1%	-5.7%	-4.3%
	Drawdown	-10.5%	-8.3%	-7.2%	-5.4%	-3.8%
	<i>Improvement</i>	0.7%	0.3%	0.0%	-0.3%	-0.5%
63-Day	Protected	-17.0%	-14.5%	-13.1%	-10.6%	-8.1%
	Drawdown	-18.4%	-14.7%	-12.8%	-10.0%	-7.3%
	<i>Improvement</i>	1.4%	0.2%	-0.2%	-0.7%	-0.8%
125-Day	Protected	-24.4%	-20.5%	-18.3%	-14.8%	-11.4%
	Drawdown	-25.2%	-20.1%	-17.7%	-13.9%	-10.4%
	<i>Improvement</i>	0.8%	-0.5%	-0.6%	-0.9%	-1.0%
250-Day	Protected	-33.7%	-28.2%	-25.1%	-20.3%	-15.6%
	Drawdown	-32.9%	-27.3%	-24.1%	-19.0%	-14.4%
	<i>Improvement</i>	-0.8%	-0.9%	-1.1%	-1.3%	-1.2%

Source: AQR. For illustrative purposes only.

they may also hope that this protection extends over longer or shorter periods. For example, perhaps an investor purchases monthly options every 20 business days but would like to see some protection over one-week periods or over six-month periods. Demonstrating the effect of this type of misalignment, the bottom two panels of Exhibit 11 scatter plot portfolio returns, serially protected with 20-day options, against stock returns over 125-day and 5-day holding periods. Where is the protection?

### Peak-to-Trough Drawdowns

Exhibit 12 reports peak-to-trough drawdowns measured over 5, 10, 20, 63, 125, and 250 business days at the 1st, 5th, 10th, 25th, and 50th percentiles, and Exhibit 13 plots the empirical probability density functions. I report results over the different window lengths for the *protected equity* portfolio and the daily-rebalanced *divested equity* portfolio, which holds 78% stock and 22% cash. Having sized the two approaches to provide the same average geometric return, I can fairly compare their drawdown characteristics.

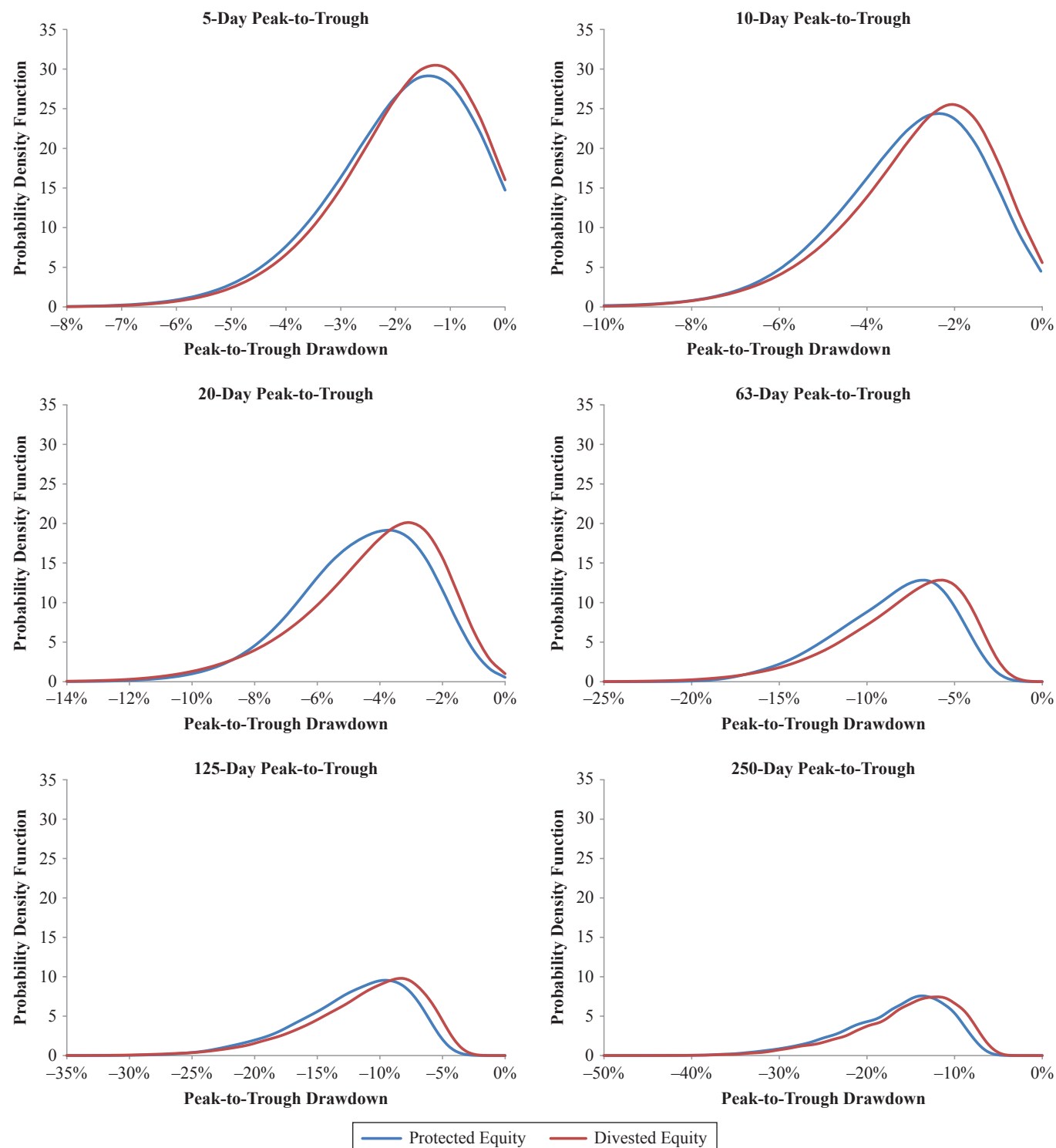
Interestingly and perhaps surprisingly, the median drawdown tends to be worse for the protected portfolio than for the divested portfolio over each of the drawdown evaluation windows. For example, over 20-day windows, the median peak-to-trough drawdown is -4.3% for the protected portfolio versus -3.8% for the divested approach. Exhibit 14 plots the probability that divesting outperforms protection buying across different peak-to-trough window horizons. At the 20-day horizon, divesting outperforms approximately 80% of the time.

The results are mixed for even the most extreme drawdowns. For instance, over the 20-day horizon, the protected portfolio's 1st percentile peak-to-trough drawdown is -9.8%. This is 0.7% better than divesting, whose 1st percentile drawdown is -10.5%, a marginal improvement. However, over 250-day horizons, the protecting portfolio's 1% worst drawdowns are -33.7% versus -32.9% for divesting. Over five-day horizons, the magnitude of the two approaches' drawdowns is similar, although protecting slightly outperforms.

Arguably, investors should care more about their largest drawdowns than their typical drawdowns. It is

## EXHIBIT 13

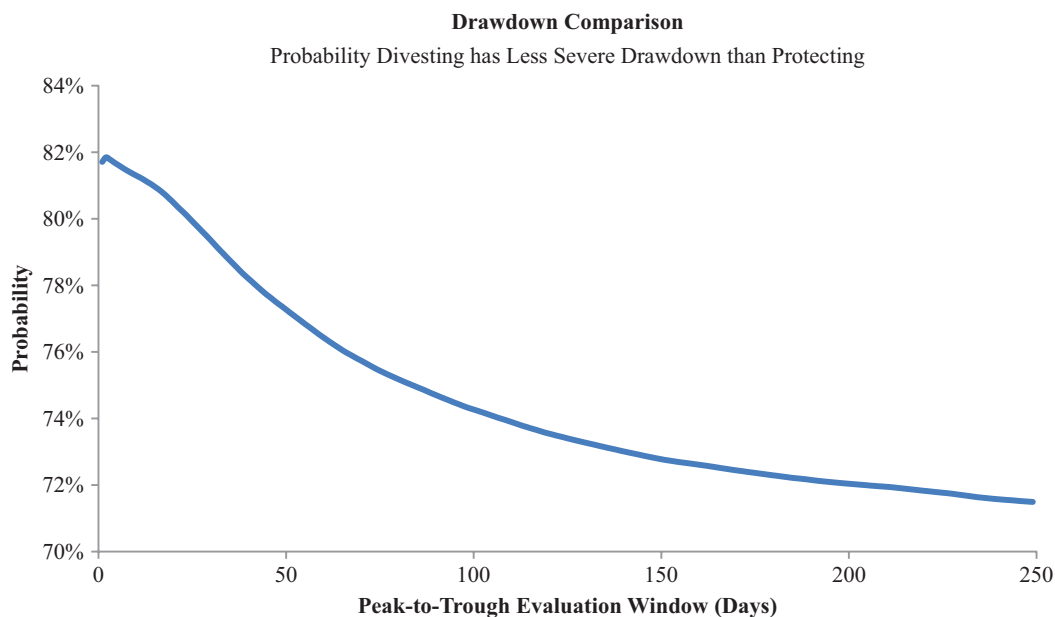
### Simulated Peak-to-Trough Drawdowns (no volatility risk premium)



Source: AQR. For illustrative purposes only.

## EXHIBIT 14

### Drawdown Comparison (no volatility risk premium)



Source: AQR. For illustrative purposes only.

hard to get excited about these results. Even when there is no volatility risk premium and options are not expensively priced, those who buy options with the hope of seeing economically meaningful reductions in left-tail risk may likely find themselves disappointed by how little benefit they actually realize.

### Upside Participation

Exhibit 15 reports trough-to-peak drawups at the 99th, 95th, 90th, 75th, and 50th percentiles, and Exhibit 16 plots the probability density functions. The findings for upside participation are similar to those for drawdowns, albeit more pronounced due to compounding. In the case of modest equity rallies, the divested portfolio outperforms the protected portfolio. But when equity markets realize their strongest performance, the protected portfolio tends to outperform.

Over 20-day evaluation horizons, the 99th percentile trough-to-peak rallies were 15.0% for the protected portfolio versus 12.3% for the divested portfolio. Over one-year horizons, the protected portfolio saw rallies of 67.9% at the 99th percentile level versus

57.0% for the divested portfolio, an 11% improvement. Buying protection provides modest improvements in upside participation versus divesting during the largest equity rallies.

It is worth re-emphasizing that these two portfolios are constructed to have the same average realized return. Buying protection changes the shape of the return distribution relative to divesting. Sometimes, individual investors may get “lucky” and see significant benefit to the protective put purchases if they are appropriately timed around drawdowns. However, my analysis shows that the differences in performance in these different environments, on average, are uninspiring. These are not game-changing improvements in downside risk or upside participation.

### SIMULATIONS—WITH VOLATILITY PREMIUM

I will now pull the rug out from under the protective put by realistically pricing crash risk in the options. The Cboe S&P 500 5% Put Protection Index realized  $-1.8\%$  of alpha because of the volatility risk premium.

## EXHIBIT 15

### Simulated Trough-to-Peak Returns (no volatility risk premium)

		Percentiles				
		99th	95th	90th	75th	Median
5-Day	Protected	6.7%	4.9%	4.0%	2.7%	1.6%
Drawup	Divested	5.6%	4.2%	3.5%	2.5%	1.6%
	<i>Improvement</i>	1.1%	0.7%	0.5%	0.2%	0.0%
10-Day	Protected	10.0%	7.5%	6.2%	4.4%	2.7%
Drawup	Divested	8.3%	6.3%	5.4%	3.9%	2.6%
	<i>Improvement</i>	1.7%	1.2%	0.9%	0.5%	0.1%
20-Day	Protected	15.0%	11.3%	9.5%	6.8%	4.4%
Drawup	Divested	12.3%	9.5%	8.1%	6.0%	4.2%
	<i>Improvement</i>	2.6%	1.8%	1.3%	0.8%	0.3%
63-Day	Protected	29.0%	21.9%	18.7%	13.7%	9.4%
Drawup	Divested	24.1%	18.7%	16.1%	12.1%	8.6%
	<i>Improvement</i>	4.9%	3.3%	2.6%	1.6%	0.8%
125-Day	Protected	43.7%	32.8%	27.9%	20.8%	14.4%
Drawup	Divested	36.6%	28.2%	24.3%	18.4%	13.1%
	<i>Improvement</i>	7.2%	4.7%	3.6%	2.4%	1.3%
250-Day	Protected	67.9%	50.7%	42.9%	31.6%	22.2%
Drawup	Divested	57.0%	43.8%	37.5%	28.2%	20.2%
	<i>Improvement</i>	11.0%	6.9%	5.4%	3.4%	2.0%

Source: AQR. For illustrative purposes only.

If crash risk will be similarly priced going forward, this alpha should not be ignored.

I update the parameters of my simulations by pricing options with 22% implied volatility. Equity prices continue to be drawn from a lognormal distribution with 4% annualized growth rate and 20% annualized return volatility. The volatility risk premium is 2.0% (i.e., 2.0%/20% = 10% of realized volatility). The risk-free return and dividend yield are set to zero. Put options are purchased 5% out of the money with 20 business days until expiration and held until they expire. Then, the cycle is repeated. I simulate one million daily returns.

I report the regression of the simulated protected portfolio on simulated equity returns:

$$r_{\text{protected},t} = -0.76 \text{ bps} + 0.82r_{\text{equity},t} \quad R^2 = 0.94$$

Buying the put option reduces equity exposure by 0.18, consistent with prior analysis as expected, (i.e., the put option has an average delta of -0.18). The put option's annualized alpha is -1.9% (the -0.76 bps is a daily alpha) and is highly significant with a -28.6 *t*-statistic.

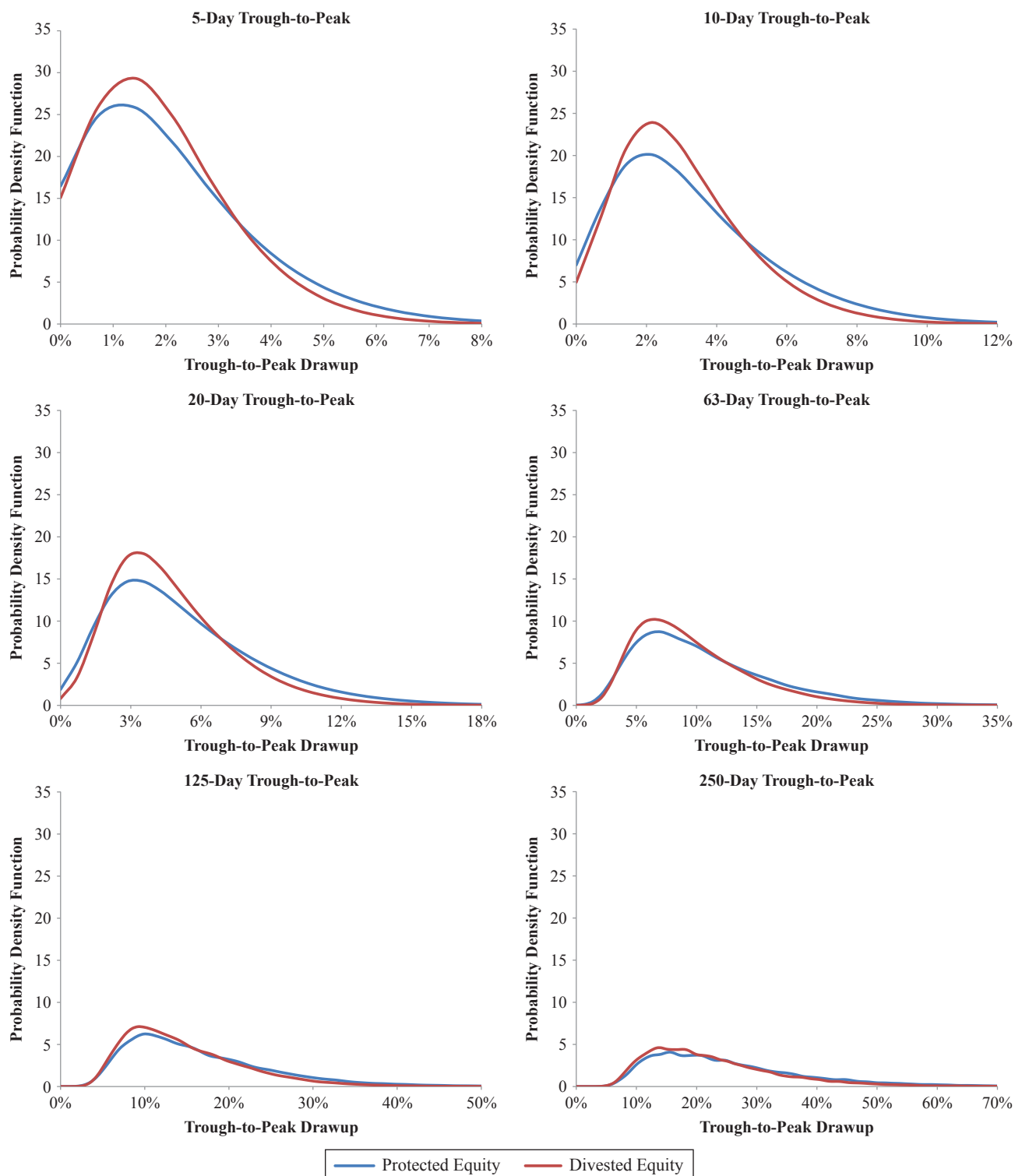
I compare the properties of this protected portfolio to a divested portfolio that has the same expected return to provide an apples-to-apples comparison. With a 0.82 beta, the protected portfolio earns 4.8% in equity risk premium and loses 1.9% in volatility risk premium. Its geometric return can be matched by a portfolio that invests 29% of its NAV in equity and 71% of its NAV in cash. These two portfolios, one with 5% out-of-the-money put options and the second that is less than one-third invested in the market, have the same expected returns. Which is less risky?

### Downside Protection and Upside Participation

Using the updated simulations, I compute peak-to-trough drawdowns over rolling overlapping windows of the following sizes: 5, 10, 20, 63, 125, and 250 business days. Similar to Exhibits 12 and 13, Exhibits 17 and 18 report the empirical cumulative and probability density functions over the different window lengths for the protected equity portfolio and the divested equity portfolio.

## EXHIBIT 16

### Simulated Trough-to-Peak Drawups (no volatility risk premium)



Source: AQR. For illustrative purposes only.

The protected portfolio consistently experiences larger drawdowns than the divested portfolio. The 1st percentile 20-day horizon peak-to-trough drawdowns are -9.9% for the protected portfolio versus -4.0% for the divested portfolio. The disparity grows over longer periods. Over the 250-day horizon, the 1st percentile drawdowns are -34.2% for the protected portfolio and -13.4% for the divested portfolio. Sized to earn similar returns, divesting has significantly better downside risk properties than buying put options in the presence of a volatility risk premium.

Exhibit 19 plots the percentage of time divesting had better peak-to-trough drawdowns than protecting with put options: 97% over very short horizons and 100% over longer horizons. These results are similar to those presented in the analysis of the PPUT. As shown in Exhibits 17, 18, and 19, in terms of downside risk, this horserace is not close.

There is a sharp contrast in upside participation, as shown in Exhibits 20 and 21. The 99th percentile trough-to-peak equity rally over a 20-day period is 14.7% for the protected portfolio versus 4.4% for the divested portfolio. Over 250 days, the protected portfolio's 99th percentile rally is 65.4% versus 18.2% for the divested portfolio.

As was the case for the PPUT, differences in beta drive the stark differences in downside risk and upside participation. The protected portfolio has nearly three times the beta of the divested portfolio. Because of this large difference in equity exposure, the protected portfolio is significantly more volatile. Its annualized volatility is 16.9% versus 5.8% for the divested portfolio. This substantially higher volatility nearly assures that protection will have worse peak-to-trough drawdowns and better trough-to-peak rallies than divesting.

## SIMULATIONS—PUT OPTION MATURITY

I have shown that the misalignment of option holding periods with realized peak-to-trough drawdowns is problematic. My analysis thus far has focused on one-month options. I will now demonstrate how option maturity affects hedging efficacy. I consider options with the following maturities (in business days): 20 days, 63 days, and 250 days.

Arguably, the moneyness of the options purchased by protection seekers should depend on their maturity. The strikes of longer maturity options should probably

be selected to be more out of the money. I select option out-of-the-moneyness to be equal to the median peak-to-trough drawdown of the underlying equity measured over a horizon equal to the option's holding period. Applying this methodology, my simulations purchase options that are 4.8%, 9.2%, and 18.2% out of the money for the 20, 63, 250 day options, respectively. Note that the moneyness of the 20-day option using this criterion is similar to that used for the simulations in previous sections.

A successful protection program should reduce the magnitude of peak-to-trough drawdowns relative to expected returns. I compute the ratio of 5th percentile peak-to-trough drawdowns to average log returns across measurement periods spanning 10 days up to 250 days. I then compute the percentage difference of this ratio between the protected strategy and the underlying equity.

Exhibit 22 plots results for the simulations, with unpriced crash risk on the left side and priced crash risk on the right side. In the case of unpriced crash risk, buying protection offers modest improvements in drawdowns across all the maturities considered. The maximal benefits for each maturity tend to occur for measurement periods that are most closely aligned with option lifespans. We can't know *ex ante* the duration of the next drawdown, but if we know the drawdown duration that we are most concerned with, that can help to guide option maturity selection.

Before investing any effort in selecting option maturity to best match drawdown horizon avoidance preferences, it is worth considering the more realistic scenario in which options do in fact price crash risk. As in previous sections, the simulations price options with a 22% implied volatility when the underlying equity's volatility is 20%. Whereas the "fairly" priced options provided benefits of up to 20% lower drawdowns per unit of return, the more realistically priced options had significantly higher drawdowns per unit of return.

Oftentimes, those who buy protective put options will look to longer maturities because they are more concerned with longer-period drawdowns, or the volatility risk premium may be lower for longer-dated options, or time decay is inversely related to option maturity. My simulations show that longer-dated options do a less bad job of protecting a portfolio against long-term drawdowns than shorter-dated options. Less bad, but not good.



## EXHIBIT 17

### Simulated Peak-to-Trough Drawdowns (volatility risk premium)

		Percentiles				
		1st	5th	10th	25th	Median
5-Day Drawdown	Protected	-5.3%	-4.1%	-3.4%	-2.5%	-1.6%
	Divested	-1.9%	-1.4%	-1.2%	-0.9%	-0.5%
	<i>Improvement</i>	-3.4%	-2.6%	-2.2%	-1.6%	-1.1%
10-Day Drawdown	Protected	-7.3%	-5.8%	-5.0%	-3.9%	-2.7%
	Divested	-2.8%	-2.1%	-1.8%	-1.3%	-0.9%
	<i>Improvement</i>	-4.5%	-3.6%	-3.2%	-2.5%	-1.8%
20-Day Drawdown	Protected	-9.9%	-8.0%	-7.2%	-5.8%	-4.3%
	Divested	-4.0%	-3.1%	-2.7%	-2.0%	-1.4%
	<i>Improvement</i>	-5.9%	-4.9%	-4.5%	-3.8%	-2.9%
63-Day Drawdown	Protected	-17.2%	-14.8%	-13.3%	-10.8%	-8.2%
	Divested	-7.1%	-5.6%	-4.8%	-3.7%	-2.7%
	<i>Improvement</i>	-10.2%	-9.2%	-8.4%	-7.1%	-5.5%
125-Day Drawdown	Protected	-24.5%	-20.8%	-18.6%	-15.2%	-11.7%
	Divested	-9.7%	-7.7%	-6.8%	-5.2%	-3.9%
	<i>Improvement</i>	-14.8%	-13.1%	-11.9%	-9.9%	-7.8%
250-Day Drawdown	Protected	-34.2%	-28.7%	-25.9%	-20.9%	-16.3%
	Divested	-13.4%	-10.6%	-9.3%	-7.3%	-5.4%
	<i>Improvement</i>	-20.8%	-18.2%	-16.6%	-13.7%	-10.9%

Source: AQR. For illustrative purposes only.

## SIMULATIONS—CRASH PROTECTION

Put options help to protect against something that is not present in my simulated returns: equity crashes. I now consider the tail hedging benefit that occurs specifically during equity crashes.

I compare the protected portfolio with and without a volatility risk premium to its respective divested portfolio (78% invested for the no volatility risk premium study and 29% invested for the volatility risk premium study). On each day in the backtest, I compute the returns of the protected and divested equity portfolios in the case of -20%, -10%, and -5% one-day equity crashes.

The protected portfolios' losses during crashes depend on the path leading up to the crash. Losses on the day following option expiration dates are always the same, by construction. On other days, protected portfolio losses depend on equity returns since the last option expiration date. If the equity market rallied preceding the crash, then the protected portfolio is more exposed because the put is further out of the money. In

the other case of an equity market decline preceding the crash, the protected portfolio offers greater protection. The "ideal" scenario in terms of quality of the crash protection hedge is that equities trend down considerably prior to the crash date. In these cases, the protected portfolio may lose little to nothing at all.

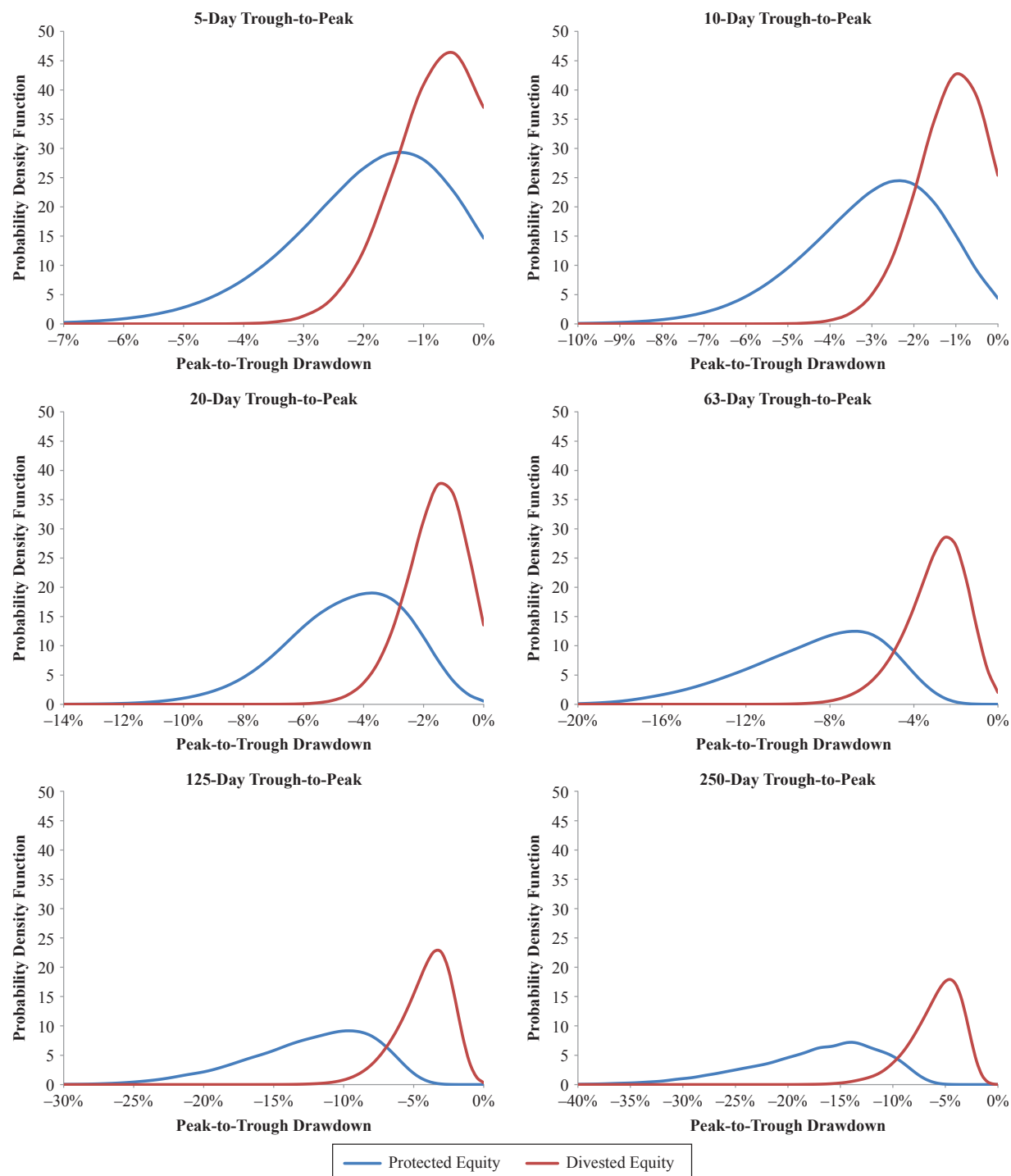
Exhibit 23 plots the empirical probability density functions for portfolio returns on crash days.<sup>17</sup> Protection generally outperforms divesting, even when options are priced with a volatility risk premium. For instance, in the case of no volatility risk premium, the protected portfolio outperformed the divested portfolio's 15.7% loss 99.6% of the time for -20% daily crashes, outperformed the divested portfolio's 7.8% loss 84.3% of the time for -10% crashes, and outperformed the divested portfolio's 3.9% loss 57.8% of the time for -5% crashes.

When options are priced to include a volatility risk premium, the protected portfolio outperformed the divested portfolio's 5.7% loss 56.2% of the time for -20%

<sup>17</sup> The density mass displayed at one-day protected losses of 0% is an unfortunate byproduct of kernel density smoothing.

## EXHIBIT 18

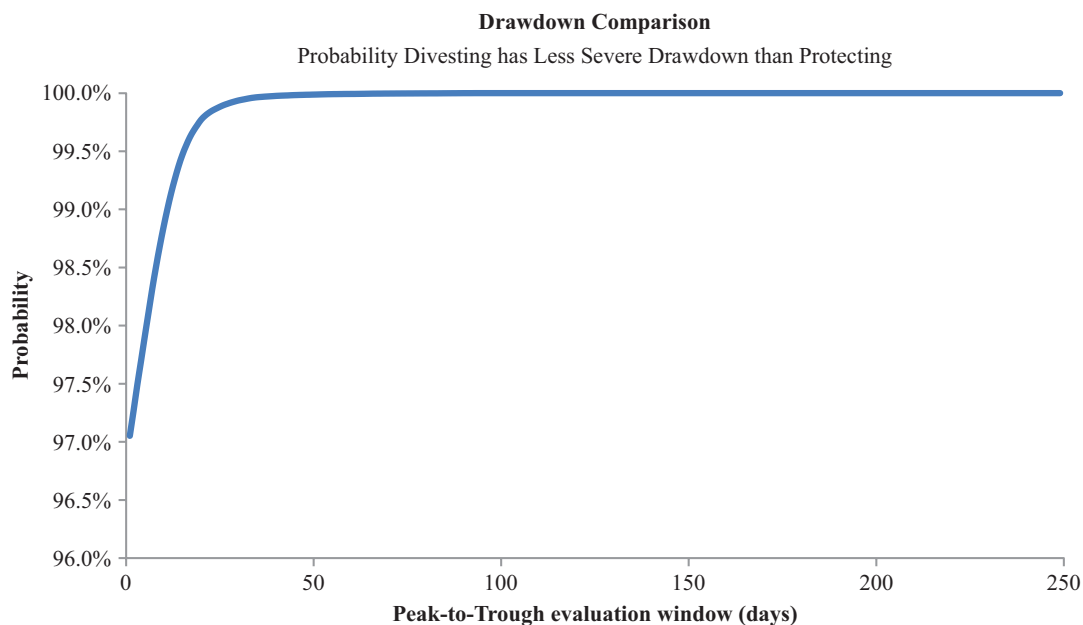
### Simulated Peak-to-Trough Drawdowns (volatility risk premium)



Source: AQR. For illustrative purposes only.

## EXHIBIT 19

### Drawdown Comparison (volatility risk premium)



Source: AQR. For illustrative purposes only.

## EXHIBIT 20

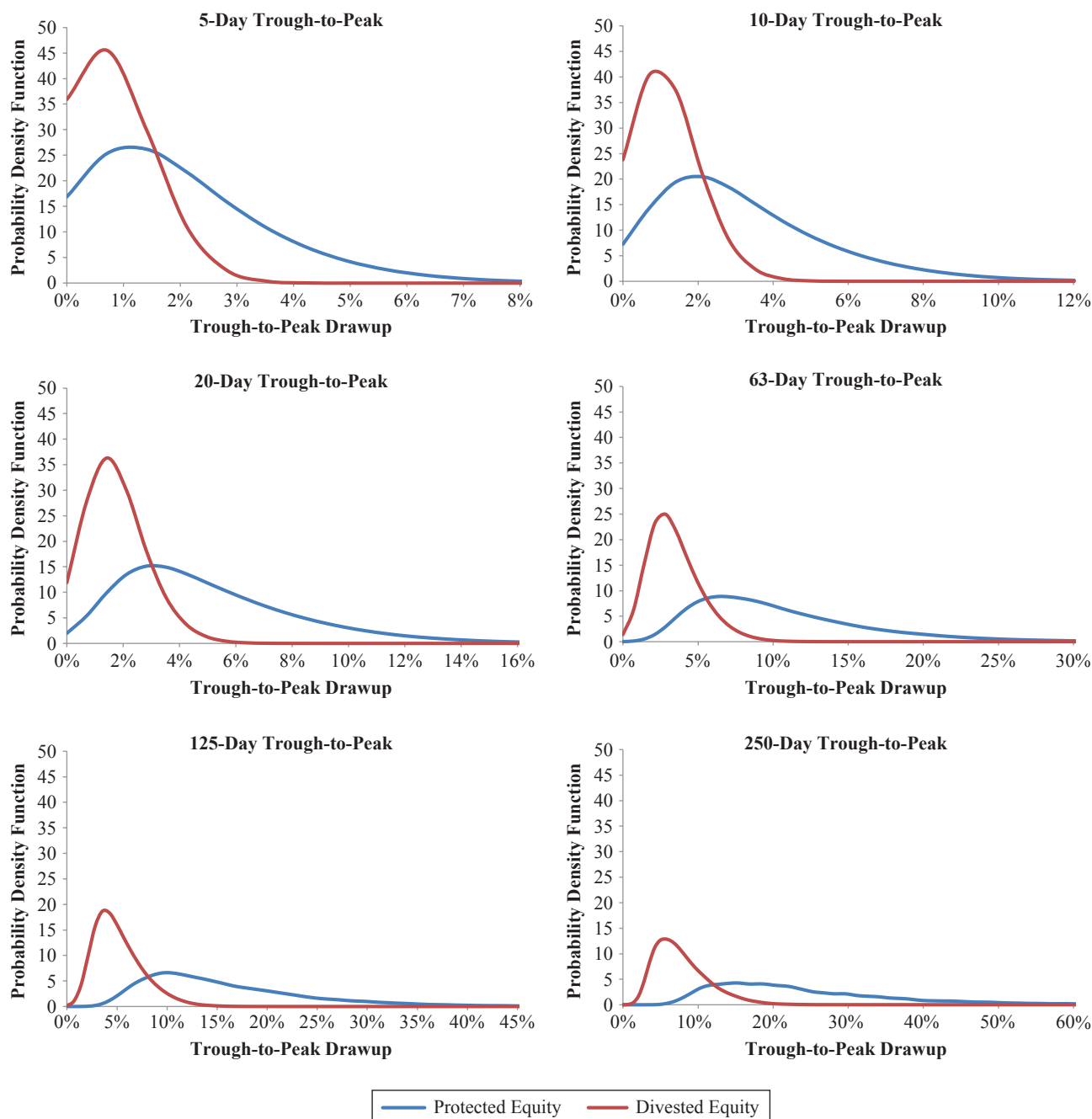
### Simulated Trough-to-Peak Returns (volatility risk premium)

		Percentiles				
		99th	95th	90th	75th	Median
5-Day Drawup	Protected	6.7%	4.9%	4.0%	2.7%	1.5%
	Divested	2.0%	1.5%	1.3%	0.9%	0.6%
	<i>Improvement</i>	4.6%	3.3%	2.7%	1.8%	1.0%
10-Day Drawup	Protected	9.9%	7.4%	6.1%	4.3%	2.7%
	Divested	3.0%	2.3%	1.9%	1.4%	1.0%
	<i>Improvement</i>	7.0%	5.1%	4.2%	2.8%	1.7%
20-Day Drawup	Protected	14.7%	11.1%	9.3%	6.7%	4.3%
	Divested	4.4%	3.4%	2.9%	2.2%	1.5%
	<i>Improvement</i>	10.4%	7.7%	6.4%	4.5%	2.8%
63-Day Drawup	Protected	28.6%	21.5%	18.1%	13.3%	9.1%
	Divested	8.3%	6.5%	5.6%	4.3%	3.1%
	<i>Improvement</i>	20.3%	14.9%	12.5%	9.0%	6.0%
125-Day Drawup	Protected	42.7%	32.1%	27.1%	19.9%	13.7%
	Divested	12.2%	9.7%	8.4%	6.4%	4.6%
	<i>Improvement</i>	30.6%	22.4%	18.7%	13.5%	9.1%
250-Day Drawup	Protected	65.4%	48.8%	41.1%	30.1%	20.9%
	Divested	18.2%	14.5%	12.6%	9.7%	7.0%
	<i>Improvement</i>	47.1%	34.2%	28.5%	20.4%	13.9%

Source: AQR. For illustrative purposes only.

## EXHIBIT 21

### Simulated Trough-to-Peak Drawups (volatility risk premium)



Source: AQR. For illustrative purposes only.

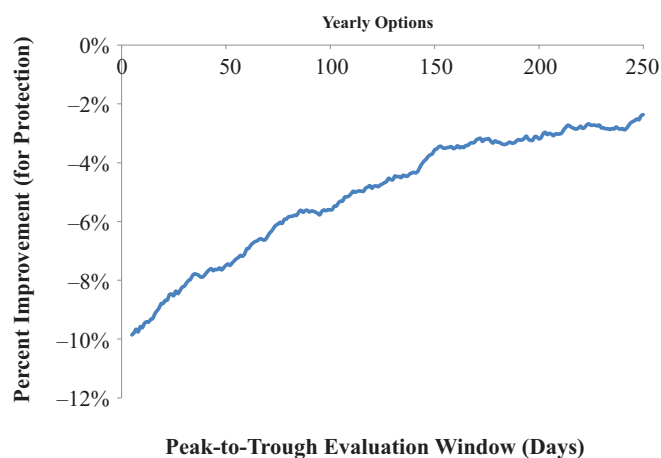
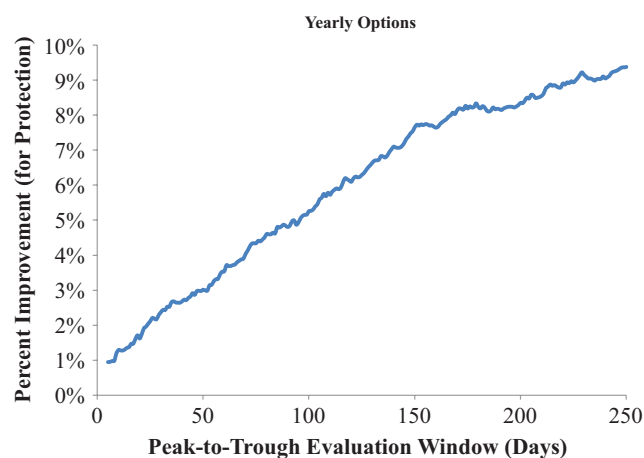
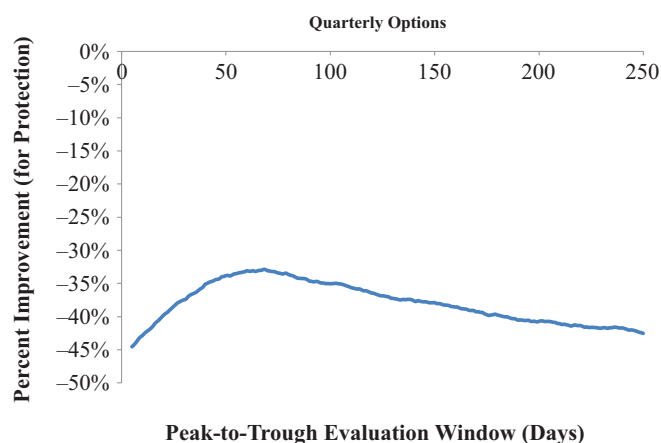
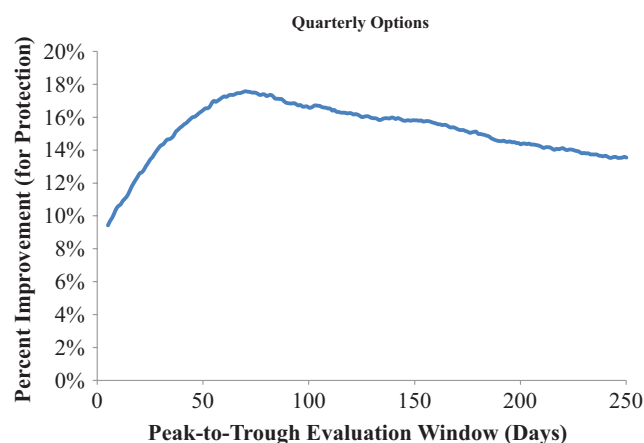
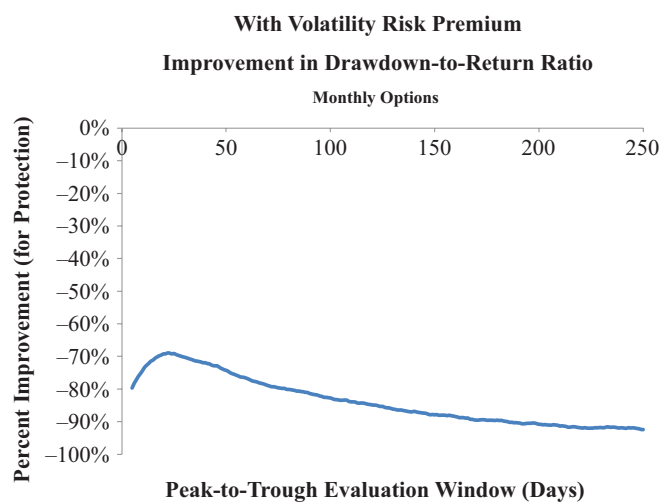
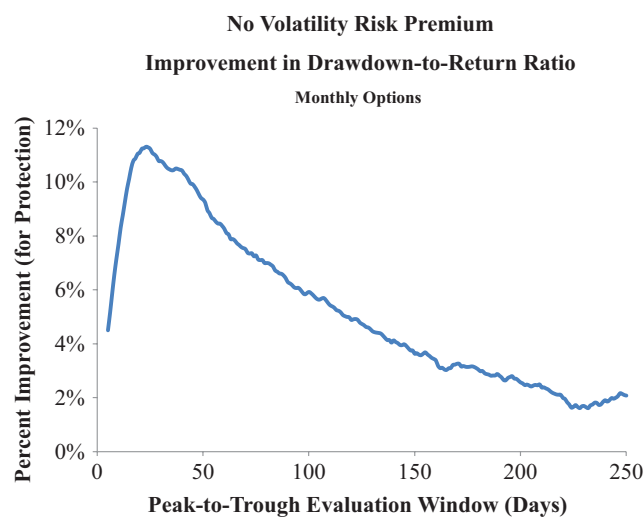
daily crashes, outperformed the divested portfolio's 2.9% loss 17.6% of the time for -10% crashes, and outperformed the divested portfolio's 1.4% loss 8.6% of the time for -5% crashes. Volatility risk premium reduces protec-

tion's edge over divesting, but for large crashes, protective puts offer better crash protection than divesting.

*Protecting against extreme crashes is clearly where buying protective puts shines.*

## EXHIBIT 22

### "Benefit" of Protection across Maturities



Source: AQR. For illustrative purposes only.

Those who purchase options specifically for crash protection may benefit by constructing a less path-dependent portfolio to provide more consistent crash hedging exposures. Variance swaps may be a potential solution. A variance swap is effectively the portfolio of delta-hedged options that is constant gamma.<sup>18</sup> However, crash protection comes at a steep cost, just to get a better result once in a blue moon: a lower Sharpe ratio and potentially worse peak-to-trough drawdowns per unit of expected return.

## CONCLUSION

Put options are usually presented as the most direct approach to protecting a portfolio against large losses. Many have rightly criticized this approach as being too costly because equity index options have historically been richly priced. Index options include a volatility risk premium as a form of compensation to option sellers for their insurance provision. This volatility risk premium eats away the expected returns of a protected portfolio.

However, the supposed benefits of protective put options have not received similar scrutiny. Many of us naturally expect that those who are willing to pay the cost will obtain meaningful downside protection. I find that this simply is not so. It is not safe to assume that a protective put will protect your portfolio against large drawdowns.

Buying a put option can effectively protect a portfolio over a well-defined period that begins when the option is purchased and ends when the option expires. Option liquidity has historically centered around 3rd Friday of the month expirations, limiting the periods over which investors can effectively protect their portfolios. Increased liquidity in recently introduced end-of-month and weekly expiration options increases the set of defined periods over which investors can purchase protection.

However, drawdowns can occur at any time and over any horizon, and path dependence weakens the put option's protective armor, particularly for those who are concerned more about peak-to-trough drawdowns than about returns over specific pre-defined periods. Systematically buying put options offers a very modest improvement over the simple alternative of reducing

the underlying equity position, if the options are priced with no volatility risk premium.

However, if options are priced to include volatility risk premium, then the outcome flips dramatically. For those who are concerned about their equity's downside risk, reducing their equity position is significantly more effective than buying protection. Sized to achieve the same average return, divesting has lower drawdowns, lower volatility, lower equity beta, and a higher Sharpe ratio than does buying put options. The one case where buying put options shines relative to holding a reduced equity position, even if options are priced to include volatility risk premium, is when a very large crash occurs prior to the options' expiration.

The results are clear. Buying protection more often than not and on average leads to worse drawdowns than does divesting the equity position to match the average return. This is particularly true when options are priced with a volatility risk premium. *The outcome is precisely the opposite of what is intended.*

There are those who will continue to seek a way to make the protective strategy work—testing different approaches by turning some knobs, pulling different levers, flipping a few switches, and maybe even clicking their heels three times. I have no doubt that with enough effort, an enticing backtest can be constructed, but I worry about robustness and out-of-sample properties. My analysis does not rule out the possibility that there exists a protection strategy that can be effective, but I think a heavy dose of caution and skepticism is in order.

Some protection seekers turn to exotic options, such as those with knock-out provisions because they typically have lower prices than vanillas. Given how mightily vanilla put options struggle to meaningfully reduce drawdown risk in even the most ideal settings, I am not optimistic about the downside-mitigating performance of exotics. Exotic provisions are likely to lead to payoffs that are even less aligned with peak-to-trough drawdowns than their vanilla cousins. For example, if a crash does occur, your protection may disappear at the worst possible time with a knock-out put. Knock-out puts have lower prices for a reason. And although their prices may be lower than vanilla options, they are likely to be even more expensively priced.

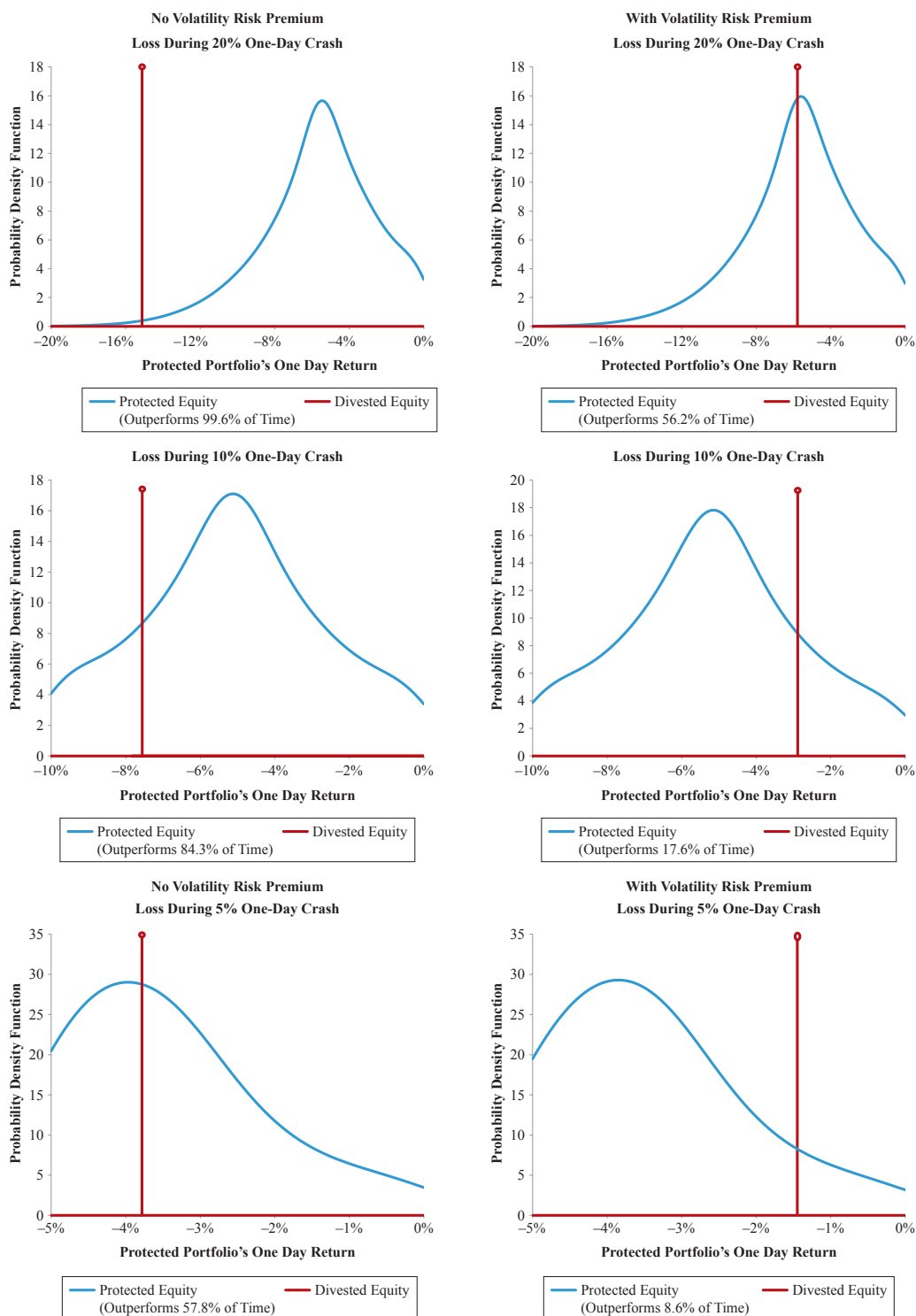
These conclusions may be viewed as discouraging. I prefer to see them as liberating, because once we accept them, we can redirect our limited resources to research that may actually improve portfolio outcomes. A more

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<sup>18</sup> Gamma measures the exposure to realized variance.

## EXHIBIT 23

### Protected Portfolio's One-Day Returns Coinciding with One-Day Crashes



Source: AQR. For illustrative purposes only.



efficient way to reduce the risk of large drawdowns for most investors is simply to reduce their long-term strategic allocation to equities. This necessarily also reduces expected returns, but can be addressed with an old idea: diversification. Equities are not the only asset class with positive expected returns—incorporating additional sources of returns can improve risk-adjusted portfolio returns, allowing for a better possibility of achieving an investor’s objective while mitigating downside risk.

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