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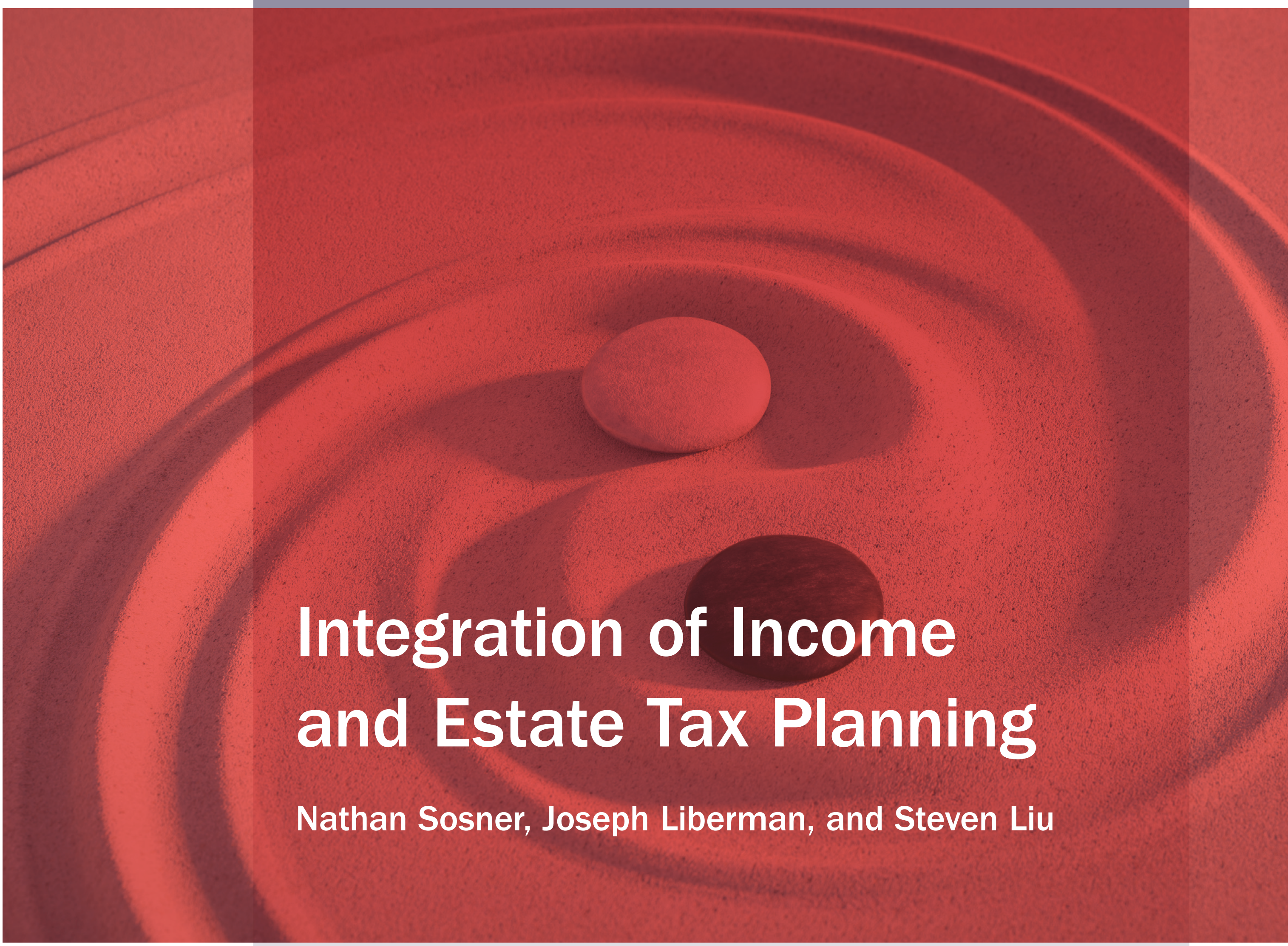

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KEY FINDINGS

- Using a stylized theoretical model and Monte Carlo simulations, this article quantifies the benefits of income and estate tax planning for growing wealth over generations. The article shows that a family that invests with income and estate tax efficiency in mind can achieve substantially higher wealth levels than a family oblivious to taxes.
- The article demonstrates that a significant value accrues from integrating income tax efficiency and estate tax planning: Becoming efficient with respect to one tax should make the family even more eager to become efficient with respect to the other.
- Our conclusions are robust to moderate declines in pre-tax returns, which might occur in pursuit of tax-efficient investing, and to significant increases in statutory annuity rates involved in estate tax planning techniques.

ABSTRACT

The preservation and transfer of wealth to future generations are among the central financial goals for most high-net-worth families. Using a stylized theoretical model and Monte Carlo simulations, this article quantifies the benefits of income and estate tax planning for growing wealth over generations. The article shows that a family that invests with income and estate tax efficiency in mind can achieve substantially higher wealth levels than a family oblivious to taxes. More important, the article demonstrates that a significant value accrues from integrating income tax efficiency and estate tax planning: Becoming efficient with respect to one tax should make the family even more eager to become efficient with respect to the other.

TOPICS

Wealth management, simulations, portfolio management/multi-asset allocation, performance measurement*

The preservation and transfer of wealth to future generations are among the central financial goals for most high-net-worth (HNW) families. Diligent planning for explicit and implicit costs, such as taxes and inflation, can help HNW families increase the expected real value of intergenerational transfers. In this article, we model and quantify the benefits of income and estate tax planning for growing wealth over generations. More important, our study is the first, to our knowledge, to model and demonstrate the powerful interaction effects between income and estate tax planning.

Apart from a few notable exceptions (Brunel 2001; Paulson 2003, 2009; Paulson and Tavel 2005; and Lucas 2020), the taxation of gifts and estates and the taxation of investment profits are typically studied in isolation. We indicate that a significant

*All articles are now categorized by topics and subtopics. **View at PM-Research.com**.

value accrues from integrating income tax efficiency and estate tax planning. More specifically, we demonstrate that as the family becomes more efficient with respect to estate tax planning, it benefits increasingly more from income tax efficiency and vice versa (i.e., the more efficient the family becomes about income tax planning, the more it benefits from estate tax efficiency).

The last point is a critical one. Let us state it differently: Being efficient about one source of taxation—income or estate—should not be viewed as an excuse for being less efficient about the other source of taxation. Rather, becoming efficient with respect to one tax should make the family even more eager to become efficient with respect to the other.

The explanation for the interaction between income tax planning and estate tax planning is intuitive. Over long investment horizons, income tax efficiency and the resulting higher after-tax returns lead to greater wealth for the current generation of the family and thus increase the potential burden of estate tax liability, which in turn makes estate tax planning more valuable. Similarly, a systematic and methodical implementation of estate tax planning over time can shield a significant portion of the family's assets from estate taxation, which makes increasing estate-tax–exempt assets through income tax efficiency more beneficial for future generations.[1]

To model this important interaction, we employ two alternative approaches. First, we derive a theoretical model that shows the intuition behind our calculations and allows the replication of our calculations using a simple spreadsheet.[2] Second, we develop and use a Monte Carlo simulation similar to the one described in Weinreb and Litman (2009), which implements estate tax planning through the use of grantor-retained annuity trusts (GRATs).[3]

As a brief preview of our results, under reasonable assumptions about the model and simulation parameters that we use in the base case, at the end of a 40-year investment horizon, the post-liquidation after-tax wealth of a family using a tax-efficient strategy with respect to both income and estate taxes is almost three times greater than that of a family with tax-inefficient approaches to both.

Before diving into the details of our calculations, in the next two sections, we explain the wealth preservation problem faced by HNW families and discuss how this study is related to the previous literature.

THE WEALTH PRESERVATION PROBLEM

Hughes (1998) makes an insightful observation on wealth preservation across generations of HNW families: Although some of the America's greatest wealth creators, including Warren Buffet and Bill Gates, have declared that they would leave their vast fortunes to philanthropy, their decision is obviously not because they are indifferent to the wealth and wellbeing of their descendants but rather because they operate in the belief that human, and not financial, capital constitutes the critical

[1] Note that a long horizon is a key component of any coherent wealth management program as the benefits of both income and estate tax planning accrue gradually over time. An additional perk delivered by a long investment horizon is the ability to tolerate short-term volatility of investment returns, which in turn creates the opportunity to enjoy long-term appreciation of investments. Lucas (2020, Ch. 5) also makes this point.

[2] Our model is based in part on Horan (2002) and incorporates the relevant aspects of trusts and estates, such as the step-up in the cost basis when assets pass through an estate, estate tax on assets passing through an estate, probability of death, and transfer of assets from a taxable estate to an estate-tax–exempt trust.

[3] Our Monte Carlo simulation framework can be further extended to jointly solve asset location-allocation problems while simultaneously considering income and estate tax efficiency. We leave this extension for subsequent research.

EXHIBIT 1

Punitive Effect of Taxes on the Real Value of Intergenerational Wealth Transfer

	No Income or Estate Tax (1)	Estate Tax Only (2)	Income Tax Only (3)	Income and Estate Tax (4)	Lower Income and Estate Tax (5)	Higher Pre-Tax Return (6)
Nominal Pre-Tax Return	7.0%	7.0%	7.0%	7.0%	7.0%	9.0%
Rate of Inflation	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%
Real Pre-Tax Return	4.9%	4.9%	4.9%	4.9%	4.9%	**6.8%**
Horizon in Years	35	35	35	35	35	35
Effective Income Tax Rate	0%	0%	20%	20%	**10%**	20%
Effective Transfer Tax Rate	0%	40%	0%	40%	**20%**	40%
Nominal Value of Transfer	10.7	6.4	6.7	4.0	6.8	6.8
Real Value of Transfer	5.3	3.2	3.4	2.0	3.4	3.4

component of successful family wealth preservation. Under this human capital paradigm, future older generations of the family continue investing in the human capital of future younger generations, and financial success is replicated by every new generation endowed with human capital.

However, rather than choosing whether to bestow financial or human capital, families can harmonize the two. In a chapter titled "Reinforce Positive Family Culture through Financial Design," Lucas (2020) discusses the virtuous cycle created by personal development and monetary wealth. For the younger generations, wealth offers the opportunity to develop risk-taking skills, financial independence, resourcefulness, and self-confidence. These traits then make them great stewards of preserving and growing the family's wealth and replicating for future generations the same opportunities they themselves had.

To this analysis, we add that many wealth creators would agree that achieving their exceptional level of financial success required—in addition to hard work, persistence, and dedication—a good portion of sheer luck. Thus, while wealth creators can share their work ethic and aptitude by investing in their descendants' human capital, financial capital represents the only way to convey the luck they happened to enjoy.

However, transferring financial capital is easier said than done. Families that seek to transfer financial assets to future generations face four significant headwinds: poor pre-tax investment performance; taxation of investment profits through income taxes; taxation of intergenerational transfers through gift, estate, and/or generation-skipping transfer taxes; and inflation, which is an effective tax on those who save and invest rather than borrow and spend, famously dubbed by Milton Friedman as "taxation without representation."[4]

Most HNW families and their advisers intuitively understand the burden of taxation and heavily discount nominal pre-tax returns. For example, Lucas and Sanz (2017) argue that for a hedge fund to provide a 5% return net of fees to a tax-exempt investor, it must generate a 7.8% gross return, but accruing the same 5% return net of both fees *and taxes* for a taxable investor requires a gross return of 12.9%—almost double the 7.8% return required to cover a 5% return net of only fees for a tax-exempt investor. Moreover, this analysis only accounts for income taxes. Gift and estate taxation and inflation further chip away at the ability to transfer financial capital.

Consider a simple example summarized in Exhibit 1. Suppose a family seeks to double its real after-tax financial wealth over the course of a generation, which, for

[4] In this list of challenges, we assume that family members act as prudent and responsible investors, so we do not even mention potential wealth-destroying behaviors such as reckless risk-taking and wasteful spending by future generations.

the purpose of this example, we assume to be 35 years. Such a goal is reasonable for a family planning to distribute wealth among several descendants.[5] Assume that the annual inflation rate is 2% and that the family selects an investment with an annual nominal pre-tax return of 7%, which translates into an annual real pre-tax return of 4.9%. We compute real pre-tax return as $real\ \ return = \frac{1+nominal\ \ return}{1+inflation\ \ rate} - 1$. if we ignore all the taxes, as in Column 1 of Exhibit 1, the family apparently could easily exceed the goal of doubling its wealth. In fact, the real value of wealth transfer to the descendants would be 5.3 times the initial investment. This is computed as $real\ \ value = \frac{(1+nominal\ \ return)^{35}}{(1+inflation\ \ rate)^{35}}$.

Now, consider the effect of taxes on our calculation. If a transfer tax (i.e., gift and estate tax) of 40% is imposed, the real value of the wealth transfer is only 3.2 times the original investment (Column 2 in Exhibit 1). Alternatively, if an effective income tax of 20% on ongoing investment profits is in place, the real transfer value is only 3.4 times the original investment (Column 3 in Exhibit 1). If both transfer and income taxes are levied on the invested assets, as shown in Column 4 of Exhibit 1, the real value of the transfer is merely double the original investment. This is computed as $real\ \ value = \frac{(1+nominal\ \ return \times (1-income\ \ tax))^{35} \times (1-transfer\ \ tax)}{(1+inflation\ \ rate)^{35}}$. Thus, if annual real pre-tax return happens to fall short of 4.9%, the family will not be able to achieve its goal of doubling the wealth over the course of a generation.

Note the striking difference between Columns 1 and 4: When taxes are accounted for, rather than enlarging the real value of intergenerational wealth transfer by more than five times, the family barely reaches its goal of doubling the transferred wealth. To increase the likelihood of accomplishing the goal of doubling the wealth, the family must either become more tax efficient or seek a higher pre-tax return on its investments. We consider these possibilities in Columns 5 and 6. In Column 5, the effective income and estate tax rates are reduced by a factor of two because of more tax-efficient investing and estate tax planning—and as a result of the higher tax efficiency, the real value of the wealth transfer rises substantially compared to the outcome of the less tax-efficient approach in Column 4. In Column 6, the same real value of transfer noted in Column 5 is achieved by increasing the annual pre-tax real return from 4.9% to 6.8%, a 40% increase in real return.

As we illustrate in the example in Exhibit 1, although superior pre-tax returns in theory can substitute for tax efficiency, historically, the latter has proved much more reliable than the former. A quest for higher investment returns is always aspirational and, unfortunately, might depend on luck more than skill, whereas tax efficiency primarily relies on competency and can yield much more certain benefits to the family. For this reason, sophisticated advisers to HNW families have long focused on identifying tax-efficient investments and estate tax planning opportunities. In this article, we document that such efforts are well spent: Income and estate tax planning can produce significant benefits for long-term wealth accumulation and transfer, and when these disciplines are integrated into one comprehensive wealth management program, the benefits become strikingly large.

RELATIONSHIP TO PREVIOUS LITERATURE

Asset *location* is a critical variable in optimizing intergenerational wealth transfers. Locating investments within estate planning vehicles—such as generation-skipping

[5] Although most HNW families actively pursue charity and philanthropy, to keep our example simple, we do not consider planning for charitable giving. If anything, charitable goals in addition to family inheritance goals only increase the required rate of growth in family wealth.

trusts (GSTs), GRATs, charitable lead trusts (CLTs), and family limited partnerships (FLPs)—accrues substantial benefits from an estate tax perspective. Another critical variable for wealth accumulation and transfer is income tax efficiency, which is solved through asset *allocation*. Over the years, several authors offer insights into solving a joint location-allocation problem for HNW families.[6]

Brunel (2001) describes an example of allocation to asset classes and strategies while simultaneously locating the investments across different entities—a personal taxable account, a tax-deferred account, various trusts, and a variable life insurance policy. In this analysis, economic risk and return, tax-efficiency of assets, and family goals determine the proposed asset location-allocation mix. Brunel (2001) also mentions GRATs as part of a dynamic asset location program but does not incorporate them in his example of a static location-allocation model output. In contrast, we do not explicitly model asset allocation and instead focus on dynamic asset location.

In a series of papers, Paulson (2003), Paulson and Tavel (2005), and Paulson (2009) discuss the use of hedge funds and separately managed loss-harvesting equity accounts inside trusts, but they do not quantify the interaction between income and estate tax efficiency (as we do in this study), instead providing conceptual considerations related to using assets with different levels of tax efficiency for estate planning purposes.[7] These papers point out that loss-harvesting strategies can be used to alleviate the tax burdens of hedge funds or that, at the very least, hedge funds should be placed in grantor trusts where the grantor, and not the trust, is the taxpayer.

The latter viewpoint is a long-accepted tenet in the legal community. As Gortz et al. (2016) put it, "… your payment of the trust's income tax essentially is an additional tax-free gift to your children and can further decrease the value of your estate." This is a correct conclusion when income tax is taken as given. However, the key conclusion of our study is that merely paying taxes on tax-inefficient investments outside of the trust is not enough. We demonstrate that a family is likely to achieve larger intergenerational wealth transfers by combining income tax efficiency and estate tax efficiency rather than focusing solely on either one or the other.

The latter conclusion is consistent with Lucas (2020, p. 90), who writes that "tax-efficient investing makes sense on a stand-alone basis, but its effects are even more powerful when paired with thoughtful estate planning." According to Lucas (2020), the value for a taxable investor is created at the intersection of investment, income tax efficiency, and estate tax planning—and these three disciplines need to work in harmony to maximize the compound effect. Our analysis complements the conceptual discussion in Lucas (2020) by developing quantitative methods to estimate the value of managing the intersection between income and estate tax planning.

[6] Note that for HNW investors, the notion of asset location and allocation is very different than it is for retail investors, whose primary objective is optimally locating tax-efficient and tax-inefficient assets among taxable and tax-deferred accounts. The latter is discussed, for example, by Reichenstein (2001); Shoven and Sialm (2004); and Dammon, Spatt, and Zhang (2004).

[7] In the words of Paulson (2009), "We don't have an eloquent definition to describe (and sell) asset location and integration; nor do we have all the answers on this emerging but important topic. We hope what we share with you heightens your awareness of certain issues and the overall importance of building custom portfolios for wealth transfer structures."

MODELING THE INTERACTION BETWEEN INCOME AND ESTATE TAX EFFICIENCY

In this section, we first develop a simple stylized model that would enable us to quantify the interaction between income and estate tax efficiency. We then explain our simulation methodology, which uses GRATs as a representation of estate tax planning.[8] In subsequent sections, we employ both the model and the simulation to illustrate our main idea—as a family becomes more efficient about one type of tax planning, whether income or estate, that family derives greater benefits from the other type of tax planning. Because inflation affects all of the scenarios equally and because we always compare results across scenarios, we simplify the exposition by ignoring the effects of inflation.

Stylized Model

In this section, we develop a stylized model of income and estate taxation that we later apply to numerical examples. Here, we only address the key equations of the model. The full derivation is relegated to Appendix A.

We assume for simplicity that a wealth creator is one person rather than a married couple. The wealth creator seeks to transfer to future generations a set of assets with an initial fair market value of V_0. The wealth creator starts with assets in a *personal account*, which will be subject to an estate tax upon death, and over time gradually shifts them in a gift-tax–free way to an *estate-tax–exempt trust* for the benefit of the descendants.[9] The assets are expected to generate an annual pre-tax return r. However, this is not the rate at which the assets will appreciate. To calculate that rate, we need to make an adjustment for tax costs.

To adjust for taxes, we define taxable income and gain amounts. The assets are expected to realize low-taxed long-term capital gains and qualified dividend income g—and highly taxed short-term capital gains and ordinary income i—where all the quantities are fractions of the value of the assets. In addition, the assets are expected to earn tax-exempt income x, also defined as a fraction of the value of the assets. Using these quantities and the tax rates applicable to low-taxed and highly taxed items (denoted by t_G and t_I, respectively), we can define the pre-liquidation after-tax return of the assets as

$$r^* = r - gt_G - it_I \tag{1}$$

and the rate of accumulation of unrealized gains, or the incremental one-period unrealized gain, as

$$u = r - x - g - i \tag{2}$$

[8] The shortcoming of GRATs is that they cannot transfer their estate-tax–exempt residuals into a trust that is exempt from a generation-skipping transfer tax (GSTT). Thus, GRATs can be used as planning tools for the generation of children but not for the generation of grandchildren and beyond. We employ GRATs as an example of a simple and widely used estate planning technique. For the purposes of GSTTs, families might rely on other wealth transfer techniques, such as installment (or leveraged) sales, to intentionally defective granter trusts (IDGTs) to which the GSTT exemption has been applied (e.g., Brunel 2001, Paulson 2003, and Weinreb and Litman 2009). Such techniques are outside of the scope of this article.

[9] As we say above, here we simply assume that techniques exist that allow a gift-tax–free transfer to the trust. In the simulation analysis, we use GRATs (Appendix C) to operationalize such a gift-tax–free transfer.

The rates in Equations 1 and 2 describe the tax efficiency of the assets.

Now, we define the variables related to estate taxation. First, we assume that the probability of death of the wealth creator in any given period is q.[10] Second, we define parameter λ, with values between 0 and 1, which measures the rate of asset retention in the personal account and thus captures estate tax inefficiency. At time i, a fraction λ^i of the assets is still held in the personal account of the wealth creator, and a fraction $1 - \lambda^i$ has been transferred in a gift-tax–free way to the estate-tax–exempt trust. The former fraction of the assets is potentially subject to estate tax if the wealth creator dies, but the latter is shielded from estate taxes. For example, if the wealth creator does not engage in any estate tax planning, the value of λ is 1, so in every period, all of the assets remain in the personal account and thus are potentially subject to estate tax. On the other hand, if the value of λ is 0, all the family assets are in the estate-tax–exempt trust and thereby are always shielded from estate taxes. Finally, we denote the estate tax rate by t_E.

We assume that taxes are paid from the profits realized on invested assets, so if assets are in a trust, the trust is the entity paying the income tax.[11] This assumption allows us to grow the assets in the trust and in the personal account at the same rate of after-tax return r^* defined in Equation 1. Thus, the value of assets in period t, whether held in the personal account of the wealth creator or in the trust, is

$$V_t = V_0(1 + r^*)^t \tag{3}$$

We also assume that the assets are invested for a fixed number of years n, irrespective of whether the wealth creator is still alive or already deceased at time n. In other words, if the death occurs before the end of the investment horizon, the descendants inheriting the assets proceed with the investment until the end of the investment horizon. We also assume that the income tax rates before and after the assets are transferred into the trust (as well as before and after the wealth creator's death) are the same.[12] These two assumptions—a fixed investment horizon and fixed tax rates—allow us to hold the economics of the investment constant while varying the parameters that define income and estate tax efficiency.

Now, assume that the wealth creator dies in year $i \leq n$, i.e., before the end of the investment horizon. Using Equation 3, the value of assets at the time of death can be expressed as

$$V_i = V_0(1 + r^*)^i \tag{4}$$

However, as we assumed, assets continue to grow at the rate r^* until the end of the investment horizon n for $n - i$ additional years. As a result, at the end of the investment horizon, the value of the assets is

$$V_n = V_0(1 + r^*)^i(1 + r^*)^{n-i} = V_0(1 + r^*)^n \tag{5}$$

[10] In actuarial mortality tables, after the very first year of life, the probability of death in a given year increases with age. To simplify the model, we assume a fixed probability of death. In our simulation analysis, we use an actual actuarial mortality table.

[11] We make this simplifying assumption to keep the model tractable, with full recognition that typically a trust would be set up as a grantor trust where the wealth creator, and not the trust, is responsible for income tax liabilities generated by assets held in the trust. Our simplifying assumption can be justified as follows. Whether income tax is paid by the trust or by the wealth creator, the tax reduces the amount of capital that the wealth creator can potentially commit to the trust. As a result, in the long run, the income tax paid outside of the trust indirectly reduces the assets of the trust.

[12] Assuming that no tax law changes occur during the investment horizon, these assumptions are reasonable for HNW families.

Note that this is the same value of assets that results when the wealth creator dies after the end of the investment horizon. This conclusion satisfies our objective of keeping the economics of the investment constant irrespective of the period when the death occurs.

The time of death matters in our model because when assets held outside of the trust pass to descendants through the estate, the assets are subject to a step-up in the cost basis, which is increased to the value of the assets at the time of death. That is, if the death occurs in year *i*, at that time, the cost basis is stepped up to the value V_i.

The following equations represent the key components of the model. First, the post-liquidation value of assets at the end of the investment horizon *n* if the wealth creator dies in year $i \leq n$, is

$$W_n^i = \lambda^i W_n^{i,PA} + (1 - \lambda^i) W_n^{i,TR} \tag{6}$$

where $W_n^{i,PA}$ and $W_n^{i,TR}$ are the post-liquidation values of assets at time *n* in the personal account of the wealth creator and in the estate-tax–exempt trust, respectively.

The post-liquidation values in Equation 6 are defined as

$$W_n^{i,PA} = (1 - t_E)(V_n(1 - T^*) + V_i T^*) \tag{7}$$

and

$$W_n^{i,TR} = V_n(1 - T^*) + V_0 T^* \tag{8}$$

where V_0 is the initial value of the wealth creator's assets; V_n and V_i are defined in Equations 4 and 5, respectively; and T^* is the effective liquidation tax defined as

$$T^* \equiv t_G \frac{u}{r^*} \tag{9}$$

Equations 8 and 9, originally derived in Horan (2002), show that the post-liquidation value is the sum of the pre-liquidation value V_n reduced by the effective liquidation tax and the tax credit for the value of the initial cost basis V_0. We derive an additional Equation 7 where the tax credit is applied to the stepped-up cost basis V_i. If the after-tax return r^* is positive, the stepped-up cost basis V_i is greater than the initial cost basis V_0, which follows directly from Equation 4 above.[13]

A comparison of Equations 7 and 8 where assets pass through an estate and a trust, respectively, is highly instructive. On one hand, a step-up in basis increases the after-tax value of the assets passing through the estate by $(V_i - V_0)T^*$ (or the difference between the fair market value at the time of death and the original cost basis multiplied by the effective liquidation tax rate). On the other hand, the estate tax lowers the value of assets passing through the estate. Which of these two effects dominates? We show in Appendix B that under plausible assumptions

[13] Equation 7 is also interesting in the following sense. It is generally well understood that assets in a trust should ideally appreciate at or close to a pre-tax rate of return, which can be achieved by investing the trust's assets tax efficiently, paying taxes on the trust's investments outside of the trust, or a combination of the two. The reasoning is clear: A dollar taken from the trust to pay income tax is directly taken away from the future generations of the family. From Equation 7, it follows that assets kept in a personal account should also be managed tax efficiently despite being eventually reduced by the estate tax. This is easy to see. Suppose that we have already concluded that managing assets tax efficiently in the trust is beneficial, which means that $V_n(1 - T^*) + V_0T^*$ in Equation 8 increases in

$$W_n^{i,TR} > W_n^{i,PA}$$

Thus, despite the step-up in the cost basis applicable to the assets remaining in the personal account when they pass through the estate, descendants of the wealth creator benefit from moving assets out of the personal account and into the estate-tax–exempt trust. As a result, from the family's perspective, it is generally optimal to move as many assets as possible into the trust.[14]

Furthermore, if the wealth creator dies after the end of the investment horizon, the assets in the personal account are liquated without a step-up in the cost basis, so Equation 7 becomes

$$W_n^{\infty,PA} = (1-t_E)(V_n(1-T^*)+V_0T^*) \tag{7′}$$

We use the superscript ∞ to indicate that death occurs at an unknown time after the liquidation of assets. Combining Equation 7′ for the personal account with Equation 8 for the trust assets, we can rewrite Equation 6 for the case when death occurs after the liquidation as

$$W_n^{\infty} = \lambda^n W_n^{\infty,PA} + (1-\lambda^n)W_n^{\infty,TR} = (1-\lambda^n t_E)(V_n(1-T^*)+V_0T^*) \tag{6′}$$

The fraction of assets in the personal account λ^n has a power of n because we assume that both the investment and the planning cease at liquidation.

Using Equations 6 (death before liquidation) and 6′ (death after liquidation), we can now define the expected wealth of the family. The probability of death in period i, conditional on surviving until period i, is $(1-q)^{i-1}q$ while the probability of surviving past the liquidation period n is $(1-q)^n$. As a result, the expected value of the transferred family wealth is

$$E(W_n) = \sum_{h=1}^{n}(1-q)^{h-1}qW_n^h + (1-q)^n W_n^{\infty} \tag{10}$$

Equation 10 is the main equation that we use in our numerical examples below.

Simulation Methodology

Our simulation methodology follows the same principles as those of the model above, with a few modifications that the flexibility of a simulation environment allows us to implement. First, rather than simply assuming a rate of transfer from a personal account to an estate-tax–exempt trust, we use GRATs as the estate planning tool. We model 5-year consecutive, 2-year consecutive, and 2-year rolling GRATs scenarios, alternatively (as explained in Appendix C). All the GRATs are zeroed-out and are described in more detail in Appendix C. The GRATs are implemented until the end of the investment horizon. For example, a 40-year horizon is associated with eight consecutive 5-year GRATs, 20 consecutive 2-year GRATs, and

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the after-tax return r^*. Assuming that u and r^* are positive, which generally would be the case for tax efficient investments, the expression $V_n(1-T^*)+V_iT^*$ in Equation 7 is also increasing in r^* because the difference between $V_i = V_0(1+r^*)^i$ and V_0 increases in r^*. This means that, despite the compression of the income tax efficiency benefit by the estate tax, as long as the multiplier $(1-t_E)$ in Equation 7 is greater than zero or—in other words, the estate tax is less than 100%—the family benefits from increasing the after-tax return of assets held in the wealth creator's personal account.

[14] An additional tax optimization can be accomplished by swapping out highly appreciated assets of the trust for low-appreciated assets of the personal account so that the trust would hold less appreciated assets while the personal account would include more appreciated assets.

39 rolling 2-year GRATs. Second, to the extent possible, taxes are paid outside of the remainder trust, which is also described in Appendix C. Third, we use a probability of death that increases with age, based on the 2012 individual annuity mortality (IAM) information for a male, using the age at nearest birthday (ANB), produced by the Society of Actuaries (2013).[15]

We then proceed with the simulations as follows. First, the wealth of the family accumulates at the after-tax rate of return, which is determined by the pre-tax return and the tax characteristics of the investment process: the character of the realized income, gains and losses, and the level of unrealized gains. We draw annual pre-tax returns from an independent and identically distributed (i.i.d.) normal distribution with a specified mean and standard deviation. Tax characters are applied as a fixed fraction of the pre-tax return. The pre-tax returns and the tax efficiency of the investment represent the key parameters of interest in our study, and we vary these parameters as explained further below. For each set of return parameters, we draw 20,000 40-year return histories.

Second, we apply mortality to the return histories. For example, if the probability of death in a given year is 1%, we apply death event logic to 200 out of our 20,000 return histories every year. In the event of death, the value of the assets passing through the estate is reduced by the estate tax; their cost basis is stepped up to the fair post–estate-tax market value; and the assets are reinvested by the family until the end of the 40-year investment horizon. Assets are unaffected if they are already in the estate-tax–exempt trust at the time the grantor dies. This situation is described in our theoretical model in Equation 8. Note that we assume investment in the same assets both inside and outside of the trust and thus use the same time series of simulated returns—one of the 20,000—for both.

Third, at the end of the 40-year investment horizon, all assets are liquidated, and liquidation taxes are paid. If the assets at the time of death are in the personal account of the grantor and thus receive a step-up in the cost basis at death, they face a lower liquidation tax than assets in the remainder trust at the time of death, which do not receive a step-up in their cost basis. We describe these two scenarios in our theoretical model in Equations 7 and 8, respectively. Clearly, if the wealth creator is still alive at the end of the 40-year investment horizon, none of the assets, either in the personal account or the trust, benefit from the post-death step-up in cost basis before liquidation. This situation is encompassed by the theoretical model in Equation 9.

By averaging across our 20,000 return histories with their respective estate tax and liquidation tax situations, we obtain an expectation that conceptually corresponds to Equation 10 in our model. Therefore, we compare below the results of the theoretical model with the results of the simulations.

Why do we show both the model and the simulation? The theoretical model has some advantages. It clearly shows the logic behind the calculations, and the calculation can easily be replicated by substituting our assumed parameter values into Equation 10. The simulation carries its own benefits. It allows for randomness in returns, easily incorporates age-dependent probability of death, and implements

[15] "2012 IAM Basic Table–Male, ANB" is available on the Society of Actuaries website (Society of Actuaries 2013). Although the Internal Revenue Service (IRS) provides its own mortality table (Table 2000CM), IRS mortality rates are arguably inappropriate for estimating the life expectancy of wealthy taxpayers, who exhibit lower-than-average mortality rates (Krueger 2011). Yeoman (2014) uses the 2012 IAM Basic Table in his analysis of charitable remainder trusts. The difference in mortality rates between the IRS and Society of Actuaries IAM tables can be substantial. For example, the probability of death by the age of 85 is 65.5% according to IRS Table 2000CM but only 45.8% according to the 2012 IAM Basic Table (Society of Actuaries 2013).

EXHIBIT 2
Return Assumptions for the Base Case

	Income Tax Scenario				
	Least Tax-Efficient	2	3	4	Most Tax-Efficient
Pre-Tax Return	6.8%	6.8%	6.8%	6.8%	6.8%
Pre-Tax Volatility	10.0%	10.0%	10.0%	10.0%	10.0%
Tax Characters (Fraction of Pre-Tax Return)					
Non-Taxable Income	11.8%	11.8%	11.8%	11.8%	11.8%
Low-Taxed Income and Gains	17.6%	17.6%	17.6%	17.6%	17.6%
Highly Taxed Income and Gains	*70.6%*	*52.9%*	*35.3%*	*17.6%*	*0.0%*
Unrealized Gain	0.0%	17.6%	35.3%	52.9%	70.6%
Low Tax Rate	23.8%	23.8%	23.8%	23.8%	23.8%
High Tax Rate	40.8%	40.8%	40.8%	40.8%	40.8%
Character-Weighted Tax Rate	33.0%	25.8%	18.6%	11.4%	4.2%
Effective Liquidation Tax Rate (T^*)	0.0%	5.7%	10.3%	14.2%	17.5%
Pre-Liquidation After-Tax Return (r^*)	4.6%	5.0%	5.5%	6.0%	6.5%
Post-Liquidation After-Tax Return	4.6%	4.9%	5.3%	5.7%	6.0%

actual widely used estate tax planning techniques rather than assuming, as the model does, that assets “miraculously” move from the personal account to the trust.

Base-Case Return Assumptions

Next, consider some examples. In the base case, we assume that the pre-tax return on assets is not affected by the level of tax efficiency. This assumption is consistent, for instance, with Sialm and Zhang (2020), who find that on the pre-tax basis, tax-efficient actively managed US equity mutual funds do not underperform, but rather surprisingly outperform, their tax-inefficient counterparts.[16] Later in the article, we perform robustness checks where pre-tax return declines with tax efficiency.

Exhibit 2 displays the assumptions for the base case on the level of pre-tax and after-tax investment returns ranging from the least to the most tax-efficient. The pre-tax return is assumed to have a mean of 6.8% and a volatility of 10.0%. Tax inefficiency is modeled by varying the fraction of highly taxed income and gains from 70.6% for the least tax-efficient investment to 0% for the most tax-efficient investment. Because we hold the other tax characters constant, the unrealized gains increase from the least to the most tax-efficient investment.

We assume that the tax rates correspond to the federal tax rates at the time of this writing in 2020—23.8% for low-taxed gains and income and 40.8% for highly taxed gains and income. However, as the fraction of highly taxed income and gains decreases, the character-weighted tax rate declines from 33.0% to 4.2%. On the other hand, because of the rise in unrealized gains, the effective liquidation tax, defined in Equation 9, increases with tax efficiency from 0% to 17.5%.

Using these tax assumptions, we can compute after-tax returns, which increase sharply with tax efficiency, as shown in Exhibit 2. The pre-liquidation after-tax return corresponds to the return defined in Equation 1 and, as Equation 3 indicates, directly enters model-based calculations and by extension our simulation results.

[16] This result is unrelated to the relative performance of active and passive funds because Sialm and Zhang (2020) specifically screen their sample to include only actively managed funds.

The post-liquidation result is more complex as it depends on the relative fractions of the assets passing through the trust and the estate and constitutes the core of our results, detailed later in the article. In Exhibit 2, we compute the simplest version of post-liquidation return based on Equation 8 and a 40-year investment horizon.[17] As seen in Exhibit 2, although the post-liquidation after-tax return increases by less than the pre-liquidation after-tax return with tax efficiency, the effect of the liquidation tax on post-liquidation returns is quite small. This is true even for the most tax-efficient investment, which defers 70.6% of its pre-tax return every year.

Additional Parameters Used in Calculations

For the model-based calculations, we need to define the investment horizon n (Equation 5), asset retention rate λ (Equation 6), and probability of death q (Equation 10). We assume an investment horizon of 40 years. For the retention rate, we assume the values of 1.0000, 0.9703, 0.9606, and 0.9441, alternatively. These values correspond, respectively, to 0%, 70%, 80%, and 90% of the assets transferred out of the personal account into the trust at the end of the 40-year horizon. The 0% transfer is the scenario for no estate planning; 70%, 80%, and 90% over 40 years approximate the level of transfer achieved in our simulations with eight consecutive 5-year GRATs, 20 consecutive 2-year GRATs, and rolling 2-year GRATs, respectively.

We must also define the probability of death. In the simulations, we use the probability of death from the Society of Actuaries (2013), which increases with age, and we assume that the wealth creator starts the process of investing and estate tax planning at the age of 40 and continues it for 40 years until the age of 80. Over this age range, the probability of death increases from 0.10% to 3.69%, with an average of 0.95%. Based on these data, in the model, we use a constant probability of death equal to 0.95%.

For the purpose of modeling GRATs in the simulations, we need to define the Internal Revenue Code Section 7520 rate, which determines the GRAT annuity payments (Appendix C includes details). In the base case, we set this rate at 2.4%. This value approximately corresponds to the average 2.2% Section 7520 rate over the past decade (from 2010 to 2019). Later in the article, we perform a robustness check, tripling the annuity rate to 7.2%.

The final parameter needing a definition is the initial investment V_0. We set this value at $100.

MAIN RESULT: STRIKINGLY LARGE BENEFITS OF INTEGRATED TAX PLANNING

Exhibit 3 summarizes the results of expected family wealth calculations under the base case assumptions. We begin by substituting these assumptions into Equation 10. The results of the model-based calculations are noted in Exhibit 3, Panel A. Estate tax efficiency increases across the rows, from top to bottom, and income tax efficiency rises across the columns, from left to right. The effect of taxes on family wealth is markedly strong. At the 40-year investment horizon, on the initial $100 investment, the least tax-efficient scenario from both income and estate tax perspectives, in the left top corner of the table, leaves the family with $357 after accounting for all of

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[17] We compute the post-liquidation after tax return as $((1+r^*)^n(1-T^*)+T^*)^{\frac{1}{n}}-1$ where the investment horizon n is 40 years.

EXHIBIT 3

Expected Post-Liquidation After-Tax Family Wealth at the End of the 40-Year Investment Horizon under Base Case Return Assumptions

Estate Tax Planning	Income Tax Scenario					Most vs Least Tax-Efficient
	Least Tax-Efficient	2	3	4	Most Tax-Efficient	
Panel A: Model Calculations						
No Transfer (λ = 1.0000)	357	411	475	551	639	283
70% Transfer (λ = 0.9703)	501	575	663	766	887	386
80% Transfer (λ = 0.9606)	523	601	692	799	925	402
90% Transfer (λ = 0.9441)	547	628	723	835	967	419
90% Transfer vs. No Transfer	191	217	248	284	327	137
Panel B: Average of 20,000 Simulated Histories						
No Planning	359	415	482	561	654	295
Consecutive 5-Year GRATs	559	639	730	834	953	393
Consecutive 2-Year GRATs	580	665	762	876	1,007	427
Rolling 2-Year GRATs	586	673	774	891	1,027	441
Rolling GRATs vs. No Planning	227	258	292	330	373	146

the taxes. At the same time, the most tax-efficient scenario from both income and estate tax perspectives almost triples after-tax family wealth to $967.

However, an even more important result lies in the strong interaction between income and estate tax efficiency. This interaction effect appears in the last row and column of the table. For the least income-tax–efficient scenario, the difference in family after-tax wealth between the most efficient estate tax planning and no estate tax planning is $191; for the most income-tax–efficient scenario, this difference is $327. Along the other dimension, if no estate tax planning is performed, the difference between the most and the least income-tax–efficient scenarios is $283, whereas for the most estate-tax–efficient scenario, this difference is $419.

Although the formula in Equation 10 allows us to easily visualize the determinants of after-tax wealth appreciation, we had to make some heroic assumptions about the rate of gift-tax–free transfer of assets from the personal account to the estate-tax–exempt trust. To make our example realistic, in simulations, we implement a widely used estate tax planning technique—GRATs. Other observers note that shortening the term of the GRATs and rolling the GRATs increase the success of a transfer (e.g., Weinreb and Singer 2008). We use consecutive 5-year GRATs, consecutive 2-year GRATs, and rolling 2-year GRATs to model the transfer. We find that the average transfer at the end of the 40-year investment horizon is approximately 70%, 80%, and 90% for the 5-year, 2-year, and rolling GRATs, respectively. These levels of transfers are modeled using the retention parameter λ in Exhibit 3, Panel A.

The results of the simulations where GRATs facilitate the transfer from the personal account to the estate-tax–exempt trust are listed in Exhibit 3, Panel B. When we use this actual estate tax planning technique, we obtain results similar to those in Panel A. First, the least income-tax–efficient investment without estate tax planning results in $359 of post-liquidation after-tax wealth, whereas the most income-tax–efficient investment with rolling GRATs yields a wealth of $1,027—an almost three-fold difference in the after-tax wealth of the family over the 40-year investment horizon.

EXHIBIT 4

Percentiles of Post-Liquidation After-Tax Family Wealth at the End of the 40-Year Investment Horizon under Base Case Return Parameters

	Income Tax Scenario					
Estate Tax Planning	**Least Tax-Efficient**	**2**	**3**	**4**	**Most Tax-Efficient**	**Most vs Least Tax-Efficient**
Panel A: 10th Percentile of 20,000 Simulated Histories						
No Planning	195	212	229	249	270	74
Consecutive 5-Year GRATs	266	285	306	329	353	86
Consecutive 2-Year GRATs	295	315	338	362	388	93
Rolling 2-Year GRATs	309	332	355	381	408	100
Rolling GRATs vs. No Planning	114	120	126	132	139	25
Panel B: 90th Percentile of 20,000 Simulated Histories						
No Planning	554	663	796	955	1,146	592
Consecutive 5-Year GRATs	904	1,069	1,262	1,490	1,740	836
Consecutive 2-Year GRATs	911	1,080	1,280	1,515	1,796	885
Rolling 2-Year GRATs	914	1,083	1,285	1,527	1,808	894
Rolling GRATs vs. No Planning	360	420	489	571	663	302

Second, similar to what we observed in Panel A with model-based calculations, income tax efficiency contributes more to after-tax wealth when estate tax planning efficiency increases from no planning to rolling GRATs, and estate tax planning efficiency contributes more when income tax efficiency increases. For example, without estate planning, the increase in wealth attributable to tax efficiency is $295, whereas for rolling GRATs, the increase in wealth derived from tax efficiency is $441. Along the dimension of estate planning, for the least income-tax–efficient strategy, moving from no estate tax planning to rolling GRATs yields a $227 increase in wealth, whereas for the most income-tax–efficient strategy, this improvement is $373.

The simulations also enable us to compute the percentiles of the family wealth distribution, which are listed in Exhibit 4. Panel A shows the 10th percentile of wealth and Panel B the 90th percentile of wealth. The pattern observed in Exhibit 3 for the model (Panel A) and the mean of the simulations (Panel B) is also evident for the tails of the distribution. Both income and estate tax efficiency contribute to the tails of the wealth distribution, and the more tax efficiency increases along one dimension, the more tax efficiency along the other dimension contributes to the family wealth.

ROBUSTNESS TESTS

We test the robustness of our results to the level of pre-tax return and the GRAT annuity rate. For the sake of brevity, we summarize below the key findings of the tests, relegating the numerical exhibits to Appendix D.

What If Tax Efficiency Reduces the Pre-Tax Return?

In the previous section, we assume that the tax efficiency of a strategy does not reduce its pre-tax return. As we already discussed, this assumption is consistent with the findings in Sialm and Zhang (2020) and also is supported by strategy simulations in recent articles by Israel and Moskowitz (2012); Sialm and Sosner (2018); Goldberg, Hand, and Cai (2019); and Israel et al. (2019)—all demonstrating little to no degradation in strategy returns resulting from tax awareness.[18] In this section, we discuss two scenarios where pre-tax returns decline as a result of tax awareness—a mild and sharp return degradation.

In the first scenario, we assume a mild degradation in pre-tax return. The pre-tax return for the most tax-efficient investment is 60 basis points (bps) lower than that for the least tax-efficient investment. Compared to our base case assumptions in Exhibit 2, where the post-liquidation after-tax return of the most tax-efficient investment was 1.4% higher than that of the least tax-efficient investment, this difference in favor of the most tax-efficient investment now declines to 0.9%.

We find that in this scenario, the results remain qualitatively similar to our base case results (as detailed in Appendix D). A sum of $100 invested for a 40-year horizon in the least income-tax–efficient option and without any estate tax planning results in $359 of post-liquidation after-tax family wealth, whereas the most income-tax–efficient investment with rolling GRATs yields more than double that amount of wealth, or $824. Also, the interaction between income and estate tax efficiency (which we observe previously) remains, with an increase in one type of tax efficiency (income or estate) making the other type of tax efficiency more valuable in terms of after-tax wealth at the end of the investment horizon.

In the second scenario of sharp pre-tax return degradation, we perform a thought experiment where tax efficiency is so punitive for the pre-tax return that the family is better advised to stick with an income-tax–inefficient investment. The pre-tax return declines sharply as income tax efficiency increases so that the most income-tax–efficient investment has a pre-tax return a full 1.8% lower than the least income-tax–efficient investment. In other words, in this scenario, a quarter of the pre-tax return is lost due to pursuit of tax efficiency. Despite this large decrease in pre-tax return, based on the post-liquidation after-tax return, the most income-tax–efficient investment underperforms the least income-tax–efficient investment only by 0.2%.

In this more extreme return degradation scenario, our findings (detailed in Appendix D) indicate that the eventual after-tax wealth still increases in the most income-tax–efficient and estate-tax–efficient scenario relative to the least income-tax–efficient and estate-tax–efficient scenario—to $529 for the former from $359 for the latter—however, all of this increase is attributable to estate planning, which is intuitive.

To conclude, we show that although income tax efficiency can be an important contributor to the growth in after-tax family wealth, investors should be mindful of the post-liquidation after-tax returns of the potential investment choices. If income tax efficiency leads to such a severe degradation in pre-tax return that the post-liquidation return of the income-tax–efficient investment dips below that of the income-tax–inefficient alternative, an income-tax–efficient investment might lead to inferior wealth outcomes. We point out that empirically this does not seem to be the case, so most investors would find tax-efficient investment options more attractive, even if they might lead to a moderate degradation of pre-tax return.

Impact of GRAT Annuity Rate

We also test the robustness of our results with respect to the GRAT annuity rate. Conventional wisdom claims that the effectiveness of estate tax planning with GRATs is reduced when the rate used for calculating GRAT annuity payments, or the Section 7520 rate, increases. With interest rates in the past decade at historic lows, this begs

[18] This point is further refined in Israel and Moskowitz (2012) and Israel et al. (2019), who show that capital gains awareness is not punitive for pre-tax returns, but dividend aversion can significantly alter portfolio composition and adversely impact expected pre-tax performance.

a question: Would our conclusions still hold if the rates were to increase? To answer this question, we triple the Section 7520 rate from 2.4% to 7.2%. To be conservative, the return parameters remain at their base case level (shown in Exhibit 2), despite the large increase in interest rates.[19]

Our findings (as specified in Appendix D) suggest that although the effectiveness of estate tax planning for growing family wealth is somewhat reduced compared to the 2.4% Section 7520 rate scenario, qualitatively the results remain similar: Income and estate tax efficiency results in a significant increase in family wealth at the end of the 40-year investment horizon. Compared to a wealth of $359 for the scenario that is both least income-tax–efficient and least estate-tax–efficient, the scenario that is most income-tax–efficient and most estate-tax–efficient generates a wealth of $948. As a reminder, under the 2.4% Section 7520 rate assumption, the latter value is $1,027, so despite the tripling of the rate, the final wealth sits only 8% lower than in the base case.

The resilience of GRAT benefits to the level of the Section 7520 rate can be attributed to the "heads you win, tails you break even" feature described in Gortz et al. (2016).[20] We also conclude (based on the details in Appendix D) that rolling GRATs utilizing this feature more effectively over the course of the 40-year investment horizon are particularly advantageous to the family when the Section 7520 rate is high.

To summarize, based on conventional wisdom, the effectiveness of GRATs as an estate tax planning technique varies inversely with the level of the Section 7520 rate. We confirm this conventional conclusion, but with an important twist to this plot: The more efficient the GRAT planning becomes by shortening the term and rolling, the less impact the Section 7520 rate exerts on the effectiveness of estate tax planning with GRATs.[21]

Before concluding this section, we should clarify that the purpose of this study is neither to advocate for or against the use of GRATs in estate tax planning nor to study the optimal ways of employing GRAT strategies, but rather to demonstrate quantitatively the critical importance of systematic and methodical estate tax planning for the growth of family wealth, especially when it is combined with income tax efficiency.

CONCLUSION

The wealth management literature has generally considered techniques for income and estate tax planning in isolation. The literature on goal-based wealth management, which includes intergenerational transfers as one of the goals, either shied away from taxation altogether or offered a limited discussion of the importance of income and estate tax efficiency. To our knowledge, this study is the first one to model the integration of income and estate tax planning and demonstrate its value numerically.

We begin by showing the deleterious effects of income and estate taxes on a family's wealth. We argue that without a thoughtful management of income and estate tax burdens, under reasonable return expectations, it is unlikely that the family could achieve its wealth growth objectives after taxes and inflation. We then develop a stylized theoretical model and a simulation environment that allow us to analyze the effects of increasing income and estate tax efficiency on the family's ability to achieve and exceed its financial goals.

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[19] One could argue that in environments with high interest rates, risky asset returns should on average be higher.

[20] Appendix C includes more details.

[21] While outside the scope of this study, our conjecture is that the dependence on the level of interest rates can be further reduced by seeding several parallel GRATs with low correlated investments.

Our model and simulations produce a number of conclusions. First, over the term of one generation, a family that invests with income and estate tax efficiency in mind can achieve significantly higher wealth levels than a family oblivious to taxes—and when we say, "significantly higher," we are talking about two to three times greater than expected wealth levels for a tax-conscious family. Second, the benefits of income tax planning increase (decrease) as the estate tax efficiency increases (decreases).

We offer a word of caution that in pursuit of tax efficiency, investors should not disregard expected pre-tax returns. The prime determinant in the rate of wealth appreciation is post-liquidation after-tax return—pre-tax return in excess of all tax costs.

Third, we test the robustness of estate planning techniques used in our simulation analysis with respect to an important input parameter—the annuity rate. Our conclusions on the importance of integrating income and estate tax planning survive this test.

APPENDIX A

DERIVATION OF A STYLIZED MODEL OF WEALTH TRANSFER

A family's wealth creator seeks to transfer to future generations assets with an initial fair market value of V_0. The assets are expected to generate an annual pre-tax return r. The assets are also expected to realize tax-exempt income x, low-taxed long-term capital gains and qualified dividend income g, and highly taxed short-term capital gains and ordinary income i where all the quantities are fractions of the value of the assets. Using these quantities and tax rates applicable to low-taxed and highly taxed items, denoted by t_G and t_I, respectively, we can define the pre-liquidation after-tax return of the assets as

$$r^* = r - gt_G - it_I \tag{A1}$$

and the rate of accumulation of unrealized gains, or the incremental one-period unrealized gain, as

$$u = r - x - g - i \tag{A2}$$

These assets are held for n years, at which point they are liquidated irrespective of whether they are held by the wealth creator in the personal account, by the descendants of the wealth creator in their personal account, or by the trust set up for the benefit of the descendants.

We define two additional parameters: the rate of asset retention in the personal account λ and the probability of death of the wealth creator in any given year q. These parameters allow us to define the allocation of assets between the personal account and the trust in year i as λ^i and $(1 - \lambda^i)$, respectively, and the probability of death in year i conditional on survival until year i as $(1 - q)^i q$.

Now, suppose that the wealth creator dies in year i. The value of assets at time of death is given by

$$V_i = V_0(1 + r^*)^i \tag{A3}$$

where r^* is defined in Equation A1. If the assets are held in the personal account and pass through the estate, their value is reduced by the estate tax to $(1 - t_E)V_i$.

We define the value of assets at the end of the investment horizon lasting n years as

$$V_n = V_0(1 + r^*)^n \tag{A4}$$

Note that Equation A4 can be rewritten in two ways as

$$V_n = V_i(1+r^*)^{n-i} = V_i + V_i\sum_{h=0}^{n-i-1}(1+r^*)^h(r - gt_G - it_I) \qquad \text{(A5)}$$

and

$$V_n = V_0(1+r^*)^n = V_0 + V_0\sum_{h=0}^{n-1}(1+r^*)^h(r - gt_G - it_I) \qquad \text{(A6)}$$

Using Equations A5 and A6, we can define the pre-liquidation value of assets that pass through the estate and the assets that were in the trust at the time of death, respectively, as

$$V_n^{i,PA} = (1-t_E)V_i + (1-t_E)V_i\sum_{h=0}^{n-i-1}(1+r^*)^h(r - gt_G - it_I) \qquad \text{(A7)}$$

and

$$V_n^{i,TR} = V_0 + V_0\sum_{h=0}^{n-1}(1+r^*)^h(r - gt_G - it_I) \qquad \text{(A8)}$$

The cost basis of the descendants in the assets held in their personal account and of the trust in the assets held by the trust are, respectively,

$$B_n^{i,PA} = (1-t_E)V_i + (1-t_E)V_i\sum_{h=0}^{n-i-1}(1+r^*)^h(x + g + i - gt_G - it_I)\,. \qquad \text{(A9)}$$

and

$$B_n^{i,TR} = V_0 + V_0\sum_{h=0}^{n-1}(1+r^*)^h(g + i - gt_G - it_I) \qquad \text{(A10)}$$

Subtracting Equation A9 from A7 and Equation A10 from A8, and observing that

$$(r - gt_G - it_I) - (x + g + i - gt_G - it_I) = r - x - g - i = u$$

with the latter equality defined in Equation A2, yields liquidation gains at time n of

$$U_n^{i,PA} \equiv V_n^{i,PA} - B_n^{i,PA} = (1-t_E)V_i\sum_{h=0}^{n-i-1}(1+r^*)^h u \qquad \text{(A11)}$$

and

$$U_n^{i,TR} \equiv V_n^{i,TR} - B_n^{i,TR} = V_0\sum_{h=0}^{n-1}(1+r^*)^h u \qquad \text{(A12)}$$

Equations A11 and A12 can be further simplified by observing that

$$\sum_{h=0}^{n-i-1}(1+r^*)^h u = \frac{u}{r^*}((1+r^*)^{n-i} - 1)$$

and

$$\sum_{h=0}^{n-1}(1+r^*)^h u = \frac{u}{r^*}((1+r^*)^n - 1)$$

which expresses the unrealized gains as

$$U_n^{i,PA} = (1-t_E)\frac{u}{r^*}(V_n - V_i) \qquad \text{(A13)}$$

and

$$U_n^{i,TR} = \frac{u}{r^*}(V_n - V_0) \qquad \text{(A14)}$$

Equations A11 and A12 show that, if the after-tax rate of return r^* is positive, and thus $V_i \geq V_0$, the liquidation gain on the assets held in the personal account, is reduced by the step-up in the cost basis when assets pass through the estate and then is further reduced by liquidating a fraction t_E of the assets to satisfy the payment of estate tax.

Following Horan (2002), we define the effective liquidation tax rate as

$$T^* \equiv t_G \frac{u}{r^*} \tag{A15}$$

Then, using the definition in Equation A15, the pre-liquidation values derived in Equations A7 and A8, and the liquidation gains in Equations A13 and A14, we can define the post-liquidation values as

$$W_n^{i,PA} = V_n^{i,PA} - t_G U_n^{i,PA} = (1 - t_E)(V_n - T^*(V_n - V_i)) \tag{A16}$$

and

$$W_n^{i,TR} = V_n^{i,TR} - t_G U_n^{i,TR} = V_n - T^*(V_n - V_0) \tag{A17}$$

Equations A16 and A17 can be slightly rearranged as

$$W_n^{i,PA} = (1 - t_E)(V_n(1 - T^*) + V_i T^*). \tag{A18}$$

and

$$W_n^{i,TR} = V_n(1 - T^*) + V_0 T^* \tag{A19}$$

The total post-liquidation result is a combination of assets passing through the estate and through the trust, with their respective weights determined at the time of death i by

$$W_n^i = \lambda^i W_n^{i,PA} + (1 - \lambda^i) W_n^{i,TR} \tag{A20}$$

The probability of outcome W_n^i is $(1 - q)^i q$. In addition, with probability $(1 - q)^n$, the investor lives past liquidation date n; in that case, the post-liquidation value of the portfolio is expressed as

$$W_n^\infty = (\lambda^n(1 - t_E) + (1 - \lambda^n))(V_n(1 - T^*) + V_0 T^*) \tag{A21}$$

because the portfolio is liquidated before the step-up in basis for the assets held in the personal account. We use the superscript ∞ to indicate that death occurs at an unknown time after liquidation of the assets. Equation A21 can be simplified to

$$W_n^\infty = (1 - \lambda^n t_E)(V_n(1 - T^*) + V_0 T^*) \tag{A22}$$

The expected post-liquidation after-tax wealth is thus

$$E(W_n) = \sum_{h=1}^{n} (1 - q)^{h-1} q W_n^h + (1 - q)^n W_n^\infty \tag{A23}$$

APPENDIX B

RELATIVE VALUE OF ASSETS IN THE PERSONAL ACCOUNT AND THE TRUST

We show that under plausible assumptions, the value of assets in the trust is greater than the value of assets passing through the estate, despite the step-up in the cost basis available for the assets passing through the estate. In other words, we show that

$$W_n^{i,TR} > W_n^{i,PA} \tag{B1}$$

Assumptions

1. Annual gains and income are all greater or equal zero: $x \geq 0, g \geq 0, i \geq 0$.
2. Annual unrealized gain is greater or equal zero: $u \equiv r - x - g - i \geq 0$.
3. Annual after-tax return is positive: $r^* \equiv r - gt_G - it_I > 0$.

First consider the ratio $\frac{u}{r^*}$. Using the definitions in Equations A1 and A2, we note

$$\frac{u}{r^*} = \frac{r - x - g - i}{r - gt_G - it_I} \tag{B2}$$

Because tax rates are less than 100%, that is, $t_G < 1$ and $t_I < 1$, it follows from Assumption 1 that

$$x + g + i \geq gt_G + it_I \tag{B3}$$

which, combined with Assumption 2, in turn implies that

$$r - gt_G - it_I \geq r - x - g - i \geq 0 \tag{B4}$$

Combining Equations B4 and B2, we obtain

$$1 \geq \frac{u}{r^*} \geq 0 \tag{B5}$$

Since, for an HNW family typically $t_E > t_G$, Equation B5 implies that

$$t_E > t_G \frac{u}{r^*} \geq 0 \tag{B6}$$

which, using the definition $T^* \equiv t_G \frac{u}{r^*}$, in turn implies that

$$t_E > T^* \tag{B7}$$

By subtracting $t_E T^*$ from each side of Equation B7 and rearranging we obtain

$$t_E(1 - T^*) > T^*(1 - t_E) \tag{B8}$$

Second, under Assumption 3, $r^* > 0$, therefore, for $i \leq n$ we find

$$V_n \geq V_i. \tag{B9}$$

Thus, combining Equations B8 and B9, we obtain

$$V_n t_E(1 - T^*) > V_i T^*(1 - t_E) \tag{B10}$$

We subtract $V_0 T^*$ from the right-hand side of the inequality in Equation B10 to get

$$V_n t_E(1 - T^*) > V_i T^*(1 - t_E) - V_0 T^* \tag{B11}$$

Note that the sign of the inequality is preserved from Equations B10 to B11 because we subtract a non-negative amount $V_0 T^*$ from the right-hand side, which is already smaller than the left-hand side, as shown in Equation B10. ($V_0 T^*$ is non-negative because the initial capital V_0 is positive and, under our assumptions, the effective liquidation tax rate T^* is non-negative, as we show in Equation B6.) We then add $V_n(1 - T^*)$ to both sides of Equation B11 and rearrange to obtain

$$V_n(1-T^*)+V_0T^* > (1-t_E)(V_n(1-T^*)+V_iT^*) \quad \text{(B12)}$$

Now, if the wealth creator dies before liquidation of the assets, i.e., $i \leq n$, from Equations A19 and A18 in Appendix A, respectively, the result is

$$W_n^{i,TR} = V_n(1-T^*)+V_0T^*$$

and

$$W_n^{i,PA} = (1-t_E)(V_n(1-T^*)+V_iT^*).$$

Therefore, Equation B12 means that

$$W_n^{i,TR} > W_n^{i,PA}$$

and thus, the inequality in Equation B1 holds for $i \leq n$.

Finally, if the wealth creator dies after the assets are liquidated, i.e., $i > n$, the inequality holds trivially because the post-liquidation value of the investment is $V_n(1-T^*)+V_0T^*$ whether the assets are held in a personal account or in a trust, but the assets in the personal account are eventually reduced by the estate tax.

APPENDIX C

GRANTOR-RETAINED ANNUITY TRUSTS (GRATS)

GRAT Basics[22]

A GRAT is an irrevocable trust. A grantor contributes property to a GRAT and retains a stream of annuity payments from the GRAT for a fixed number of years. The annuity payments are determined using the Internal Revenue Code Section 7520 rate, which equals the IRS-stipulated federal midterm rate multiplied by 1.2 and rounded to the nearest 20 bps. To satisfy annuity payments, the GRAT can use income from the property or transfers of the property back to the grantor.[23]

Additional property cannot be contributed to a GRAT and, other than through annuity payments to the grantor, cannot be distributed from the GRAT. Property, and income generated by the property, that remain in the GRAT after the last annuity payment pass estate-tax–free to the beneficiary of the GRAT, which in our case is a remainder trust, discussed further below in this appendix. If the value of the property in the GRAT is insufficient to make annuity payments, all of the property is returned to the grantor, so the GRAT fails to make an estate-tax–free transfer to its beneficiary.

The GRAT can fail to achieve estate-tax–free transfer in two ways. First, the assets in the GRAT appreciate at a rate lower than the Section 7520 rate. In this case, the GRAT simply returns all the assets to the grantor as if the GRAT never existed, with the only cost being legal fees to set up the GRAT. Hence, estate tax planning with GRATs is described as "heads you win, tails you break even" in Gortz et al. (2016). Second, if the grantor dies before the maturity of the GRAT, all the GRAT assets join the estate of the grantor, again as if the GRAT never existed.

As a grantor-retained trust, a GRAT is not subject to income tax—the grantor pays all of the income taxes on gains and income realized by the property held by the GRAT. Simply put, from an income tax perspective, the GRAT does not exist.

[22] See Gortz et al. (2016) for further details.

[23] These annuity payments can be set up as equal payments, as in our examples, but can alternatively be structured to increase annually by not more than 20% per year.

EXHIBIT C1

Example of Rolling Zeroed-Out GRATs

	June 1, 2020	June 1, 2021	June 1, 2022
GRAT-2020			
Contribution	$100.00		
FMV Before Distribution		$110.00	$63.51
Distribution		$52.26	$52.26
FMV After Distribution		$57.74	$11.25
Remainder			$11.25
GRAT-2021			
Contribution		$52.26	
FMV Before Distribution			$57.49
Distribution			$27.31
FMV After Distribution			$30.17
GRAT-2022			
Contribution			$79.57

NOTE: FMV stands for fair market value.

A "Zeroed-Out" GRAT

A contribution to a GRAT in excess of the present value of the annuity, which is calculated as the sum of annuity payments discounted using the Section 7520 rate, is considered a taxable gift. To avoid such a taxable gift, a grantor can contribute to a GRAT a property with fair market value exactly equal to the present value of the GRAT annuity. A 2000 court decision in Walton v. Commissioner upheld such a strategy, referred to colloquially as a "zeroed-out" GRAT.[24]

Consecutive GRATs

In our consecutive GRATs methodology, annuity payments made by a GRAT are held by the grantor in the personal account until the GRAT's maturity and are only contributed to a new GRAT after the previous GRAT matures. For example, a 5-year GRAT makes five annuity payments to the grantor, who holds on to the payments until year 5 (when the last annuity payment is made) and contributes the total sum of all previous payments grown with the appropriate rate of return to a new 5-year GRAT.[25]

"Rolling" GRATs

A "rolling" GRAT is a series of separate GRATs where new GRATs are seeded every year by using annuity payments from the old GRATs. In a rolling GRAT, rather than waiting to collect all the annuity payments from an old GRAT before contributing them to a new GRAT, the grantor contributes annuity payments to new GRATs immediately when received.

An Example of Rolling Zeroed-Out GRATs

The following example illustrates the operation of rolling zeroed-out GRATs. Exhibit C1 supports the illustration. Suppose that the grantor has an asset that appreciates by 10% every year and that the Section 7520 rate is constant at 3%. On June 1, 2020, a grantor creates a 2-year GRAT-2020 by contributing $100.00. The GRAT will make two annuity payments of $52.26 on June 1, 2021, and June 1, 2022. The present value of the annuity payments discounted by the 3% Section 7520 rate is $100.00, which makes GRAT-2020 a zeroed-out GRAT. GRAT-2020 will transfer the remainder, if there is any, to a remainder trust.

A year later, the value of assets in GRAT-2020 increases by 10% to $110.00; the GRAT distributes $52.26 worth of assets to the grantor; and assets with the value of $57.74 (110.00 minus 52.26) remain in the GRAT. The grantor immediately creates a new GRAT by contributing the distributed assets to GRAT-2021. GRAT-2021 is also a 2-year zeroed-out GRAT with two annuity payments of $27.31, with a present value adding up exactly to the value of contributed assets, or $52.26. Same as GRAT-2020, GRAT-2021 transfers the remainder, if there is any, to a remainder trust.

[24] 115 T.C. 589 (2000). Audrey J. Walton, Petitioner v. Commissioner of Internal Revenue, Respondent. United States Tax Court. Filed December 22, 2000.

[25] Assets in the personal account appreciate by the after-tax rate of return. As a result, if a GRAT makes an annuity payment of P dollars and if assets in the personal account appreciate at an after-tax rate of return r^*, the total contribution to a new GRAT at the end of the 5-year period is $P \times ((1 + r^*)^4 + (1 + r^*)^3 + (1 + r^*)^2 + (1 + r^*)^1 + 1)$.

Another year passes, and on June 1, 2022, GRAT-2020—with assets that now appreciate from $57.74 on June 1, 2021, to $63.51—makes the last payment of $52.26 to the grantor and transfers the remaining $11.25 to a remainder trust. Again, the grantor immediately creates a new GRAT by contributing the assets distributed by GRAT-2020 and GRAT-2021, $79.57 in total, to GRAT-2022. GRAT-2022 is a 2-year zeroed-out GRAT, and it also will transfer the remainder, if there is any, to a remainder trust.

The process will be repeated on June 1, 2023, when a 2-year zeroed-out GRAT-2023 will be created using the last annuity payment from GRAT-2021 and the first annuity payment from GRAT-2022, and so on each year thereafter.

A Remainder Trust

We assume that the remainder trust that receives GRAT residuals is an intentionally defective grantor trust (IDGT) created by the wealth creator for the benefit of the next generation of the family. Intentionally defective in this case means that the trust is not recognized for income tax purposes and, as a result, all the income taxes on the gains and income realized by the assets of the trust are paid by the grantor, in our case the wealth creator. Upon the death of the wealth creator, no estate taxes are levied on the assets already in the remainder trust.

APPENDIX D

ROBUSTNESS TESTS

Income Tax Efficiency Reduces the Pre-Tax Return

We consider two scenarios where pre-tax returns decline as a result of income tax efficiency. Exhibit D1 summarizes our return assumptions. In Panel A, we show a scenario of mild pre-tax return decline. The pre-tax return for the most income-tax–efficient investment is 60 bps lower than that for the least income-tax–efficient investment. Compared to our base case assumptions in Exhibit 2, both the pre- and post-liquidation after-tax returns of the most income-tax–efficient strategy are also reduced by approximately 60 bps. This result constitutes an approximately 10% reduction in after-tax returns relative to the base case level of the most income-tax–efficient investment returns in Exhibit 2.

In Panel B, the pre-tax return declines sharply as income tax efficiency increases. The most income-tax–efficient strategy has a pre-tax return a full 1.8% lower than the least tax-efficient strategy; in other words, a quarter of the pre-tax return is lost due to pursuit of tax efficiency. Despite this, the pre-liquidation after-tax return is still marginally higher for the most tax-efficient strategy than for the least tax-efficient strategy—4.7% for the former versus 4.6% for the latter. However, when the liquidation tax is accounted for, the most tax-efficient investment slightly underperforms the least tax-efficient investment—4.4% versus 4.6% (last row of Panel B, Exhibit D1).

In calculations not shown here for the sake of brevity, we find that under the new return assumptions, similar to the base case, the results of substituting the new return parameters in Equation 10 closely match the results of simulations. For this reason, here we only show the simulation evidence where we model actual estate planning techniques. Exhibit D2 replicates Exhibit 3, Panel B, using the new return assumptions shown in Exhibit D1. Panels A and B of Exhibit D2 show the small and large pre-tax return degradation scenarios, respectively.

Now, first review the scenario in Panel A of small pre-tax return degradation. The decrease in pre-tax return reduces the expected post-liquidation wealth of the family relative to what we saw in the base case example in Exhibit 3. However, there is still a substantial increase in the family's wealth attributable to tax efficiency. This is because after-tax returns—and more specifically, post-liquidation after-tax returns—still

EXHIBIT D1

Return Assumptions and Reduction in Pre-Tax Return because of Income Tax Efficiency

	Income Tax Scenario				
Estate Tax Planning	Least Tax-Efficient	2	3	4	Most Tax-Efficient
Panel A: Small Reduction in Pre-Tax Return because of Income Tax Efficiency					
Pre-Tax Return	6.8%	6.7%	6.5%	6.4%	6.2%
Pre-Tax Volatility	10.0%	10.0%	10.0%	10.0%	10.0%
Tax Characters (Fraction of Pre-Tax Return)					
Non-Taxable Income	11.8%	12.0%	12.3%	12.6%	12.9%
Low-Taxed Income and Gains	17.6%	18.0%	18.5%	18.9%	19.4%
Highly Taxed Income and Gains	*70.6%*	*54.1%*	*36.9%*	*18.9%*	*0.0%*
Unrealized Gain	0.0%	15.8%	32.3%	49.6%	67.7%
Low Tax Rate	23.8%	23.8%	23.8%	23.8%	23.8%
High Tax Rate	40.8%	40.8%	40.8%	40.8%	40.8%
Character-Weighted Tax Rate	33.0%	26.4%	19.5%	12.2%	4.6%
Effective Liquidation Tax Rate (T^*)	0.0%	5.1%	9.5%	13.4%	16.9%
Pre-Liquidation After-Tax Return (r^*)	4.6%	4.9%	5.2%	5.6%	5.9%
Post-Liquidation After-Tax Return	4.6%	4.8%	5.0%	5.2%	5.5%
Panel B: Large Reduction in Pre-Tax Return because of Income Tax Efficiency					
Pre-Tax Return	6.8%	6.4%	5.9%	5.5%	5.0%
Pre-Tax Volatility	10.0%	10.0%	10.0%	10.0%	10.0%
Tax Characters (Fraction of Pre-Tax Return)					
Non-Taxable Income	11.8%	12.6%	13.6%	14.7%	16.0%
Low-Taxed Income and Gains	17.6%	18.9%	20.3%	22.0%	24.0%
Highly Taxed Income and Gains	*70.6%*	*56.7%*	*40.7%*	*22.0%*	*0.0%*
Unrealized Gain	0.0%	11.8%	25.4%	41.3%	60.0%
Low Tax Rate	23.8%	23.8%	23.8%	23.8%	23.8%
High Tax Rate	40.8%	40.8%	40.8%	40.8%	40.8%
Character-Weighted Tax Rate	33.0%	27.6%	21.4%	14.2%	5.7%
Effective Liquidation Tax Rate (T^*)	0.0%	3.9%	7.7%	11.5%	15.1%
Pre-Liquidation After-Tax Return (r^*)	4.6%	4.6%	4.6%	4.7%	4.7%
Post-Liquidation After-Tax Return	4.6%	4.5%	4.5%	4.4%	4.4%

increase with income tax efficiency. The most estate-tax–efficient and income-tax–efficient scenario provides a more than two-fold increase in family wealth compared to the least estate-tax–efficient and income-tax–efficient scenario. Notably, the same interaction remains between the income and estate tax efficiency that we observed in the base case scenario.

Panel B shows that with large pre-tax return degradation, income tax efficiency detracts from family wealth. Nonetheless, efficient estate tax planning still benefits the family. As we move from no planning to consecutive GRATs and then to rolling GRATs, the family's wealth increases by close to $200 on average.

A Higher GRAT Annuity Rate

In the simulation results reported in Exhibit D3, we triple the Section 7520 rate from 2.4% in the base case to 7.2%. Panels A, B, and C of Exhibit D3 show the mean, 10th, and 90th percentiles of the simulations, respectively. Although the effectiveness of estate tax planning for growing family wealth is somewhat reduced compared to the 2.4% Section 7520 rate scenario in Exhibits 3 and 4, the results are still qualitatively similar: income

EXHIBIT D2

Expected Post-Liquidation After-Tax Wealth of the Family at the End of the 40-Year Investment Horizon, Average of 20,000 Simulated Histories

	Income Tax Scenario					
Estate Tax Planning	**Least Tax-Efficient**	**2**	**3**	**4**	**Most Tax-Efficient**	**Most vs Least Tax-Efficient**
Panel A: Small Reduction in Pre-Tax Return Attributable to Income Tax Efficiency						
No Planning	359	394	433	477	526	168
Consecutive 5-Year GRATs	559	604	651	701	754	194
Consecutive 2-Year GRATs	580	630	684	742	804	225
Rolling 2-Year GRATs	586	638	695	757	824	238
Rolling GRATs vs. No Planning	227	245	262	280	298	70
Panel B: Large Reduction in Pre-Tax Return Attributable to Income Tax Efficiency						
No Planning	359	355	350	346	342	–16
Consecutive 5-Year GRATs	559	540	518	494	470	–89
Consecutive 2-Year GRATs	580	566	550	531	510	–69
Rolling 2-Year GRATs	586	575	561	546	529	–57
Rolling GRATs vs. No Planning	227	220	211	200	187	–40

EXHIBIT D3

Post-Liquidation After-Tax Wealth of the Family at the End of the 40-Year Investment Horizon, High GRAT Annuity Rate

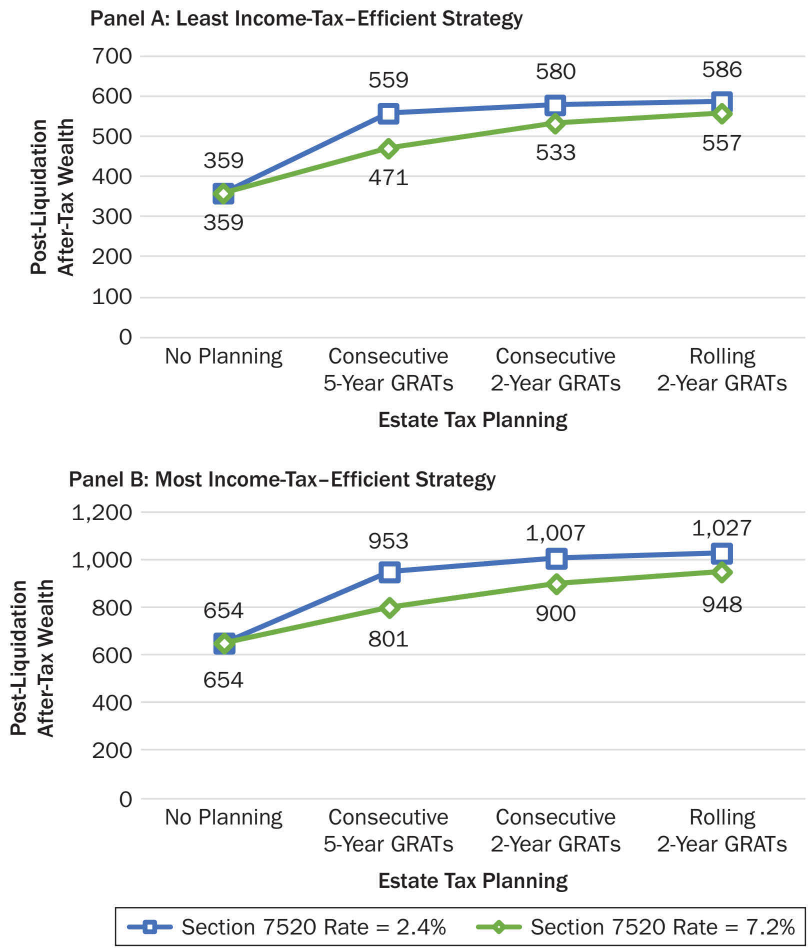
	Income Tax Scenario					
Estate Tax Planning	**Least Tax-Efficient**	**2**	**3**	**4**	**Most Tax-Efficient**	**Most vs Least Tax-Efficient**
Panel A: Average of 20,000 Simulated Histories						
No Planning	359	415	482	561	654	295
Consecutive 5-Year GRATs	471	535	610	698	801	330
Consecutive 2-Year GRATs	533	606	690	787	900	367
Rolling 2-Year GRATs	557	635	725	829	948	392
Rolling GRATs vs. No Planning	198	220	243	268	295	97
Panel B: 10th Percentile of 20,000 Simulated Histories						
No Planning	195	212	229	249	270	74
Consecutive 5-Year GRATs	217	234	253	274	296	79
Consecutive 2-Year GRATs	250	270	290	312	335	85
Rolling 2-Year GRATs	266	285	306	329	352	87
Rolling GRATs vs. No Planning	70	73	76	80	83	12
Panel C: 90th Percentile of 20,000 Simulated Histories						
No Planning	554	663	796	955	1,146	592
Consecutive 5-Year GRATs	801	926	1,073	1,251	1,463	662
Consecutive 2-Year GRATs	888	1,036	1,205	1,399	1,632	744
Rolling 2-Year GRATs	904	1,068	1,260	1,475	1,724	820
Rolling GRATs vs. No Planning	350	405	464	520	578	228

and estate tax efficiency results in a significant increase in family wealth at the end of the 40-year investment horizon.

In Exhibit D4, we demonstrate that rolling GRATs are particularly beneficial when the Section 7520 rate is high. Panels A and B of the exhibit show the least and most income-tax–efficient scenarios, respectively. As can be seen from the exhibit, more effective estate tax planning techniques, such as shorter-term GRATs and rolling GRATs, add value in all income tax efficiency scenarios, and the benefit of using more efficient estate tax planning techniques is greater when the Section 7520 rate is higher.

EXHIBIT D4

Comparison of Low and High GRAT Annuity Rate Results



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