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THE VALUE OF DURATION AS A RISK MEASURE FOR CORPORATE DEBT

ANTTI ILMANEN, DONALD MCGUIRE, AND ARTHUR WARGA

Duration is the primary risk measure for fixed-income portfolio managers. It is well-known that duration is an exact measure of an asset's interest rate sensitivity only if all term structure shifts are parallel and infinitesimally small. While large yield shifts and yield curve reshapings are often observed, empirical evidence suggests that duration works reasonably well for government bonds. Specifically, Ilmanen [1992] shows that during the 1980s duration explains 80%-90% of the cross-sectional variation in government bond returns.

This article analyzes duration's ability to measure the risk in non-callable corporate bonds. Non-callable bond issuance has risen dramatically recently, primarily because of investors' negative perceptions of call features in the face of high call activity in today's low interest rate environment.

The adequacy of duration as a risk measure for corporate bonds is topical because of the current "reach for yield" phenomenon. Historically low yield levels have caused many bond fund managers to shift their portfolios toward higher-yielding sectors, at the same time that their risk measurement tools lag behind.

That is, while duration is a convenient overall measure of a portfolio's interest rate risk, no such simple measure is available for quantifying a portfolio's default risk. This leaves duration as the only quantified risk measure for many bond fund managers. How well does it do in a "reach for yield" environment?

The usefulness of duration as a risk measure for non-government bonds is not clear a priori. The question is mainly empirical. If the yields of corporate bonds covary closely with those of government bonds, duration is likely to be a good risk measure. But if gen-

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eral or bond-specific default spreads are very variable relative to marketwide movements in yield levels, duration will perform badly.

We analyze the relative importance of 1) the marketwide parallel yield shifts (split into duration and convexity effects), 2) a shift in the general default spread, and 3) the bond-specific component in causes of cross-sectional dispersion in corporate bond returns. Our main results are that duration is able to explain nearly 90% of the cross-sectional variation in Aaa-rated bonds, and almost 80% in portfolios composed of Aaa and Aa bonds. When the "reach for yield" extends to A and Baa issues, duration's explanatory power drops to around 35%. Adding a default spread variable to the model increases explanatory power by at most four and a half percentage points.

I. DATA DESCRIPTION

In any study of corporate bonds a researcher must first confront the issue of access to bond data. Investment-grade bonds, which are the focus of this study, are largely traded in the over-the-counter dealer market. Outside the dealer market, the largest exchange for corporate bonds is the Automated Bond System (ABS) of the New York Stock Exchange, which trades around $40 million (face value) of bonds daily. According to the NYSE, 27% of the listed issues are below investment-grade. These issues account for over 57% of the trades, so the ABS accounts for a small percentage of total trading in investment-grade debt. The dealer market thus becomes the only data source that can provide a comprehensive reference for investment-grade non-callable corporate debt.

Warga [1991] compares month-end dealer bid quotes from Lehman Brothers with actual month-end transactions from the NYSE, and concludes that price discrepancies are random and on average insignificant for investment-grade debt. This suggests that trader-quoted bond prices do not contain any biases, so we collected monthly trader bid quotes from Lehman Brothers for all non-callable (and non-convertible) corporate debt traded at the firm to form our data set.

We access these data from Lehman's historical data tapes archived at the Fixed Income Research Program at the University of Wisconsin-Milwaukee. While the data go back to 1973, we chose 1985 as our starting date because there are very few Aaa-rated non-callable corporate bonds before this year.[1] Currently the archived data extend to December 1991.

In Exhibit 1 we list the number of bonds by rating (Moody's) in each month for which we have trader-quoted data. It is important to note that we exclude any prices that are matrix-based.[2] Our final exclusion criterion is to leave out bonds with less than one year to maturity. We do this because the change in duration over a period of one month cannot be considered trivial for short-maturity issues, and the constancy of duration over the return measurement interval is an assumption we will need to make.

II. EMPIRICAL METHODOLOGY

Ilmanen [1992] proposes a simple measure to assess the adequacy of duration as a risk measure: its ability to explain cross-sectional variation in bond returns. Assuming infinitesimally small parallel shifts of a flat yield curve, there is a simple relation among bond price change, yield change, and duration:

$$\frac{dP}{P} = D(-dY) \tag{1}$$

where P is the bond price, D is the (modified) duration, and Y is the yield to maturity.

If all yield changes are unexpected, the percentage price change (dP/P) is equal to unexpected bond return. Given these assumptions, a cross-sectional regression of unexpected bond return on the product of bond duration and minus yield change should give a perfect fit, zero intercept, and unit slope. Formally, $E(a_t) = 0$ and $E(b_t) = 1$ in Equation (2).

$$\begin{aligned} U(R_{it}) &= R_{it} - E(R_{it}) \\ &= a_t + b_t D_{it}(-dY_t) + e_{it} \end{aligned} \tag{2}$$

Because expected returns are unobservable, we use excess bond return over the one-month Treasury bill rate R_F as the dependent variable.

$$R_{it} - R_{Ft} = a_t^* + b_t D_{it}(-dY_t) + e_{it} \tag{3}$$

These assumptions imply that $E(a_t^*) = \overline{E(R_{it})} - R_{Ft}$ and $E(b_t) = 1$. Cross-sectional regressions (3) are

EXHIBIT 1 ■ Number of Non-Callable Corporate Bonds Available by Month

Date	Aaa	Aa	A	Baa	Date	Aaa	Aa	A	Baa
8503	53	64	85	18	8808	61	171	222	124
8504	54	64	87	19	8809	62	172	223	124
8505	55	67	92	21	8810	62	148	217	124
8506	56	66	94	24	8811	62	139	223	126
8507	55	68	94	26	8812	62	137	221	127
8508	57	94	102	25	8901	20	125	198	117
8509	58	84	115	25	8902	19	125	202	110
8510	57	83	116	25	8903	19	125	195	113
8511	57	83	119	25	8904	18	122	203	101
8512	55	84	121	26	8905	21	123	205	100
8601	55	84	124	31	8906	21	131	198	98
8602	55	81	128	34	8907	28	125	193	98
8603	55	82	130	34	8908	26	94	164	86
8604	55	89	133	39	8909	25	95	165	83
8605	54	79	110	44	8910	23	98	176	84
8606	55	80	117	45	8911	21	96	184	84
8607	57	83	127	47	8912	21	97	183	89
8608	57	84	130	48	9001	20	96	179	80
8609	59	86	132	56	9002	20	93	176	79
8610	59	83	132	65	9003	22	94	188	87
8611	60	107	142	66	9004	20	98	201	88
8612	60	100	152	70	9005	24	99	196	98
8701	59	100	149	69	9006	26	100	225	93
8702	59	96	153	66	9007	26	105	232	99
8703	62	113	178	71	9008	25	112	246	118
8704	61	110	175	71	9009	25	118	260	118
8705	61	124	160	70	9010	26	119	260	127
8706	60	124	162	68	9011	26	114	260	129
8707	59	124	167	68	9012	25	114	271	130
8708	63	125	172	69	9101	28	118	285	140
8709	63	128	171	74	9102	30	81	335	150
8710	63	131	172	73	9103	29	81	350	147
8711	62	133	169	75	9104	32	96	373	162
8712	64	160	177	78	9105	33	98	402	164
8801	64	165	179	77	9106	33	108	418	170
8802	62	154	178	94	9107	34	109	443	186
8803	63	159	185	99	9108	30	110	457	195
8804	61	155	183	103	9109	30	86	468	179
8805	60	152	179	100	9110	29	94	488	185
8806	63	161	186	108	9111	30	102	517	203
8807	61	166	190	112	9112	34	114	563	228

run each month, using bond duration multiplied by the negative of the change in an index interest rate as the independent variable. Time series averages are then computed for the monthly regression coefficients and the coefficient of determination (R^2) (see Fama and MacBeth [1973]).

The R^2 value in each regression shows how much of the cross-sectional differences in bond returns are due to their duration differences. If yield changes are exactly parallel (and not too large), excess returns are linear in durations, and the R^2 is 1. Ilmanen [1992] notes that there is a strong positive relation between the R^2 value and the size of the monthly yield change. So duration tends to be a better risk measure when the market is more volatile (and cross-sectional differences are large).

In the Fama-MacBeth methodology, each month is given equal weight when the time series average of monthly R^2 values is computed. An alternative is to weigh the more volatile months more heavily. Ilmanen [1992] suggests that the weighted average R_w^2 reflects duration's risk measurement ability better than a simple average R^2. The R_w^2 is computed by weighting each month's R^2 value by the same month's cross-sectional variance in bond returns (regression total sum of squares).

Second Risk Factor

We can view Equation (3) not as a cross-sectional regression equation but instead as an equation that holds for each individual bond. The interpretation of the intercept on a per bond basis is now

$$E\left(a_{i,t}^*\right) = E\left(R_{i,t}\right) - R_{Ft} \tag{4}$$

In other words, the intercept should equal an individual bond's expected return in excess of the riskless rate. This implies that an expected default spread variable should provide additional power in the cross-sectional regressions.

While the exact variable $(E[R_i - R_F])$ is not observable, including a proxy for it will allow us to assess the relative importance of default risk versus interest rate risk across bonds. We choose as our proxy for this variable a measure of the expected default spread given by the yield spread two months prior to the current month.[3]

Using the spread from two months in the past eliminates potential bias in the regressions because there will be no contemporaneous prices appearing on the left- and right-hand sides of the regression equation (see Stambaugh [1988]). Thus we will also run cross-sectional two-factor regressions where excess bond returns are regressed on their durations (*−dY) and on their default spreads. In particular, we examine the marginal increase in R^2.

Convexity

Duration provides an accurate risk measure if a linear approximation to the price-yield relationship is close. Convexity, which is the second derivative of a bond's price with respect to the yield, can be used in conjunction with duration to provide a more accurate approximation to the true price-yield relationship. Employing convexity means that Equation (1) becomes

$$\frac{dP}{P} = D(-dY) + \frac{1}{2}\left[\frac{d^2P}{dY^2}\frac{1}{P}\right](dY)^2 \tag{4}$$

with convexity equalling the term in brackets.

This suggests that we add a factor equal to convexity times the change in yield squared. Such a factor has a very high degree of correlation with the duration factor in the model (each month's sample correlation between the duration and convexity factors across bonds almost always exceeds 97%).

By construction, convexity can be expected to add significantly less explanatory power to the regression equation than duration. This suggests that the resulting collinearity may mask the value of convexity, so it will be important to gauge convexity's contribution by the marginal increase in R^2. Our third model variant will therefore include the convexity factor, and we shall see that it plays an important but tertiary role in the model.

III. EMPIRICAL RESULTS

Our empirical tests analyze the time series averages of regression coefficients and R^2 values from monthly cross-sectional regressions based on Equation (3), with additional default spread and convexity factors added in later regressions. Exhibit 2 reports results from regressions of individual bonds on their modified durations (multiplied by the index yield change).

To address the "reach for yield" issue, we analyze the data by seeing how the regression results change as a high-quality bond portfolio is extended to include bonds from lower rating classes. This is the relevant perspective for a bond fund manager who is currently reaching for yield and whose portfolio is therefore going through such extensions.

Exhibit 2 shows the one-factor regression results for Aaa, Aaa + Aa, Aaa + Aa + A, and Aaa + Aa + A + Baa-rated bonds. The duration coefficient is significantly less than 1.0, and the intercept in the model is significantly positive. While this is at odds with the model's predictions of a duration coefficient of 1.0 and a zero intercept, we will see that the refinements given by the default spread and convexity factors can explain away these results. What is important to note from this exhibit is the explanatory power that a single duration factor provides.

In the Aaa regressions, over 88% of the cross-sectional variation in bond return spreads is explained. Moving to portfolios including Aa bonds reduces the cross-sectional explanatory power to just over 76%. The big decrease in explanatory power comes in the reach for yield portfolios including A (35%) and Baa (32+%) bonds.

In Exhibit 3 we provide results for a two-factor model that includes the default risk factor. If this factor behaves according to our model's predictions, we should expect to see the model intercept go to zero [see discussion of Equation (3)]. The intercept remains statistically significant for the Aaa-only results, and gradually loses significance afterward.

Default spread is statistically significant in all versions of the model. Relative to the duration factor-only version of the model (Exhibit 2), the default factor adds 0.5 percentage points to the Aaa model, 2.8 percentage points to the Aaa + Aa model, 3.1 percentage points to the Aaa + Aa + A model, and 4.5 percentage points to the explanatory power of the Aaa + Aa + A + Baa model.

Clearly the value of the default factor rises as the portfolio includes riskier and riskier bonds. It is also worth noting that the actual duration factor coefficients change by very little relative to Exhibit 2 values, suggesting that the default factor is unrelated to the type of risk explained by the duration factor.

Finally, in Exhibit 4 we add the convexity factor to the duration and default spread factors. The inter-

EXHIBIT 2 ■ Regressing Individual Corporate Bonds' Excess Returns on a Duration Factor

$$R_{it} - R_{Ft} = a^*_t + b_t D_{it}(-dY_t) + e_{it}$$

	a^*	$t(a^*)$	b	t(b)	t(b−1)	R^2_{adj}	R^2_w
Aaa	0.253	2.98	0.595	4.30	−2.93	0.607	0.885
+Aa	0.224	2.84	0.646	5.19	−2.85	0.497	0.764
+A	0.232	2.83	0.615	4.85	−3.03	0.324	0.350
+Baa	0.236	2.94	0.614	4.95	−3.12	0.290	0.328

In each month between 1985/2 and 1991/12, excess bond returns are regressed on modified durations (multiplied by the change in the Aaa index yield). Sample means of the cross-sectional regression coefficients and the t-statistics for those means are reported along with a t-statistic testing if the duration factor coefficient deviates from its theoretical value of 1. a^* is the mean of the monthly model intercepts, and b is the mean of the monthly duration factor coefficients. R^2_{adj} is the simple average of the monthly coefficients of determination adjusted for degrees of freedom. R^2_w is the weighted average coefficient of determination adjusted for degrees of freedom, where the weights are each month's total sum of squares.

EXHIBIT 3 ■ Regressing Individual Corporate Bonds' Excess Returns on a Duration Factor and Default Risk Factor

$$R_{it} - R_{Ft} = a_t + b_{1t} D_{it}(-dY_t) + b_{2t} SPRD_{it} + e_{it}$$

	a	t(a)	b_1	$t(b_1)$	$t(b_1 - 1)$	b_2	$t(b_2)$	R^2_{adj}	R^2_w
Aaa	0.183	1.99	0.609	4.62	−2.96	0.168	2.86	0.641	0.890
+Aa	0.134	1.65	0.627	4.96	−2.95	0.117	2.79	0.529	0.792
+A	0.091	1.05	0.603	4.91	−3.24	0.143	2.89	0.356	0.382
+Baa	0.094	1.06	0.578	4.80	−3.50	0.125	2.39	0.336	0.372

In each month between 1985/2 and 1991/12, excess bond returns are regressed on modified durations (multiplied by the change in the Aaa index yield) and the yield spread from two months before the current date. Sample means of the cross-sectional regression coefficients and the t-statistics for those means are reported along with a t-statistic testing if the duration factor coefficient deviates from its theoretical value of 1. a is the mean of the monthly model intercepts, b_1 is the mean of the monthly duration factor coefficients, and b_2 is the mean of the monthly default spread factor coefficients. R^2_{adj} is the simple average of the monthly coefficients of determination adjusted for degrees of freedom. R^2_w is the weighted average coefficient of determination adjusted for degrees of freedom, where the weights are each month's total sum of squares.

EXHIBIT 4 ■ Regressing Individual Corporate Bonds' Excess Returns on a Duration Factor, Default Risk Factor, and Convexity Factor

$$R_{it} - R_{Ft} = a_t + b_{1t}D_{it}(-dY_t) + b_{2t}SPRD_{it} + b_{3t}CNVX_{it} + e_{it}$$

	a	t(a)	b_1	$t(b_1)$	$t(b_1 - 1)$	b_2	$t(b_2)$	b_3	$t(b_3)$	R^2_{adj}	R^2_w
Aaa	0.007	0.06	1.025	3.60	0.089	0.179	2.74	–1.327	–2.35	0.688	0.925
+Aa	0.031	0.417	1.030	4.99	0.145	0.123	3.02	–0.456	–1.22	0.568	0.810
+A	0.029	0.309	1.21	4.46	0.786	0.138	2.85	–0.106	–0.224	0.383	0.403
+Baa	0.046	0.490	1.23	4.32	0.821	0.121	2.32	0.051	0.072	0.361	0.393

In each month between 1985/2 and 1991/12, excess bond returns are regressed on modified durations (multiplied by the change in the Aaa index yield), the yield spread from two months before the current date, and convexity (multiplied by the squared change in the Aaa index yield). Sample means and t-statistics for those means are reported along with a t-statistic testing if the duration factor coefficient deviates from its theoretical value of 1. a is the mean of the monthly model intercepts, b_1 is the mean of the monthly duration factor coefficients, b_2 is the mean of the monthly default spread factor coefficients, and b_3 is the mean of the monthly convexity factor coefficients. R^2_{adj} is the simple average of the monthly coefficients of determination adjusted for degrees of freedom. R^2_w is the weighted average coefficient of determination adjusted for degrees of freedom, where the weights are each month's total sum of squares.

cepts are all close to zero (as predicted), which means that our proxy for the default spread is not biased and is capable of explaining cross-sectional variation in default risk adequately.

The duration coefficients now come into line in the sense that they are all insignificantly different from their theoretical values of 1.0 (and still highly statistically significant). This suggests that duration is accurately measuring interest rate risk.

Convexity seems to be insignificant, but it is important to note that it does add some explanatory power to the model. Its high correlation with the duration factor means that the convexity coefficient estimates each month are subject to substantial variation that may mask their true contribution.[4] While on average convexity appears to add nothing to the model, it does explain variation in individual months. Weighted R^2s increase explanatory power relative to Exhibit 3 by 3.5 percentage points, 1.8 percentage points, 2.1 percentage points, and 2.1 percentage points, respectively.[5]

Ignoring the issue of statistical significance for a moment, we see that in all but the last line in Exhibit 4 the mean coefficient is negative. Convexity and yield squared (which it is multiplied by to get the factor used in the model) probably serve a role as proxies for misspecification in the model. A likely source of the misspecification is the failure of older bonds to have their values updated in as timely a manner as on-the-run or younger bonds. The concentration of older bonds will generally lie in the longer issues, and therefore cause some one-sided bias in recorded prices for longer-maturity instruments.[6]

The convexity factor, which is closely related to the square of the duration factor, will serve as an excellent proxy for picking up the resulting bias in the data. Because the bias will generally be random each month (i.e., of the same sign each month but random over time as the term structure shifts randomly), the significance of this factor should wash out when averaged over time (and we see it does).

The drop in explanatory power in Exhibit 4 (as measured by R^2_w) that happens below Aa-rated bonds suggests that duration remains a very good risk measure (i.e., parallel yield shifts are important causes of cross-sectional return differences relative to other factors) if a corporate bond portfolio contains Aaa- and Aa-rated bonds. Duration still explains about a third of the cross-sectional variation in portfolios with lower-rated debt. Most importantly, we may conclude that duration explains about ten times more cross-sectional variation than default risk does.

IV. CONCLUSIONS

Duration's ability to explain cross-sectional variation in option-free government bond returns has been established in past research. Its value as a summary risk measure for corporate debt is not as well understood.

We show empirically in this article that duration provides an accurate, though by no means complete, measure of risk for option-free corporate debt. In a "reach for yield" environment, where bond portfolio managers seek higher returns by adding lower-rated debt to their portfolios, duration's explanatory power drops in, moving from Aaa + Aa debt (around 80%) to mixes of debt that include Aaa through Baa bonds (around 35%). While we are unable to test bonds with options (call features, for example) we can expect that a properly option-adjusted duration measure will also serve as a good measure of relative risk.

Default risk appears to be adequately captured in a model by measuring it with the yield spread of a bond two months prior to the current month and by the addition of a convexity factor. The marginal contribution of default risk in explaining cross-sectional variation of bond returns is of an order of magnitude less than that provided by duration alone. Finally, convexity adds a small amount of explanatory power (somewhat less than default spread) and seems to play an important role in the specification of individual monthly regressions.

ENDNOTES

[1]Our starting date is February 1985. Data on corporate bonds are not currently available for December 1984 because of translation problems from an old mainframe computer. This prevents the calculation of monthly return for January 1985.

[2]"Matrix" prices are not based on trader quotes but on the price of a frequently traded government or corporate bond that "benchmarks" the bond in question. See Warga [1991] and Warga and Welch [1993] for illustrations of the mistakes in valuation that are possible in studies using matrix prices.

[3]The measure of "default factor sensitivity" is each bond's default spread relative to a matching-duration government bond.

[4]This is usually referred to as the ill effects of collinearity in the econometrics literature.

[5]In regressions not reported here, convexity adds less than the default spread variable in explanatory power when run as a second factor along with duration (except for the Aaa-only regressions).

[6]Most bonds in this study were issued in the early to late 1980s, so the bonds with the longest seasoning are also the bonds with long original maturities. Thus the longer end of the term structure will be dominated by less liquid issues. Sarig and Warga [1989] study the effects of seasoning on government bonds and conclude that older bonds are more likely to suffer from a failure of their prices to be updated in a timely manner than are younger bonds.

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